

Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.1.3-g-tan^p-a+b-sin^m

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3.152	$\int \cot(c + dx)(a + b \sin(c + dx))^2 dx$	570
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3.157	$\int \cot^2(c + dx)(a + b \sin(c + dx))^2 dx$	590
3.158	$\int \cot^4(c + dx)(a + b \sin(c + dx))^2 dx$	593
3.159	$\int \cot^6(c + dx)(a + b \sin(c + dx))^2 dx$	597
3.160	$\int (a + b \sin(c + dx))^3 \tan^3(c + dx) dx$	602
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3.162	$\int \cot(c + dx)(a + b \sin(c + dx))^3 dx$	609
3.163	$\int \cot^3(c + dx)(a + b \sin(c + dx))^3 dx$	612
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3.165	$\int (a + b \sin(c + dx))^3 \tan^4(c + dx) dx$	618
3.166	$\int (a + b \sin(c + dx))^3 \tan^2(c + dx) dx$	622
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3.168	$\int \cot^4(c + dx)(a + b \sin(c + dx))^3 dx$	630
3.169	$\int \cot^6(c + dx)(a + b \sin(c + dx))^3 dx$	634
3.170	$\int \frac{\tan^5(c+dx)}{a+b \sin(c+dx)} dx$	639
3.171	$\int \frac{\tan^3(c+dx)}{a+b \sin(c+dx)} dx$	643
3.172	$\int \frac{\tan(c+dx)}{a+b \sin(c+dx)} dx$	646
3.173	$\int \frac{\cot(c+dx)}{a+b \sin(c+dx)} dx$	649
3.174	$\int \frac{\cot^3(c+dx)}{a+b \sin(c+dx)} dx$	652
3.175	$\int \frac{\cot^5(c+dx)}{a+b \sin(c+dx)} dx$	655
3.176	$\int \frac{\tan^4(c+dx)}{a+b \sin(c+dx)} dx$	658
3.177	$\int \frac{\tan^2(c+dx)}{a+b \sin(c+dx)} dx$	662
3.178	$\int \frac{\cot^2(c+dx)}{a+b \sin(c+dx)} dx$	666
3.179	$\int \frac{\cot^4(c+dx)}{a+b \sin(c+dx)} dx$	670
3.180	$\int \frac{\cot^6(c+dx)}{a+b \sin(c+dx)} dx$	674
3.181	$\int \frac{\tan^5(c+dx)}{(a+b \sin(c+dx))^2} dx$	679
3.182	$\int \frac{\tan^3(c+dx)}{(a+b \sin(c+dx))^2} dx$	683
3.183	$\int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^2} dx$	687
3.184	$\int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^2} dx$	690
3.185	$\int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^2} dx$	693
3.186	$\int \frac{\cot^5(c+dx)}{(a+b \sin(c+dx))^2} dx$	696
3.187	$\int \frac{\tan^4(c+dx)}{(a+b \sin(c+dx))^2} dx$	700
3.188	$\int \frac{\tan^2(c+dx)}{(a+b \sin(c+dx))^2} dx$	705
3.189	$\int \frac{\cot^2(c+dx)}{(a+b \sin(c+dx))^2} dx$	709
3.190	$\int \frac{\cot^4(c+dx)}{(a+b \sin(c+dx))^2} dx$	713
3.191	$\int \frac{\cot^6(c+dx)}{(a+b \sin(c+dx))^2} dx$	718
3.192	$\int \frac{\tan^5(c+dx)}{(a+b \sin(c+dx))^3} dx$	724
3.193	$\int \frac{\tan^3(c+dx)}{(a+b \sin(c+dx))^3} dx$	728

3.194	$\int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^3} dx$	732
3.195	$\int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^3} dx$	735
3.196	$\int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^3} dx$	738
3.197	$\int \frac{\cot^5(c+dx)}{(a+b \sin(c+dx))^3} dx$	741
3.198	$\int \frac{\tan^4(c+dx)}{(a+b \sin(c+dx))^3} dx$	745
3.199	$\int \frac{\tan^2(c+dx)}{(a+b \sin(c+dx))^3} dx$	750
3.200	$\int \frac{\cot^2(c+dx)}{(a+b \sin(c+dx))^3} dx$	755
3.201	$\int \frac{\cot^4(c+dx)}{(a+b \sin(c+dx))^3} dx$	760
3.202	$\int \frac{\cot^6(c+dx)}{(a+b \sin(c+dx))^3} dx$	766
3.203	$\int (a + b \sin(e + fx))^3 (g \tan(e + fx))^p dx$	772
3.204	$\int (a + b \sin(e + fx))^2 (g \tan(e + fx))^p dx$	777
3.205	$\int (a + b \sin(e + fx)) (g \tan(e + fx))^p dx$	781
3.206	$\int \frac{(g \tan(e+fx))^p}{a+b \sin(e+fx)} dx$	784
3.207	$\int \frac{(g \tan(e+fx))^p}{(a+b \sin(e+fx))^2} dx$	787
3.208	$\int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx$	790
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [208]. This is test number [72].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.04 (206)	% 0.96 (2)
Mathematica	% 97.6 (203)	% 2.4 (5)
Maple	% 85.58 (178)	% 14.42 (30)
Maxima	% 68.27 (142)	% 31.73 (66)
Fricas	% 85.58 (178)	% 14.42 (30)
Sympy	% 1.92 (4)	% 98.08 (204)
Giac	% 67.31 (140)	% 32.69 (68)

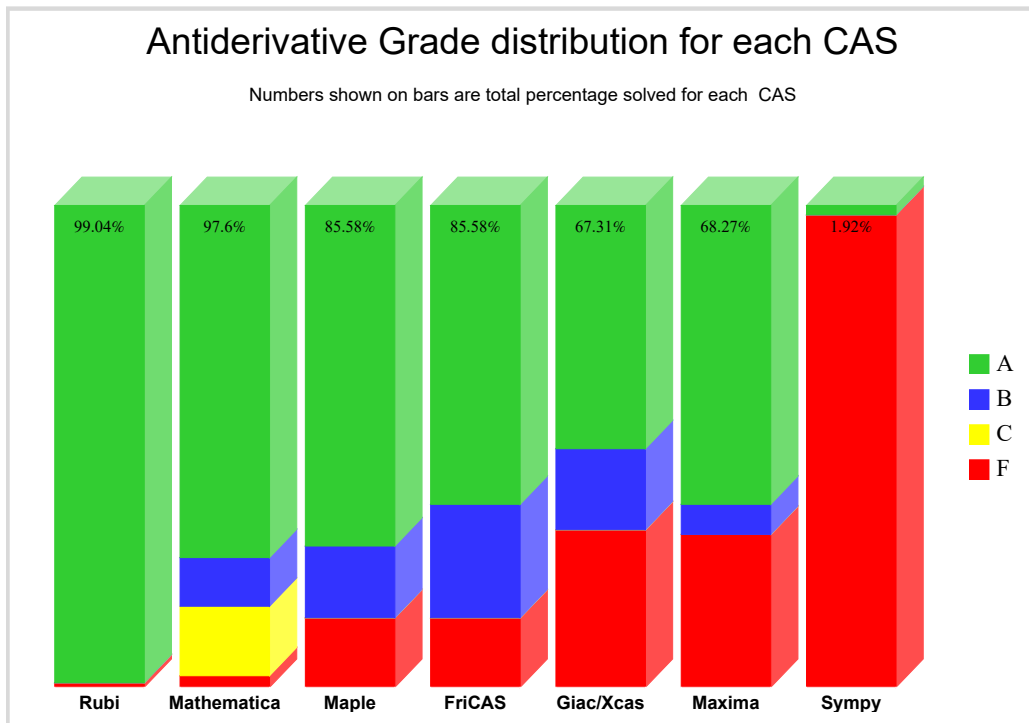
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

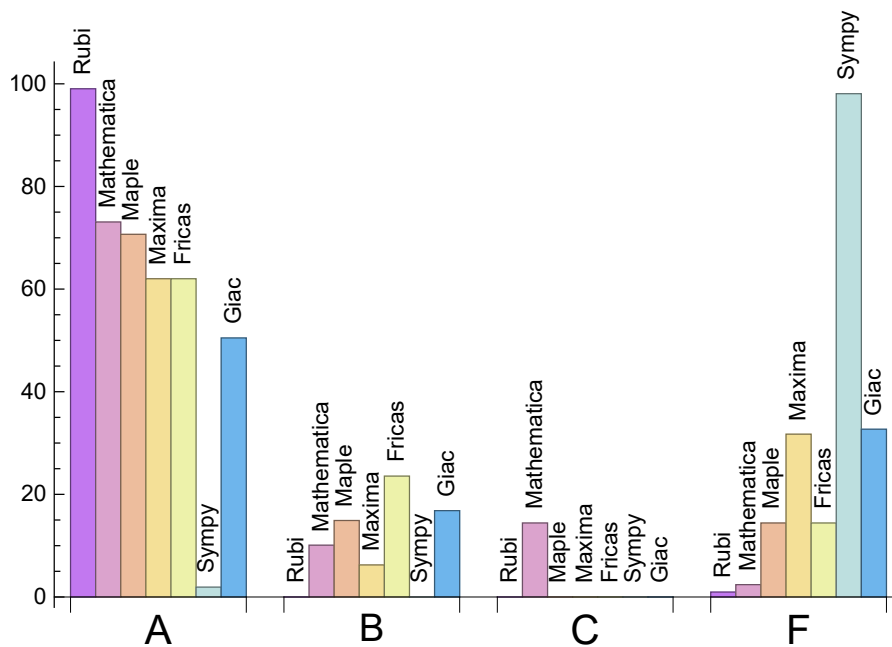
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.04	0.	0.	0.96
Mathematica	73.08	10.1	14.42	2.4
Maple	70.67	14.9	0.	14.42
Maxima	62.02	6.25	0.	31.73
Fricas	62.02	23.56	0.	14.42
Sympy	1.92	0.	0.	98.08
Giac	50.48	16.83	0.	32.69

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	133.35	1.01	113.	1.
Mathematica	2.93	548.75	4.84	113.	1.
Maple	0.15	183.64	1.39	136.	1.3
Maxima	1.63	169.07	1.56	128.	1.35
Fricas	1.83	643.02	4.51	411.	3.84
Sympy	1.29	112.5	1.85	99.5	1.83
Giac	3.02	586.31	7.02	198.5	2.

1.4 list of integrals that has no closed form antiderivative

{208}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {110, 114, 115, 117, 118, 123, 124, 126, 127, 128, 129, 136, 138, 158, 168, 179, 190, 201, 203, 204, 205, 206, 207}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

```

```

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1

```

For Sympy, called directly from Python, the following code is used

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

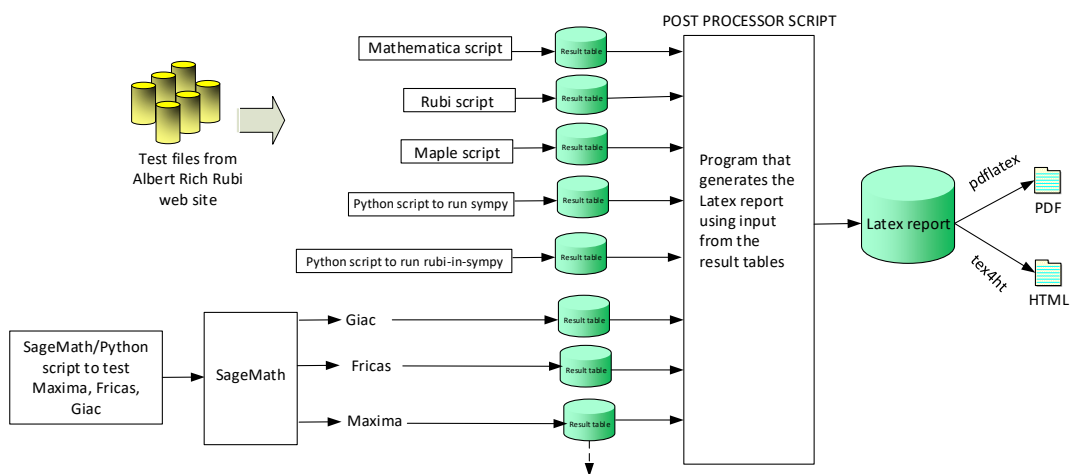
except Exception as ee:
    leafCount =1

```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208 }

B grade: { }

C grade: { }

F grade: { 206, 207 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 102, 120, 130, 131, 132, 133, 134, 137, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208 }

B grade: { 21, 55, 56, 57, 58, 59, 60, 88, 89, 90, 93, 105, 106, 110, 126, 127, 128, 129, 158, 179, 206 }

C grade: { 11, 12, 13, 91, 92, 95, 103, 104, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 136, 138, 147, 148, 149, 203, 204, 205 }

F grade: { 121, 122, 125, 135, 139 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 19, 20, 21, 22, 23, 27, 28, 31, 32, 33, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 140, 141, 142, 143, 144, 145,

146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 192, 193, 194, 195, 196, 197, 208 }

B grade: { 14, 15, 17, 18, 24, 25, 26, 29, 30, 34, 35, 38, 39, 53, 54, 58, 59, 60, 160, 164, 178, 179, 180, 189, 190, 191, 198, 199, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 96, 100, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197, 208 }

B grade: { 53, 54, 57, 58, 59, 60, 87, 88, 89, 90, 99, 181, 192 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 97, 98, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 176, 177, 178, 179, 180, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 91, 92, 95, 96, 99, 100, 103, 107, 111, 112, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 187, 188, 198, 199, 208 }

B grade: { 10, 11, 12, 13, 17, 24, 30, 57, 58, 59, 60, 74, 84, 88, 89, 90, 93, 94, 97, 98, 101, 102, 104, 105, 106, 108, 109, 110, 113, 114, 147, 148, 149, 181, 182, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.6 Sympy

A grade: { 22, 32, 40, 56 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 12, 13, 17, 18, 22, 28, 32, 33, 37, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 142, 143, 144, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 208 }

B grade: { 3, 10, 11, 16, 21, 23, 24, 42, 53, 57, 58, 59, 60, 93, 94, 97, 98, 101, 102, 103, 104, 105, 106, 109, 110, 114, 141, 146, 147, 151, 156, 167, 168, 181, 193 }

C grade: { }

F grade: { 1, 2, 8, 9, 14, 15, 19, 20, 25, 26, 27, 29, 30, 31, 34, 35, 36, 38, 39, 91, 92, 95, 96, 99, 100, 107, 108, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 145, 150, 155, 160, 161, 165, 166, 203, 204, 205, 206, 207 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	147	128	410	0	0
normalized size	1	1.	1.07	1.28	1.11	3.57	0.	0.
time (sec)	N/A	0.072	0.446	0.064	1.049	1.552	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	77	96	69	224	0	0
normalized size	1	1.	1.08	1.35	0.97	3.15	0.	0.
time (sec)	N/A	0.048	0.108	0.056	0.992	1.532	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	38	29	34	65	0	1966
normalized size	1	1.	1.27	0.97	1.13	2.17	0.	65.53
time (sec)	N/A	0.021	0.021	0.031	1.076	1.371	0.	1.611

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	25	30	62	0	31
normalized size	1	1.	1.08	1.04	1.25	2.58	0.	1.29
time (sec)	N/A	0.02	0.033	0.016	1.092	1.573	0.	1.268

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	61	167	0	81
normalized size	1	1.	1.11	1.54	1.13	3.09	0.	1.5
time (sec)	N/A	0.037	0.118	0.037	1.09	1.496	0.	1.325

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.048	0.198	0.036	1.081	1.573	0.	1.389

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	111	195	123	412	0	140
normalized size	1	1.	0.97	1.7	1.07	3.58	0.	1.22
time (sec)	N/A	0.059	0.399	0.039	1.153	1.599	0.	1.387

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	110	135	117	300	0	0
normalized size	1	1.	1.09	1.34	1.16	2.97	0.	0.
time (sec)	N/A	0.092	0.065	0.071	1.659	1.467	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	88	224	0	0
normalized size	1	1.	1.12	1.36	1.22	3.11	0.	0.
time (sec)	N/A	0.074	0.048	0.057	1.825	1.476	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	53	194	0	1361
normalized size	1	1.	1.21	1.51	1.36	4.97	0.	34.9
time (sec)	N/A	0.104	0.045	0.039	1.688	1.421	0.	3.472

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.052	0.048	0.029	2.303	1.438	0.	1.25

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.08	0.053	0.034	1.691	1.504	0.	1.278

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	169	620	0	269
normalized size	1	1.	1.34	1.3	1.39	5.08	0.	2.2
time (sec)	N/A	0.096	0.071	0.038	1.63	1.538	0.	1.293

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	261	130	412	0	0
normalized size	1	1.	0.63	2.19	1.09	3.46	0.	0.
time (sec)	N/A	0.082	0.245	0.085	1.129	1.557	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	162	78	212	0	0
normalized size	1	1.	0.75	2.25	1.08	2.94	0.	0.
time (sec)	N/A	0.062	0.103	0.071	1.112	1.482	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	40	69	58	108	0	9038
normalized size	1	1.	0.77	1.33	1.12	2.08	0.	173.81
time (sec)	N/A	0.038	0.038	0.042	1.125	1.418	0.	4.572

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	94	72	173	0	63
normalized size	1	1.	0.93	3.13	2.4	5.77	0.	2.1
time (sec)	N/A	0.039	0.041	0.049	1.113	1.44	0.	1.371

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	313	144	508	0	163
normalized size	1	1.	0.65	2.37	1.09	3.85	0.	1.23
time (sec)	N/A	0.075	0.216	0.049	1.086	1.768	0.	1.496

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	174	251	205	382	0	0
normalized size	1	1.	1.17	1.68	1.38	2.56	0.	0.
time (sec)	N/A	0.165	0.838	0.078	1.602	1.701	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	159	186	162	468	0	0
normalized size	1	1.	1.32	1.55	1.35	3.9	0.	0.
time (sec)	N/A	0.203	1.306	0.071	1.625	1.471	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	113	296	0	7250
normalized size	1	1.	2.04	1.65	1.59	4.17	0.	102.11
time (sec)	N/A	0.089	0.438	0.049	1.615	1.537	0.	64.982

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	63	97	78	51
normalized size	1	1.	0.76	1.16	1.4	2.16	1.73	1.13
time (sec)	N/A	0.014	0.193	0.023	1.085	1.558	0.697	1.853

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	94	89	107	281	0	193
normalized size	1	1.	1.27	1.2	1.45	3.8	0.	2.61
time (sec)	N/A	0.102	0.58	0.036	1.64	1.524	0.	1.732

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	191	190	188	474	0	282
normalized size	1	1.	1.95	1.94	1.92	4.84	0.	2.88
time (sec)	N/A	0.161	5.581	0.044	1.647	1.703	0.	1.762

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	99	445	180	594	0	0
normalized size	1	1.	0.62	2.78	1.12	3.71	0.	0.
time (sec)	N/A	0.109	0.564	0.105	1.077	1.575	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	205	97	248	0	0
normalized size	1	1.	0.73	2.25	1.07	2.73	0.	0.
time (sec)	N/A	0.071	0.157	0.078	1.071	1.599	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	85	77	147	0	0
normalized size	1	1.	0.74	1.21	1.1	2.1	0.	0.
time (sec)	N/A	0.045	0.045	0.051	1.071	1.516	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	109	108	284	0	127
normalized size	1	1.	0.68	1.11	1.1	2.9	0.	1.3
time (sec)	N/A	0.065	0.202	0.053	1.06	1.483	0.	1.372

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	243	359	282	734	0	0
normalized size	1	1.	1.35	1.99	1.57	4.08	0.	0.
time (sec)	N/A	0.357	5.614	0.131	1.648	1.598	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	223	539	0	0
normalized size	1	1.	1.49	2.24	1.87	4.53	0.	0.
time (sec)	N/A	0.194	2.25	0.085	1.612	1.583	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	158	378	0	0
normalized size	1	1.	1.29	1.88	1.78	4.25	0.	0.
time (sec)	N/A	0.125	0.49	0.053	1.57	1.42	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	97	134	121	74
normalized size	1	1.	0.7	1.17	1.54	2.13	1.92	1.17
time (sec)	N/A	0.054	0.326	0.027	1.074	1.458	0.655	1.64

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	106	105	126	321	0	219
normalized size	1	1.	1.15	1.14	1.37	3.49	0.	2.38
time (sec)	N/A	0.137	1.065	0.043	1.637	1.566	0.	2.059

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	83	387	147	394	0	0
normalized size	1	1.	0.64	3.	1.14	3.05	0.	0.
time (sec)	N/A	0.095	0.469	0.1	1.107	1.672	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	76	245	115	293	0	0
normalized size	1	1.	0.71	2.29	1.07	2.74	0.	0.
time (sec)	N/A	0.079	0.155	0.084	1.065	1.532	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	62	101	95	182	0	0
normalized size	1	1.	0.7	1.15	1.08	2.07	0.	0.
time (sec)	N/A	0.053	0.07	0.052	1.076	1.538	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	78	125	111	324	0	130
normalized size	1	1.	0.76	1.23	1.09	3.18	0.	1.27
time (sec)	N/A	0.067	0.137	0.054	1.118	1.734	0.	1.393

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	321	620	0	0
normalized size	1	1.	1.76	2.52	2.24	4.34	0.	0.
time (sec)	N/A	0.199	1.651	0.092	1.673	1.563	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	125	231	244	456	0	0
normalized size	1	1.	1.11	2.04	2.16	4.04	0.	0.
time (sec)	N/A	0.161	1.126	0.06	1.589	1.589	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	111	146	177	224	97
normalized size	1	1.	0.66	1.28	1.68	2.03	2.57	1.11
time (sec)	N/A	0.082	0.425	0.029	1.114	1.479	2.074	1.381

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	136	127	158	360	0	262
normalized size	1	1.	1.17	1.09	1.36	3.1	0.	2.26
time (sec)	N/A	0.159	1.574	0.045	1.573	1.693	0.	1.396

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	209	190	294	560	0	370
normalized size	1	1.	1.49	1.36	2.1	4.	0.	2.64
time (sec)	N/A	0.225	5.36	0.05	1.527	1.977	0.	1.482

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	283	293	423	770	0	458
normalized size	1	1.	1.43	1.48	2.14	3.89	0.	2.31
time (sec)	N/A	0.428	1.52	0.054	1.692	2.085	0.	1.703

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	101	162	236	464	0	184
normalized size	1	1.	0.78	1.25	1.82	3.57	0.	1.42
time (sec)	N/A	0.161	1.009	0.059	1.028	2.054	0.	8.07

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	84	126	176	398	0	157
normalized size	1	1.	0.79	1.19	1.66	3.75	0.	1.48
time (sec)	N/A	0.136	0.308	0.056	1.	1.836	0.	3.934

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	90	120	338	0	130
normalized size	1	1.	0.66	1.1	1.46	4.12	0.	1.59
time (sec)	N/A	0.116	0.165	0.053	1.	1.343	0.	2.115

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	58	28	54	63	163	0	78
normalized size	1	1.57	0.76	1.46	1.7	4.41	0.	2.11
time (sec)	N/A	0.067	0.039	0.058	1.088	1.636	0.	1.341

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	42	74	0	45
normalized size	1	1.	1.	1.03	1.31	2.31	0.	1.41
time (sec)	N/A	0.039	0.019	0.023	1.047	1.407	0.	1.283

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	30	35	73	0	35
normalized size	1	1.	0.75	0.94	1.09	2.28	0.	1.09
time (sec)	N/A	0.068	0.033	0.032	1.118	1.37	0.	1.323

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	62	162	0	62
normalized size	1	1.	0.59	0.96	1.22	3.18	0.	1.22
time (sec)	N/A	0.089	0.049	0.085	1.106	1.4	0.	1.321

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	89	248	0	89
normalized size	1	1.	0.9	0.99	1.31	3.65	0.	1.31
time (sec)	N/A	0.093	0.149	0.095	1.128	1.413	0.	1.462

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	87	116	343	0	116
normalized size	1	1.	0.92	1.04	1.38	4.08	0.	1.38
time (sec)	N/A	0.1	0.209	0.115	1.108	1.517	0.	1.438

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	146	175	456	248	0	232
normalized size	1	1.	1.74	2.08	5.43	2.95	0.	2.76
time (sec)	N/A	0.097	0.319	0.057	1.095	1.673	0.	5.32

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	289	194	0	162
normalized size	1	1.	1.54	1.88	4.19	2.81	0.	2.35
time (sec)	N/A	0.092	0.32	0.055	1.081	1.511	0.	2.567

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	122	127	0	92
normalized size	1	1.	2.12	1.4	2.44	2.54	0.	1.84
time (sec)	N/A	0.088	0.164	0.047	1.095	1.371	0.	1.659

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	36	108	27	28
normalized size	1	1.	2.09	0.96	1.57	4.7	1.17	1.22
time (sec)	N/A	0.012	0.041	0.022	1.078	1.46	1.726	1.27

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	56	95	173	0	88
normalized size	1	1.	2.38	1.93	3.28	5.97	0.	3.03
time (sec)	N/A	0.051	0.218	0.068	1.087	1.539	0.	1.286

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	124	132	209	311	0	171
normalized size	1	1.	2.14	2.28	3.6	5.36	0.	2.95
time (sec)	N/A	0.088	0.491	0.075	1.027	1.416	0.	1.416

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	189	208	316	431	0	252
normalized size	1	1.	2.3	2.54	3.85	5.26	0.	3.07
time (sec)	N/A	0.106	0.743	0.088	1.02	1.396	0.	1.578

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	284	284	425	545	0	329
normalized size	1	1.	2.68	2.68	4.01	5.14	0.	3.1
time (sec)	N/A	0.127	0.914	0.102	1.125	1.579	0.	1.979

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	112	180	273	599	0	197
normalized size	1	1.	0.59	0.95	1.44	3.17	0.	1.04
time (sec)	N/A	0.149	1.693	0.096	1.106	1.728	0.	9.844

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	91	144	225	532	0	170
normalized size	1	1.	0.62	0.99	1.54	3.64	0.	1.16
time (sec)	N/A	0.108	0.45	0.084	1.054	1.528	0.	5.289

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	70	108	149	467	0	138
normalized size	1	1.	0.67	1.04	1.43	4.49	0.	1.33
time (sec)	N/A	0.086	0.311	0.083	1.055	1.531	0.	2.998

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	36	72	95	274	0	122
normalized size	1	1.	0.6	1.2	1.58	4.57	0.	2.03
time (sec)	N/A	0.048	0.081	0.082	1.025	1.456	0.	1.848

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	50	62	162	0	61
normalized size	1	1.	0.69	0.96	1.19	3.12	0.	1.17
time (sec)	N/A	0.05	0.055	0.034	1.119	1.509	0.	1.795

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	66	74	204	0	155
normalized size	1	1.	0.75	1.02	1.14	3.14	0.	2.38
time (sec)	N/A	0.059	0.07	0.106	1.384	1.548	0.	2.102

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	49	138	0	49
normalized size	1	1.	0.69	0.71	0.89	2.51	0.	0.89
time (sec)	N/A	0.051	0.074	0.092	0.956	1.466	0.	1.962

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	62	234	0	62
normalized size	1	1.	1.	0.67	0.85	3.21	0.	0.85
time (sec)	N/A	0.057	0.073	0.108	1.072	1.401	0.	2.364

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	78	79	103	331	0	103
normalized size	1	1.	0.61	0.62	0.81	2.61	0.	0.81
time (sec)	N/A	0.074	0.143	0.125	1.762	1.446	0.	2.225

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	431	0	116
normalized size	1	1.	0.61	0.61	0.8	2.97	0.	0.8
time (sec)	N/A	0.081	0.205	0.139	1.837	1.676	0.	1.931

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	118	119	157	541	0	157
normalized size	1	1.	0.59	0.6	0.79	2.72	0.	0.79
time (sec)	N/A	0.102	0.329	0.165	1.835	1.654	0.	2.161

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	102	162	254	657	0	184
normalized size	1	1.	0.6	0.95	1.49	3.84	0.	1.08
time (sec)	N/A	0.123	0.926	0.106	1.961	1.671	0.	4.824

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	126	197	586	0	154
normalized size	1	1.	0.65	1.	1.56	4.65	0.	1.22
time (sec)	N/A	0.091	0.363	0.092	1.938	1.581	0.	2.552

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	52	90	132	409	0	109
normalized size	1	1.	0.63	1.1	1.61	4.99	0.	1.33
time (sec)	N/A	0.057	0.145	0.095	2.143	1.553	0.	2.19

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	68	97	284	0	80
normalized size	1	1.	0.7	0.92	1.31	3.84	0.	1.08
time (sec)	N/A	0.058	0.175	0.037	1.769	1.607	0.	1.98

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	61	84	108	393	0	208
normalized size	1	1.	0.71	0.98	1.26	4.57	0.	2.42
time (sec)	N/A	0.07	0.191	0.129	1.951	1.547	0.	1.734

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	69	97	101	348	0	235
normalized size	1	1.	0.72	1.01	1.05	3.62	0.	2.45
time (sec)	N/A	0.071	0.328	0.126	2.056	1.652	0.	2.575

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	62	208	0	62
normalized size	1	1.	0.66	0.67	0.85	2.85	0.	0.85
time (sec)	N/A	0.057	0.099	0.119	1.539	1.461	0.	1.64

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	89	302	0	89
normalized size	1	1.	0.62	0.63	0.82	2.77	0.	0.82
time (sec)	N/A	0.069	0.072	0.136	1.212	1.476	0.	1.619

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	398	0	116
normalized size	1	1.	0.61	0.61	0.8	2.74	0.	0.8
time (sec)	N/A	0.081	0.113	0.153	1.965	1.591	0.	2.013

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	498	0	116
normalized size	1	1.	0.61	0.61	0.8	3.43	0.	0.8
time (sec)	N/A	0.078	0.118	0.174	2.733	1.669	0.	1.993

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	112	180	288	775	0	197
normalized size	1	1.	0.57	0.92	1.48	3.97	0.	1.01
time (sec)	N/A	0.135	1.489	0.088	3.476	1.734	0.	5.53

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	50	81	128	250	0	103
normalized size	1	1.	0.38	0.61	0.97	1.89	0.	0.78
time (sec)	N/A	0.089	0.115	0.088	2.939	1.354	0.	2.775

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	108	163	525	0	123
normalized size	1	1.	0.59	1.03	1.55	5.	0.	1.17
time (sec)	N/A	0.065	0.254	0.092	3.157	1.591	0.	2.141

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	73	101	139	522	0	250
normalized size	1	1.	0.69	0.95	1.31	4.92	0.	2.36
time (sec)	N/A	0.083	0.839	0.139	2.616	1.583	0.	1.819

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	89	130	128	502	0	313
normalized size	1	1.	0.66	0.96	0.95	3.72	0.	2.32
time (sec)	N/A	0.086	0.171	0.148	2.681	1.598	0.	1.93

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	124	158	481	338	0	197
normalized size	1	1.	0.98	1.24	3.79	2.66	0.	1.55
time (sec)	N/A	0.31	0.416	0.076	3.082	1.496	0.	2.539

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	315	161	389	992	0	182
normalized size	1	1.	2.92	1.49	3.6	9.19	0.	1.69
time (sec)	N/A	0.318	0.42	0.125	3.729	1.597	0.	1.945

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	589	195	385	1187	0	242
normalized size	1	1.	4.91	1.62	3.21	9.89	0.	2.02
time (sec)	N/A	0.249	6.089	0.138	3.069	1.581	0.	2.401

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	733	229	377	1191	0	275
normalized size	1	1.	5.51	1.72	2.83	8.95	0.	2.07
time (sec)	N/A	0.249	6.145	0.149	3.619	1.787	0.	1.735

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	195	394	172	0	548	0	0
normalized size	1	1.2	2.43	1.06	0.	3.38	0.	0.
time (sec)	N/A	0.923	5.54	0.464	0.	2.006	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	114	89	0	459	0	0
normalized size	1	1.	1.13	0.88	0.	4.54	0.	0.
time (sec)	N/A	0.185	0.328	0.502	0.	1.822	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	749	0	570
normalized size	1	1.	2.31	1.4	0.	8.42	0.	6.4
time (sec)	N/A	0.192	0.955	0.55	0.	1.936	0.	2.091

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	1025	0	950
normalized size	1	1.	1.9	1.04	0.	6.29	0.	5.83
time (sec)	N/A	0.377	1.568	0.701	0.	1.816	0.	2.115

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	195	141	139	0	648	0	0
normalized size	1	1.17	0.84	0.83	0.	3.88	0.	0.
time (sec)	N/A	0.975	5.562	0.574	0.	1.902	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	46	55	196	123	0	0
normalized size	1	1.	0.52	0.62	2.23	1.4	0.	0.
time (sec)	N/A	0.196	3.992	0.402	1.547	1.754	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	833	0	671
normalized size	1	1.	1.93	1.19	0.	6.88	0.	5.55
time (sec)	N/A	0.321	0.76	0.642	0.	1.846	0.	2.634

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	1123	0	1058
normalized size	1	1.	1.7	0.99	0.	5.7	0.	5.37
time (sec)	N/A	0.496	1.356	0.651	0.	1.723	0.	3.102

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	208	112	87	374	246	0	0
normalized size	1	1.38	0.74	0.58	2.48	1.63	0.	0.
time (sec)	N/A	0.979	5.459	0.404	1.764	1.432	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	60	67	258	173	0	0
normalized size	1	1.	0.51	0.57	2.19	1.47	0.	0.
time (sec)	N/A	0.214	5.463	0.412	1.663	1.423	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	261	162	0	934	0	714
normalized size	1	1.	1.73	1.07	0.	6.19	0.	4.73
time (sec)	N/A	0.429	1.193	0.733	0.	1.571	0.	2.906

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	360	222	0	1249	0	1135
normalized size	1	1.	1.59	0.98	0.	5.5	0.	5.
time (sec)	N/A	0.627	1.899	0.694	0.	1.732	0.	3.194

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	241	118	231	0	624	0	1056
normalized size	1	1.61	0.79	1.54	0.	4.16	0.	7.04
time (sec)	N/A	0.933	0.673	0.766	0.	1.639	0.	5.889

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	118	130	0	549	0	594
normalized size	1	1.	1.1	1.21	0.	5.13	0.	5.55
time (sec)	N/A	0.195	0.269	0.462	0.	1.543	0.	2.823

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	706	0	502
normalized size	1	1.	2.23	1.66	0.	11.39	0.	8.1
time (sec)	N/A	0.11	0.308	0.59	0.	1.609	0.	2.219

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	987	0	819
normalized size	1	1.	2.16	1.07	0.	7.31	0.	6.07
time (sec)	N/A	0.622	0.58	0.638	0.	1.463	0.	2.373

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	195	334	289	0	726	0	0
normalized size	1	1.1	1.89	1.63	0.	4.1	0.	0.
time (sec)	N/A	1.199	0.356	0.619	0.	1.62	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	128	202	0	644	0	0
normalized size	1	1.	0.96	1.51	0.	4.81	0.	0.
time (sec)	N/A	0.223	0.444	0.607	0.	1.635	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	206	134	0	1133	0	656
normalized size	1	1.	1.82	1.19	0.	10.03	0.	5.81
time (sec)	N/A	0.229	2.198	0.591	0.	1.764	0.	2.485

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	1003	0	828
normalized size	1	1.	2.04	1.	0.	6.97	0.	5.75
time (sec)	N/A	0.553	0.761	0.621	0.	1.572	0.	3.131

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	260	394	353	0	830	0	0
normalized size	1	1.26	1.9	1.71	0.	4.01	0.	0.
time (sec)	N/A	1.434	0.546	0.843	0.	1.901	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	751	0	0
normalized size	1	1.	1.7	1.59	0.	4.5	0.	0.
time (sec)	N/A	0.303	0.408	0.648	0.	1.697	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	451	219	0	1436	0	0
normalized size	1	1.	3.2	1.55	0.	10.18	0.	0.
time (sec)	N/A	0.346	0.718	0.57	0.	1.859	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	1504	0	975
normalized size	1	1.	1.74	0.95	0.	7.87	0.	5.1
time (sec)	N/A	0.959	2.389	0.722	0.	1.745	0.	3.16

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	982	982	318	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.271	3.273	0.128	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	290	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	2.746	0.104	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	2692	0	0	0	0	0
normalized size	1	1.	33.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	23.898	0.095	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	2796	0	0	0	0	0
normalized size	1	1.	34.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	22.599	0.102	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	128	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.494	0.796	0.119	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	100	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.513	0.105	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	12.766	0.093	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	9.129	0.105	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	5199	0	0	0	0	0
normalized size	1	1.	19.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.345	58.895	1.709	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	2054	0	0	0	0	0
normalized size	1	1.	10.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	18.053	1.421	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	2.025	0.907	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	232	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	4.033	0.178	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	667	0	0	0	0	0
normalized size	1	1.	4.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.275	14.178	0.41	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	1276	0	0	0	0	0
normalized size	1	1.	5.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	31.232	0.451	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	367	0	0	0	0	0
normalized size	1	1.	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	2.198	0.847	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.267	0.144	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.071	0.76	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.059	0.821	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	68	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.188	0.268	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	83	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.275	0.352	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	1.064	0.152	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	11184	0	0	0	0	0
normalized size	1	1.	71.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	34.458	0.12	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.181	0.286	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	47487	0	0	0	0	0
normalized size	1	1.	533.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	99.938	0.231	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.741	0.294	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	77	96	99	236	0	0
normalized size	1	1.	0.88	1.09	1.12	2.68	0.	0.
time (sec)	N/A	0.077	0.119	0.032	1.901	1.68	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	46	58	124	0	1966
normalized size	1	1.	0.69	0.84	1.05	2.25	0.	35.75
time (sec)	N/A	0.038	0.016	0.025	1.943	1.61	0.	2.827

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	43	25	30	62	0	31
normalized size	1	1.	1.79	1.04	1.25	2.58	0.	1.29
time (sec)	N/A	0.021	0.033	0.019	1.698	1.505	0.	2.282

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	61	167	0	81
normalized size	1	1.	1.11	1.54	1.13	3.09	0.	1.5
time (sec)	N/A	0.041	0.21	0.039	1.44	1.601	0.	2.007

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.052	0.239	0.04	1.281	1.536	0.	2.1

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	88	182	0	0
normalized size	1	1.	1.12	1.36	1.22	2.53	0.	0.
time (sec)	N/A	0.078	0.038	0.032	2.133	1.815	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	53	108	0	1361
normalized size	1	1.	1.24	1.55	1.39	2.84	0.	35.82
time (sec)	N/A	0.059	0.033	0.026	2.723	1.705	0.	8.562

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.053	0.031	0.029	2.207	1.836	0.	2.308

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.08	0.042	0.036	1.873	1.864	0.	2.177

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	169	620	0	269
normalized size	1	1.	1.34	1.3	1.39	5.08	0.	2.2
time (sec)	N/A	0.097	0.06	0.039	2.7	1.706	0.	2.224

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	108	172	142	348	0	0
normalized size	1	1.	0.97	1.55	1.28	3.14	0.	0.
time (sec)	N/A	0.173	0.432	0.049	1.676	1.548	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	82	95	186	0	10604
normalized size	1	1.	0.82	1.05	1.22	2.38	0.	135.95
time (sec)	N/A	0.071	0.136	0.039	1.631	1.609	0.	5.72

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	45	54	108	0	55
normalized size	1	1.	1.	0.98	1.17	2.35	0.	1.2
time (sec)	N/A	0.039	0.023	0.023	1.893	1.58	0.	2.322

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	120	93	271	0	134
normalized size	1	1.	0.83	1.43	1.11	3.23	0.	1.6
time (sec)	N/A	0.073	0.236	0.053	1.568	1.521	0.	2.313

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	220	142	437	0	186
normalized size	1	1.	0.85	1.75	1.13	3.47	0.	1.48
time (sec)	N/A	0.104	0.757	0.047	1.403	1.629	0.	2.223

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	176	185	161	281	0	0
normalized size	1	1.	1.18	1.24	1.08	1.89	0.	0.
time (sec)	N/A	0.162	0.687	0.044	2.5	1.464	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	112	190	0	10355
normalized size	1	1.	0.82	1.23	1.19	2.02	0.	110.16
time (sec)	N/A	0.122	0.457	0.039	2.52	1.37	0.	30.612

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	116	102	107	308	0	200
normalized size	1	1.	1.49	1.31	1.37	3.95	0.	2.56
time (sec)	N/A	0.086	0.416	0.039	2.595	1.65	0.	2.237

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	293	199	186	539	0	325
normalized size	1	1.	2.2	1.5	1.4	4.05	0.	2.44
time (sec)	N/A	0.149	6.233	0.049	2.453	1.827	0.	2.059

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	351	302	247	786	0	455
normalized size	1	1.	1.74	1.5	1.22	3.89	0.	2.25
time (sec)	N/A	0.169	1.093	0.052	1.676	1.639	0.	2.779

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	141	279	219	470	0	0
normalized size	1	1.	0.94	1.86	1.46	3.13	0.	0.
time (sec)	N/A	0.242	0.263	0.056	1.546	1.721	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	90	139	153	277	0	0
normalized size	1	1.	0.86	1.32	1.46	2.64	0.	0.
time (sec)	N/A	0.114	0.19	0.046	1.707	1.606	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	64	77	158	0	78
normalized size	1	1.	1.	0.96	1.15	2.36	0.	1.16
time (sec)	N/A	0.045	0.025	0.027	1.567	1.492	0.	2.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	97	165	132	358	0	177
normalized size	1	1.	0.84	1.42	1.14	3.09	0.	1.53
time (sec)	N/A	0.094	0.321	0.065	1.9	1.563	0.	2.022

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	144	316	192	536	0	250
normalized size	1	1.	0.87	1.92	1.16	3.25	0.	1.52
time (sec)	N/A	0.141	1.164	0.062	1.354	1.6	0.	2.141

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	226	268	225	369	0	0
normalized size	1	1.	1.03	1.22	1.02	1.68	0.	0.
time (sec)	N/A	0.224	0.678	0.058	1.824	1.593	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	161	271	0	0
normalized size	1	1.	0.77	1.16	1.1	1.86	0.	0.
time (sec)	N/A	0.169	0.729	0.047	3.013	1.485	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	125	128	370	0	269
normalized size	1	1.	1.4	1.23	1.25	3.63	0.	2.64
time (sec)	N/A	0.112	1.295	0.052	1.515	1.605	0.	2.628

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	355	264	252	699	0	568
normalized size	1	1.	1.83	1.36	1.3	3.6	0.	2.93
time (sec)	N/A	0.184	6.236	0.062	2.21	1.652	0.	2.652

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	346	415	340	1014	0	636
normalized size	1	1.	1.19	1.43	1.17	3.48	0.	2.19
time (sec)	N/A	0.231	2.578	0.063	2.641	1.832	0.	2.013

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	184	304	389	594	0	463
normalized size	1	1.	0.9	1.49	1.91	2.91	0.	2.27
time (sec)	N/A	0.363	1.462	0.066	1.826	2.474	0.	5.118

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	117	164	192	367	0	239
normalized size	1	1.	0.93	1.3	1.52	2.91	0.	1.9
time (sec)	N/A	0.191	0.484	0.059	1.536	1.961	0.	1.98

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	87	76	88	157	0	96
normalized size	1	1.	1.18	1.03	1.19	2.12	0.	1.3
time (sec)	N/A	0.066	0.086	0.049	1.551	1.611	0.	1.3

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	45	80	0	47
normalized size	1	1.	1.	1.03	1.32	2.35	0.	1.38
time (sec)	N/A	0.04	0.02	0.026	1.465	1.57	0.	1.695

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	104	271	0	154
normalized size	1	1.	0.77	1.26	1.24	3.23	0.	1.83
time (sec)	N/A	0.089	0.161	0.063	1.616	1.596	0.	2.011

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	115	216	188	629	0	271
normalized size	1	1.	0.78	1.46	1.27	4.25	0.	1.83
time (sec)	N/A	0.138	3.847	0.063	1.463	1.663	0.	1.538

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	195	269	0	1054	0	325
normalized size	1	1.	1.1	1.52	0.	5.95	0.	1.84
time (sec)	N/A	0.239	1.419	0.067	0.	1.639	0.	3.306

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	152	117	0	684	0	144
normalized size	1	1.	1.58	1.22	0.	7.12	0.	1.5
time (sec)	N/A	0.099	0.203	0.053	0.	1.67	0.	2.053

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	108	155	0	801	0	174
normalized size	1	1.	1.35	1.94	0.	10.01	0.	2.17
time (sec)	N/A	0.243	0.236	0.06	0.	1.742	0.	1.87

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	350	348	0	1507	0	369
normalized size	1	1.	2.27	2.26	0.	9.79	0.	2.4
time (sec)	N/A	0.426	6.119	0.069	0.	2.929	0.	1.911

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	504	629	0	2566	0	662
normalized size	1	1.	1.64	2.05	0.	8.36	0.	2.16
time (sec)	N/A	1.106	1.386	0.073	0.	4.834	0.	2.212

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	240	318	682	1231	0	667
normalized size	1	1.	0.99	1.31	2.82	5.09	0.	2.76
time (sec)	N/A	0.633	6.252	0.093	1.828	3.386	0.	4.672

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	145	182	370	869	0	335
normalized size	1	1.	0.9	1.13	2.3	5.4	0.	2.08
time (sec)	N/A	0.312	0.719	0.087	1.753	2.394	0.	1.843

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	162	132	167	463	0	211
normalized size	1	1.	1.49	1.21	1.53	4.25	0.	1.94
time (sec)	N/A	0.096	0.287	0.078	1.764	1.695	0.	1.284

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	54	63	176	0	69
normalized size	1	1.	0.79	1.02	1.19	3.32	0.	1.3
time (sec)	N/A	0.053	0.076	0.038	1.47	1.637	0.	1.499

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	96	150	157	599	0	223
normalized size	1	1.	0.84	1.32	1.38	5.25	0.	1.96
time (sec)	N/A	0.115	0.608	0.089	1.364	1.619	0.	1.976

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	187	282	255	1241	0	375
normalized size	1	1.	0.99	1.5	1.36	6.6	0.	1.99
time (sec)	N/A	0.181	6.144	0.098	1.734	2.294	0.	1.811

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	341	382	0	1790	0	548
normalized size	1	1.	1.02	1.15	0.	5.38	0.	1.65
time (sec)	N/A	0.627	1.967	0.098	0.	2.504	0.	2.784

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	169	282	0	1261	0	339
normalized size	1	1.	0.84	1.41	0.	6.3	0.	1.7
time (sec)	N/A	0.301	1.012	0.088	0.	1.958	0.	2.152

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	139	245	0	1681	0	294
normalized size	1	1.	1.21	2.13	0.	14.62	0.	2.56
time (sec)	N/A	0.453	0.746	0.086	0.	2.816	0.	1.715

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	403	527	0	2639	0	481
normalized size	1	1.	1.69	2.21	0.	11.09	0.	2.02
time (sec)	N/A	0.695	6.295	0.096	0.	3.101	0.	1.859

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	361	897	0	4674	0	805
normalized size	1	1.	0.85	2.12	0.	11.02	0.	1.9
time (sec)	N/A	1.493	1.592	0.108	0.	5.532	0.	1.37

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	304	465	986	2206	0	790
normalized size	1	1.	0.95	1.45	3.07	6.87	0.	2.46
time (sec)	N/A	0.877	6.341	0.116	1.303	6.427	0.	3.708

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	196	323	595	1727	0	626
normalized size	1	1.	0.84	1.39	2.56	7.44	0.	2.7
time (sec)	N/A	0.482	2.189	0.109	1.876	3.964	0.	2.1

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	213	198	308	995	0	347
normalized size	1	1.	1.43	1.33	2.07	6.68	0.	2.33
time (sec)	N/A	0.132	2.02	0.1	1.915	2.649	0.	1.582

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	74	109	365	0	93
normalized size	1	1.	0.8	0.99	1.45	4.87	0.	1.24
time (sec)	N/A	0.06	0.262	0.046	1.547	2.004	0.	1.746

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	121	194	211	911	0	208
normalized size	1	1.	0.83	1.34	1.46	6.28	0.	1.43
time (sec)	N/A	0.133	0.931	0.111	1.921	2.342	0.	1.8

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	195	348	319	1708	0	441
normalized size	1	1.	0.88	1.57	1.44	7.73	0.	2.
time (sec)	N/A	0.211	5.327	0.114	1.612	2.424	0.	2.104

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	351	922	0	2765	0	853
normalized size	1	1.	0.74	1.95	0.	5.83	0.	1.8
time (sec)	N/A	0.872	1.043	0.131	0.	2.853	0.	3.399

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	212	766	0	2067	0	518
normalized size	1	1.	0.61	2.19	0.	5.91	0.	1.48
time (sec)	N/A	0.539	3.291	0.115	0.	2.322	0.	1.956

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	195	729	0	3044	0	458
normalized size	1	1.	0.97	3.61	0.	15.07	0.	2.27
time (sec)	N/A	0.788	5.784	0.116	0.	5.548	0.	1.552

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	459	780	0	4578	0	609
normalized size	1	1.	1.59	2.7	0.	15.84	0.	2.11
time (sec)	N/A	1.072	6.211	0.134	0.	5.542	0.	2.051

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	448	1252	0	6091	0	987
normalized size	1	1.	0.91	2.54	0.	12.38	0.	2.01
time (sec)	N/A	2.127	1.705	0.138	0.	6.517	0.	1.85

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	4791	0	0	0	0	0
normalized size	1	1.	17.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	18.403	1.514	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	2464	0	0	0	0	0
normalized size	1	1.	13.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	14.208	1.446	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	849	0	0	0	0	0
normalized size	1	1.	6.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	8.262	0.904	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	284	0	864	0	0	0	0	0
normalized size	1	0.	3.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	13.498	0.362	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	737	0	908	0	0	0	0	0
normalized size	1	0.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	14.227	0.677	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.695	0.862	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [159] had the largest ratio of [0.4762]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	19	0.105
2	A	3	2	1.	19	0.105
3	A	3	2	1.	17	0.118
4	A	3	2	1.	17	0.118
5	A	3	2	1.	19	0.105
6	A	3	2	1.	19	0.105
7	A	3	2	1.	19	0.105
8	A	9	5	1.	19	0.263
9	A	8	5	1.	19	0.263
10	A	5	5	1.	19	0.263
11	A	7	6	1.	19	0.316
12	A	9	7	1.	19	0.368
13	A	11	7	1.	19	0.368
14	A	3	2	1.	21	0.095
15	A	3	2	1.	21	0.095
16	A	3	2	1.	19	0.105
17	A	2	2	1.	21	0.095
18	A	3	2	1.	21	0.095
19	A	14	9	1.	21	0.429
20	A	4	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	6	5	1.	21	0.238
22	A	1	1	1.	12	0.083
23	A	8	6	1.	21	0.286
24	A	12	7	1.	21	0.333
25	A	3	2	1.	21	0.095
26	A	3	2	1.	21	0.095
27	A	3	2	1.	19	0.105
28	A	3	2	1.	21	0.095
29	A	9	7	1.	21	0.333
30	A	10	7	1.	21	0.333
31	A	8	6	1.	21	0.286
32	A	7	5	1.	12	0.417
33	A	10	7	1.	21	0.333
34	A	3	2	1.	21	0.095
35	A	3	2	1.	21	0.095
36	A	3	2	1.	19	0.105
37	A	3	2	1.	21	0.095
38	A	13	7	1.	21	0.333
39	A	11	6	1.	21	0.286
40	A	10	5	1.	12	0.417
41	A	12	6	1.	21	0.286
42	A	17	8	1.	21	0.381
43	A	21	8	1.	21	0.381
44	A	8	5	1.	21	0.238
45	A	7	5	1.	21	0.238
46	A	6	5	1.	21	0.238
47	A	5	5	1.57	19	0.263
48	A	4	4	1.	19	0.21
49	A	5	4	1.	21	0.19
50	A	5	4	1.	21	0.19
51	A	6	5	1.	21	0.238
52	A	6	5	1.	21	0.238
53	A	6	5	1.	21	0.238
54	A	6	5	1.	21	0.238
55	A	5	4	1.	21	0.19
56	A	1	1	1.	12	0.083
57	A	4	4	1.	21	0.19
58	A	5	5	1.	21	0.238
59	A	6	5	1.	21	0.238
60	A	7	5	1.	21	0.238
61	A	4	3	1.	21	0.143
62	A	4	3	1.	21	0.143
63	A	4	3	1.	21	0.143
64	A	4	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	3	2	1.	19	0.105
66	A	3	2	1.	21	0.095
67	A	3	2	1.	21	0.095
68	A	3	2	1.	21	0.095
69	A	3	2	1.	21	0.095
70	A	3	2	1.	21	0.095
71	A	3	2	1.	21	0.095
72	A	4	3	1.	21	0.143
73	A	4	3	1.	21	0.143
74	A	4	3	1.	19	0.158
75	A	3	2	1.	19	0.105
76	A	3	2	1.	21	0.095
77	A	3	2	1.	21	0.095
78	A	3	2	1.	21	0.095
79	A	3	2	1.	21	0.095
80	A	3	2	1.	21	0.095
81	A	3	2	1.	21	0.095
82	A	4	3	1.	21	0.143
83	A	3	2	1.	21	0.095
84	A	4	3	1.	19	0.158
85	A	3	2	1.	21	0.095
86	A	3	2	1.	21	0.095
87	A	17	5	1.	21	0.238
88	A	14	8	1.	21	0.381
89	A	14	8	1.	21	0.381
90	A	16	6	1.	21	0.286
91	A	15	10	1.2	23	0.435
92	A	4	4	1.	23	0.174
93	A	4	4	1.	23	0.174
94	A	7	7	1.	23	0.304
95	A	14	9	1.17	23	0.391
96	A	3	3	1.	23	0.13
97	A	5	5	1.	23	0.217
98	A	8	8	1.	23	0.348
99	A	10	7	1.38	23	0.304
100	A	4	4	1.	23	0.174
101	A	6	5	1.	23	0.217
102	A	10	8	1.	23	0.348
103	A	17	9	1.61	23	0.391
104	A	4	4	1.	23	0.174
105	A	4	4	1.	23	0.174
106	A	11	7	1.	23	0.304
107	A	20	9	1.1	23	0.391
108	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	6	5	1.	23	0.217
110	A	10	6	1.	23	0.261
111	A	23	9	1.26	23	0.391
112	A	6	6	1.	23	0.261
113	A	7	6	1.	23	0.261
114	A	16	8	1.	23	0.348
115	A	10	9	1.	23	0.391
116	A	4	4	1.	23	0.174
117	A	3	3	1.	23	0.13
118	A	3	3	1.	23	0.13
119	A	8	7	1.	23	0.304
120	A	4	4	1.	23	0.174
121	A	3	3	1.	23	0.13
122	A	3	3	1.	23	0.13
123	A	10	6	1.	23	0.261
124	A	8	6	1.	23	0.261
125	A	6	5	1.	21	0.238
126	A	4	4	1.	23	0.174
127	A	10	6	1.	23	0.261
128	A	13	6	1.	23	0.261
129	A	4	3	1.	23	0.13
130	A	4	4	1.	21	0.19
131	A	3	3	1.	19	0.158
132	A	2	2	1.	19	0.105
133	A	3	3	1.	21	0.143
134	A	4	4	1.	21	0.19
135	A	6	6	1.	21	0.286
136	A	5	5	1.	21	0.238
137	A	2	2	1.	12	0.167
138	A	3	3	1.	21	0.143
139	A	3	3	1.	21	0.143
140	A	6	5	1.	19	0.263
141	A	5	4	1.	17	0.235
142	A	3	2	1.	17	0.118
143	A	3	2	1.	19	0.105
144	A	3	2	1.	19	0.105
145	A	8	5	1.	19	0.263
146	A	7	5	1.	19	0.263
147	A	7	6	1.	19	0.316
148	A	9	7	1.	19	0.368
149	A	11	7	1.	19	0.368
150	A	7	5	1.	21	0.238
151	A	6	4	1.	19	0.21
152	A	3	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	3	2	1.	21	0.095
154	A	3	2	1.	21	0.095
155	A	13	9	1.	21	0.429
156	A	11	9	1.	21	0.429
157	A	9	7	1.	21	0.333
158	A	13	9	1.	21	0.429
159	A	16	10	1.	21	0.476
160	A	7	5	1.	21	0.238
161	A	6	4	1.	19	0.21
162	A	3	2	1.	19	0.105
163	A	3	2	1.	21	0.095
164	A	3	2	1.	21	0.095
165	A	16	9	1.	21	0.429
166	A	14	10	1.	21	0.476
167	A	11	9	1.	21	0.429
168	A	17	10	1.	21	0.476
169	A	21	10	1.	21	0.476
170	A	5	3	1.	21	0.143
171	A	4	3	1.	21	0.143
172	A	3	2	1.	19	0.105
173	A	4	4	1.	19	0.21
174	A	3	2	1.	21	0.095
175	A	3	2	1.	21	0.095
176	A	13	9	1.	21	0.429
177	A	8	7	1.	21	0.333
178	A	7	7	1.	21	0.333
179	A	7	7	1.	21	0.333
180	A	9	7	1.	21	0.333
181	A	5	3	1.	21	0.143
182	A	4	3	1.	21	0.143
183	A	3	2	1.	19	0.105
184	A	3	2	1.	19	0.105
185	A	3	2	1.	21	0.095
186	A	3	2	1.	21	0.095
187	A	16	8	1.	21	0.381
188	A	12	7	1.	21	0.333
189	A	8	7	1.	21	0.333
190	A	8	7	1.	21	0.333
191	A	10	7	1.	21	0.333
192	A	5	3	1.	21	0.143
193	A	4	3	1.	21	0.143
194	A	3	2	1.	19	0.105
195	A	3	2	1.	19	0.105
196	A	3	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	3	2	1.	21	0.095
198	A	22	9	1.	21	0.429
199	A	18	8	1.	21	0.381
200	A	9	8	1.	21	0.381
201	A	9	7	1.	21	0.333
202	A	11	7	1.	21	0.333
203	A	10	6	1.	23	0.261
204	A	8	6	1.	23	0.261
205	A	6	5	1.	21	0.238
206	F	0	0	N/A	0	N/A
207	F	0	0	N/A	0	N/A
208	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (a + a \sin(c + dx)) \tan^5(c + dx) dx$

Optimal. Leaf size=115

$$\frac{a^3}{8d(a - a \sin(c + dx))^2} - \frac{a^2}{d(a - a \sin(c + dx))} + \frac{a^2}{8d(a \sin(c + dx) + a)} - \frac{a \sin(c + dx)}{d} - \frac{23a \log(1 - \sin(c + dx))}{16d} + \frac{7a}{16d}$$

[Out] $(-23*a*\text{Log}[1 - \text{Sin}[c + d*x]])/(16*d) + (7*a*\text{Log}[1 + \text{Sin}[c + d*x]])/(16*d) - (a*\text{Sin}[c + d*x])/d + a^3/(8*d*(a - a*\text{Sin}[c + d*x])^2) - a^2/(d*(a - a*\text{Sin}[c + d*x])) + a^2/(8*d*(a + a*\text{Sin}[c + d*x]))$

Rubi [A] time = 0.0716649, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 88}

$$\frac{a^3}{8d(a - a \sin(c + dx))^2} - \frac{a^2}{d(a - a \sin(c + dx))} + \frac{a^2}{8d(a \sin(c + dx) + a)} - \frac{a \sin(c + dx)}{d} - \frac{23a \log(1 - \sin(c + dx))}{16d} + \frac{7a}{16d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])* \text{Tan}[c + d*x]^5, x]$

[Out] $(-23*a*\text{Log}[1 - \text{Sin}[c + d*x]])/(16*d) + (7*a*\text{Log}[1 + \text{Sin}[c + d*x]])/(16*d) - (a*\text{Sin}[c + d*x])/d + a^3/(8*d*(a - a*\text{Sin}[c + d*x])^2) - a^2/(d*(a - a*\text{Sin}[c + d*x])) + a^2/(8*d*(a + a*\text{Sin}[c + d*x]))$

Rule 2707

$\text{Int}[(a + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)} \tan[(e_*) + (f_*)(x_*)]^{(p_*)}, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[(p + 1)/2]$

Rule 88

$\text{Int}[(a + (b_*)(x_*)^{(m_*)} * ((c_*) + (d_*)(x_*)^{(n_*)} * ((e_*) + (f_*)(x_*)^{(p_*)}))^{(q_*)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rubi steps

$$\int (a + a \sin(c + dx)) \tan^5(c + dx) dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2} dx, x, a \sin(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-1 + \frac{a^3}{4(a-x)^3} - \frac{a^2}{(a-x)^2} + \frac{23a}{16(a-x)} - \frac{a^2}{8(a+x)^2} + \frac{7a}{16(a+x)}\right) dx, x, a \sin(c + dx)\right)}{d}$$

$$= -\frac{23a \log(1 - \sin(c + dx))}{16d} + \frac{7a \log(1 + \sin(c + dx))}{16d} - \frac{a \sin(c + dx)}{d} + \frac{1}{8d(a - a \sin(c + dx))}$$

Mathematica [A] time = 0.446024, size = 123, normalized size = 1.07

$$\frac{a \sin(c + dx) \tan^4(c + dx)}{d} - \frac{a(-\tan^4(c + dx) + 2 \tan^2(c + dx) + 4 \log(\cos(c + dx)))}{4d} - \frac{5a(6 \tan(c + dx) \sec^3(c + dx))}{8d(a - a \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]

[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))) / (8*d)

Maple [A] time = 0.064, size = 147, normalized size = 1.3

$$\frac{a(\sin(dx + c))^7}{4d(\cos(dx + c))^4} - \frac{3a(\sin(dx + c))^7}{8d(\cos(dx + c))^2} - \frac{3(\sin(dx + c))^5 a}{8d} - \frac{5a(\sin(dx + c))^3}{8d} - \frac{15a \sin(dx + c)}{8d} + \frac{15a \ln(\sec(dx + c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))*tan(d*x+c)^5,x)

[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8/d*a*sin(d*x+c)^5*a-5/8/d*a*sin(d*x+c)^3-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(d*x+c)^2-1/d*a*ln(cos(d*x+c))

Maxima [A] time = 1.04939, size = 128, normalized size = 1.11

$$\frac{7a \log(\sin(dx + c) + 1) - 23a \log(\sin(dx + c) - 1) - 16a \sin(dx + c) + \frac{2(9a \sin(dx + c)^2 - a \sin(dx + c) - 6a)}{\sin(dx + c)^3 - \sin(dx + c)^2 - \sin(dx + c) + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")

[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d

Fricas [A] time = 1.55232, size = 410, normalized size = 3.57

$$\frac{16 a \cos (d x+c)^4+2 a \cos (d x+c)^2+7\left(a \cos (d x+c)^2 \sin (d x+c)-a \cos (d x+c)^2\right) \log (\sin (d x+c)+1)-23\left(a \cos (d x+c)^2 \sin (d x+c)-a \cos (d x+c)^2\right) \log (-\sin (d x+c)+1)+2\left(8 a \cos (d x+c)^2+a\right) \sin (d x+c)-6 a}{16\left(d \cos (d x+c)^2 \sin (d x+c)-d \cos (d x+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")

[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin (c+d x) \tan ^5(c+d x) d x+\int \tan ^5(c+d x) d x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**5,x)

[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x))

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")

[Out] Timed out

3.2 $\int (a + a \sin(c + dx)) \tan^3(c + dx) dx$

Optimal. Leaf size=71

$$\frac{a^2}{2d(a - a \sin(c + dx))} + \frac{a \sin(c + dx)}{d} + \frac{5a \log(1 - \sin(c + dx))}{4d} - \frac{a \log(\sin(c + dx) + 1)}{4d}$$

[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Sin[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))

Rubi [A] time = 0.048039, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 88}

$$\frac{a^2}{2d(a - a \sin(c + dx))} + \frac{a \sin(c + dx)}{d} + \frac{5a \log(1 - \sin(c + dx))}{4d} - \frac{a \log(\sin(c + dx) + 1)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]

[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Sin[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx)) \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{a^2}{2(a-x)^2} - \frac{5a}{4(a-x)} - \frac{a}{4(a+x)}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{5a \log(1 - \sin(c + dx))}{4d} - \frac{a \log(1 + \sin(c + dx))}{4d} + \frac{a \sin(c + dx)}{d} + \frac{a^2}{2d(a - a \sin(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.108376, size = 77, normalized size = 1.08

$$\frac{a \sin(c + dx) \tan^2(c + dx)}{d} + \frac{a (\tan^2(c + dx) + 2 \log(\cos(c + dx)))}{2d} - \frac{3a (\tanh^{-1}(\sin(c + dx)) - \tan(c + dx) \sec(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]

[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*a*(ArcTanh[Sin[c + d*x]] - Sec[c + d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)

Maple [A] time = 0.056, size = 96, normalized size = 1.4

$$\frac{a (\sin(dx + c))^5}{2d (\cos(dx + c))^2} + \frac{a (\sin(dx + c))^3}{2d} + \frac{3a \sin(dx + c)}{2d} - \frac{3a \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{a (\tan(dx + c))^2}{2d} + \frac{a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))*tan(d*x+c)^3,x)

[Out] 1/2/d*a*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a*sin(d*x+c)^3+3/2*a*sin(d*x+c)/d-3/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))

Maxima [A] time = 0.99153, size = 69, normalized size = 0.97

$$\frac{a \log(\sin(dx + c) + 1) - 5a \log(\sin(dx + c) - 1) - 4a \sin(dx + c) + \frac{2a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")

[Out] -1/4*(a*log(sin(d*x + c) + 1) - 5*a*log(sin(d*x + c) - 1) - 4*a*sin(d*x + c) + 2*a/(sin(d*x + c) - 1))/d

Fricas [A] time = 1.53161, size = 224, normalized size = 3.15

$$\frac{4a \cos(dx + c)^2 + (a \sin(dx + c) - a) \log(\sin(dx + c) + 1) - 5(a \sin(dx + c) - a) \log(-\sin(dx + c) + 1) + 4a \sin(dx + c) - 2a}{4(d \sin(dx + c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")

[Out] -1/4*(4*a*cos(d*x + c)^2 + (a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - 5*(a*sin(d*x + c) - a)*log(-sin(d*x + c) + 1) + 4*a*sin(d*x + c) - 2*a)/(d*sin(d*x + c) - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \tan^3(c + dx) dx + \int \tan^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.3 $\int (a + a \sin(c + dx)) \tan(c + dx) dx$

Optimal. Leaf size=30

$$-\frac{a \sin(c + dx)}{d} - \frac{a \log(1 - \sin(c + dx))}{d}$$

[Out] $-\frac{(a \log[1 - \sin[c + d*x]])}{d} - \frac{(a \sin[c + d*x])}{d}$

Rubi [A] time = 0.0212762, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2707, 43}

$$-\frac{a \sin(c + dx)}{d} - \frac{a \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x],x]

[Out] $-\frac{(a \log[1 - \sin[c + d*x]])}{d} - \frac{(a \sin[c + d*x])}{d}$

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx)) \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x}{a-x} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-1 + \frac{a}{a-x}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{a \log(1 - \sin(c + dx))}{d} - \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0212587, size = 38, normalized size = 1.27

$$-\frac{a \sin(c + dx)}{d} + \frac{a \tanh^{-1}(\sin(c + dx))}{d} - \frac{a \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x],x]

[Out] $(a*\text{ArcTanh}[\text{Sin}[c + d*x]])/d - (a*\text{Log}[\text{Cos}[c + d*x]])/d - (a*\text{Sin}[c + d*x])/d$

Maple [A] time = 0.031, size = 29, normalized size = 1.

$$-\frac{a \sin(dx + c)}{d} - \frac{a \ln(\sin(dx + c) - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(d*x+c))*tan(d*x+c),x)`

[Out] `-a*sin(d*x+c)/d-1/d*a*ln(sin(d*x+c)-1)`

Maxima [A] time = 1.07582, size = 34, normalized size = 1.13

$$-\frac{a \log(\sin(dx + c) - 1) + a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")`

[Out] `-(a*log(sin(d*x + c) - 1) + a*sin(d*x + c))/d`

Fricas [A] time = 1.37141, size = 65, normalized size = 2.17

$$-\frac{a \log(-\sin(dx + c) + 1) + a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")`

[Out] `-(a*log(-sin(d*x + c) + 1) + a*sin(d*x + c))/d`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \tan(c + dx) dx + \int \tan(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))*tan(d*x+c),x)`

[Out] `a*(Integral(sin(c + d*x)*tan(c + d*x), x) + Integral(tan(c + d*x), x))`

Giac [B] time = 1.61062, size = 1966, normalized size = 65.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")

[Out]
$$-1/2*(a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^2 - a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^2 + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2 - 4*a*\tan(1/2*d*x)^2*\tan(1/2*c) + a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(1/2*c)^2 - a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(1/2*c)^2 + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*c)^2 - 4*a*\tan(1/2*d*x)*\tan(1/2*c)^2 + a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) - a*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 4*a*\tan(1/2*d*x) + 4*a*\tan(1/2*c))/(d*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(1/2*d*x)^2 + d*\tan(1/2*c)^2 + d)$$

3.4 $\int \cot(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{a \sin(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d}$$

[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d

Rubi [A] time = 0.0202604, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2707, 43}

$$\frac{a \sin(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]

[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + a \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{a}{x}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{a \log(\sin(c + dx))}{d} + \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0331529, size = 26, normalized size = 1.08

$$\frac{a(\sin(c + dx) + \log(\tan(c + dx)) + \log(\cos(c + dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]

[Out] $(a*(\text{Log}[\text{Cos}[c + d*x]] + \text{Log}[\text{Tan}[c + d*x]] + \text{Sin}[c + d*x]))/d$

Maple [A] time = 0.016, size = 25, normalized size = 1.

$$\frac{a \ln(\sin(dx + c))}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+a*sin(d*x+c)),x)`

[Out] $a*\ln(\sin(d*x+c))/d+a*\sin(d*x+c)/d$

Maxima [A] time = 1.09179, size = 30, normalized size = 1.25

$$\frac{a \log(\sin(dx + c)) + a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")`

[Out] $(a*\log(\sin(d*x + c)) + a*\sin(d*x + c))/d$

Fricas [A] time = 1.57343, size = 62, normalized size = 2.58

$$\frac{a \log\left(\frac{1}{2} \sin(dx + c)\right) + a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")`

[Out] $(a*\log(1/2*\sin(d*x + c)) + a*\sin(d*x + c))/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \cot(c + dx) dx + \int \cot(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x)`

[Out] $a*(\text{Integral}(\sin(c + d*x)*\cot(c + d*x), x) + \text{Integral}(\cot(c + d*x), x))$

Giac [A] time = 1.26787, size = 31, normalized size = 1.29

$$\frac{a \log(|\sin(dx + c)|) + a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```


3.5 $\int \cot^3(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=54

$$-\frac{a \sin(c + dx)}{d} - \frac{a \csc^2(c + dx)}{2d} - \frac{a \csc(c + dx)}{d} - \frac{a \log(\sin(c + dx))}{d}$$

[Out] $-\frac{(a \operatorname{Csc}[c + d*x])}{d} - \frac{(a \operatorname{Csc}[c + d*x]^2)}{(2*d)} - \frac{(a \operatorname{Log}[\operatorname{Sin}[c + d*x]])}{d} - \frac{(a \operatorname{Sin}[c + d*x])}{d}$

Rubi [A] time = 0.037076, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 75}

$$-\frac{a \sin(c + dx)}{d} - \frac{a \csc^2(c + dx)}{2d} - \frac{a \csc(c + dx)}{d} - \frac{a \log(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cot}[c + d*x]^3*(a + a*\operatorname{Sin}[c + d*x]), x]$

[Out] $-\frac{(a \operatorname{Csc}[c + d*x])}{d} - \frac{(a \operatorname{Csc}[c + d*x]^2)}{(2*d)} - \frac{(a \operatorname{Log}[\operatorname{Sin}[c + d*x]])}{d} - \frac{(a \operatorname{Sin}[c + d*x])}{d}$

Rule 2707

$\operatorname{Int}[(a + (b \sin(e + f*x)))^m \tan(e + f*x)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/f, \operatorname{Subst}[\operatorname{Int}[(x^p (a + x)^{m - (p + 1)/2}) / (a - x)^{(p + 1)/2}, x], x, b \operatorname{Sin}[e + f*x]], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, m\}, x \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{IntegerQ}[(p + 1)/2]$

Rule 75

$\operatorname{Int}[(d*x)^n (a + b*x) (e + f*x)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x) (d*x)^n (e + f*x)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, d, e, f, n\}, x \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{EqQ}[b*e + a*f, 0] \&\& \operatorname{!(ILtQ}[n + p + 2, 0] \&\& \operatorname{GtQ}[n + 2*p, 0])$

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + a \sin(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-1 + \frac{a^3}{x^3} + \frac{a^2}{x^2} - \frac{a}{x}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{a \csc(c + dx)}{d} - \frac{a \csc^2(c + dx)}{2d} - \frac{a \log(\sin(c + dx))}{d} - \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.117517, size = 60, normalized size = 1.11

$$-\frac{a \sin(c + dx)}{d} - \frac{a \csc(c + dx)}{d} - \frac{a (\cot^2(c + dx) + 2 \log(\tan(c + dx)) + 2 \log(\cos(c + dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]

[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d

Maple [A] time = 0.037, size = 83, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^4}{d\sin(dx+c)} - \frac{(\cos(dx+c))^2\sin(dx+c)a}{d} - 2\frac{a\sin(dx+c)}{d} - \frac{a(\cot(dx+c))^2}{2d} - \frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c)),x)

[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)/d-1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d

Maxima [A] time = 1.08976, size = 61, normalized size = 1.13

$$\frac{2a\log(\sin(dx+c)) + 2a\sin(dx+c) + \frac{2a\sin(dx+c)+a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin(d*x + c)^2)/d

Fricas [A] time = 1.49611, size = 167, normalized size = 3.09

$$\frac{2(a\cos(dx+c)^2 - a)\log\left(\frac{1}{2}\sin(dx+c)\right) + 2(a\cos(dx+c)^2 - 2a)\sin(dx+c) - a}{2(d\cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin(c+dx)\cot^3(c+dx)dx + \int \cot^3(c+dx)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [A] time = 1.32525, size = 81, normalized size = 1.5

$$-\frac{2 a \log (|\sin (d x+c)|)+2 a \sin (d x+c)-\frac{3 a \sin (d x+c)^2-2 a \sin (d x+c)-a}{\sin (d x+c)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.6 $\int \cot^5(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=81

$$\frac{a \sin(c + dx)}{d} - \frac{a \csc^4(c + dx)}{4d} - \frac{a \csc^3(c + dx)}{3d} + \frac{a \csc^2(c + dx)}{d} + \frac{2a \csc(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d}$$

[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d

Rubi [A] time = 0.048033, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 88}

$$\frac{a \sin(c + dx)}{d} - \frac{a \csc^4(c + dx)}{4d} - \frac{a \csc^3(c + dx)}{3d} + \frac{a \csc^2(c + dx)}{d} + \frac{2a \csc(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]

[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + a \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{a^5}{x^5} + \frac{a^4}{x^4} - \frac{2a^3}{x^3} - \frac{2a^2}{x^2} + \frac{a}{x}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{2a \csc(c + dx)}{d} + \frac{a \csc^2(c + dx)}{d} - \frac{a \csc^3(c + dx)}{3d} - \frac{a \csc^4(c + dx)}{4d} + \frac{a \log(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.197611, size = 87, normalized size = 1.07

$$\frac{a \sin(c + dx)}{d} - \frac{a \csc^3(c + dx)}{3d} + \frac{2a \csc(c + dx)}{d} + \frac{a(-\cot^4(c + dx) + 2 \cot^2(c + dx) + 4 \log(\tan(c + dx)) + 4 \log(\cos(c + dx)))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]

[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c + d*x])/d

Maple [A] time = 0.036, size = 136, normalized size = 1.7

$$\frac{a(\cos(dx+c))^6}{3d(\sin(dx+c))^3} + \frac{a(\cos(dx+c))^6}{d\sin(dx+c)} + \frac{8a\sin(dx+c)}{3d} + \frac{\sin(dx+c)(\cos(dx+c))^4 a}{d} + \frac{4(\cos(dx+c))^2 \sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5*(a+a*sin(d*x+c)),x)

[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/4/d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d

Maxima [A] time = 1.08056, size = 93, normalized size = 1.15

$$\frac{12a \log(\sin(dx+c)) + 12a \sin(dx+c) + \frac{24a \sin(dx+c)^3 + 12a \sin(dx+c)^2 - 4a \sin(dx+c) - 3a}{\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d

Fricas [A] time = 1.57272, size = 292, normalized size = 3.6

$$\frac{12a \cos(dx+c)^2 - 12(a \cos(dx+c)^4 - 2a \cos(dx+c)^2 + a) \log\left(\frac{1}{2} \sin(dx+c)\right) - 4(3a \cos(dx+c)^4 - 12a \cos(dx+c)^2 + a)}{12(d \cos(dx+c)^4 - 2d \cos(dx+c)^2 + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \cot^5(c + dx) dx + \int \cot^5(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5*(a+a*sin(d*x+c)),x)

[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**5, x) + Integral(cot(c + d*x)**5, x))

Giac [A] time = 1.38853, size = 111, normalized size = 1.37

$$\frac{12 a \log(|\sin(dx + c)|) + 12 a \sin(dx + c) - \frac{25 a \sin(dx+c)^4 - 24 a \sin(dx+c)^3 - 12 a \sin(dx+c)^2 + 4 a \sin(dx+c) + 3 a}{\sin(dx+c)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin(d*x + c)^4)/d

3.7 $\int \cot^7(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=115

$$-\frac{a \sin(c + dx)}{d} - \frac{a \csc^6(c + dx)}{6d} - \frac{a \csc^5(c + dx)}{5d} + \frac{3a \csc^4(c + dx)}{4d} + \frac{a \csc^3(c + dx)}{d} - \frac{3a \csc^2(c + dx)}{2d} - \frac{3a \csc(c + dx)}{d}$$

[Out] $(-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d$

Rubi [A] time = 0.0594132, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 88}

$$-\frac{a \sin(c + dx)}{d} - \frac{a \csc^6(c + dx)}{6d} - \frac{a \csc^5(c + dx)}{5d} + \frac{3a \csc^4(c + dx)}{4d} + \frac{a \csc^3(c + dx)}{d} - \frac{3a \csc^2(c + dx)}{2d} - \frac{3a \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^7*(a + a*\text{Sin}[c + d*x]), x]$

[Out] $(-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m*\tan(e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{((p + 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

$\text{Int}[(a + b*x)^m*((c + d*x)^n*(e + f*x)^p), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \cot^7(c + dx)(a + a \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-1 + \frac{a^7}{x^7} + \frac{a^6}{x^6} - \frac{3a^5}{x^5} - \frac{3a^4}{x^4} + \frac{3a^3}{x^3} + \frac{3a^2}{x^2} - \frac{a}{x}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{3a \csc(c + dx)}{d} - \frac{3a \csc^2(c + dx)}{2d} + \frac{a \csc^3(c + dx)}{d} + \frac{3a \csc^4(c + dx)}{4d} - \frac{a \csc^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.398755, size = 111, normalized size = 0.97

$$\frac{a \sin(c + dx)}{d} - \frac{a \csc^5(c + dx)}{5d} + \frac{a \csc^3(c + dx)}{d} - \frac{3a \csc(c + dx)}{d} - \frac{a(2 \cot^6(c + dx) - 3 \cot^4(c + dx) + 6 \cot^2(c + dx) - 3)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]

[Out] $(-3*a*\text{Csc}[c + d*x])/d + (a*\text{Csc}[c + d*x]^3)/d - (a*\text{Csc}[c + d*x]^5)/(5*d) - (a*(6*\text{Cot}[c + d*x]^2 - 3*\text{Cot}[c + d*x]^4 + 2*\text{Cot}[c + d*x]^6 + 12*\text{Log}[\text{Cos}[c + d*x]] + 12*\text{Log}[\text{Tan}[c + d*x]]))/(12*d) - (a*\text{Sin}[c + d*x])/d$

Maple [A] time = 0.039, size = 195, normalized size = 1.7

$$-\frac{a(\cos(dx+c))^8}{5d(\sin(dx+c))^5} + \frac{a(\cos(dx+c))^8}{5d(\sin(dx+c))^3} - \frac{a(\cos(dx+c))^8}{d\sin(dx+c)} - \frac{16a\sin(dx+c)}{5d} - \frac{\sin(dx+c)(\cos(dx+c))^6 a}{d} - \frac{6\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c)),x)

[Out] $-1/5/d*a/\sin(d*x+c)^5*\cos(d*x+c)^8+1/5/d*a/\sin(d*x+c)^3*\cos(d*x+c)^8-1/d*a/\sin(d*x+c)*\cos(d*x+c)^8-16/5*a*\sin(d*x+c)/d-1/d*\cos(d*x+c)^6*\sin(d*x+c)*a-6/5/d*\cos(d*x+c)^4*\sin(d*x+c)*a-8/5/d*\cos(d*x+c)^2*\sin(d*x+c)*a-1/6/d*a*\cot(d*x+c)^6+1/4/d*a*\cot(d*x+c)^4-1/2/d*a*\cot(d*x+c)^2-a*\ln(\sin(d*x+c))/d$

Maxima [A] time = 1.15296, size = 123, normalized size = 1.07

$$\frac{60a \log(\sin(dx+c)) + 60a \sin(dx+c) + \frac{180a \sin(dx+c)^5 + 90a \sin(dx+c)^4 - 60a \sin(dx+c)^3 - 45a \sin(dx+c)^2 + 12a \sin(dx+c) + 10a}{\sin(dx+c)^6}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/60*(60*a*\log(\sin(d*x + c)) + 60*a*\sin(d*x + c) + (180*a*\sin(d*x + c)^5 + 90*a*\sin(d*x + c)^4 - 60*a*\sin(d*x + c)^3 - 45*a*\sin(d*x + c)^2 + 12*a*\sin(d*x + c) + 10*a)/\sin(d*x + c)^6)/d$

Fricas [A] time = 1.59933, size = 412, normalized size = 3.58

$$\frac{90a \cos(dx+c)^4 - 135a \cos(dx+c)^2 - 60(a \cos(dx+c)^6 - 3a \cos(dx+c)^4 + 3a \cos(dx+c)^2 - a) \log\left(\frac{1}{2} \sin(dx+c)\right)}{60(d \cos(dx+c)^6 - 3d \cos(dx+c)^4 + 3d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $1/60*(90*a*\cos(d*x + c)^4 - 135*a*\cos(d*x + c)^2 - 60*(a*\cos(d*x + c)^6 - 3*a*\cos(d*x + c)^4 + 3*a*\cos(d*x + c)^2 - a)*\log(1/2*\sin(d*x + c)) - 12*(5*a*\cos(d*x + c)^6 - 30*a*\cos(d*x + c)^4 + 40*a*\cos(d*x + c)^2 - 16*a)*\sin(d*x + c) + 55*a)/(d*\cos(d*x + c)^6 - 3*d*\cos(d*x + c)^4 + 3*d*\cos(d*x + c)^2 - d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.38698, size = 140, normalized size = 1.22

$$60 a \log(|\sin(dx + c)|) + 60 a \sin(dx + c) - \frac{147 a \sin(dx+c)^6 - 180 a \sin(dx+c)^5 - 90 a \sin(dx+c)^4 + 60 a \sin(dx+c)^3 + 45 a \sin(dx+c)^2 - 12 a \sin(dx+c) - 10 a}{\sin(dx+c)^6} \cdot \frac{1}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out]
$$-1/60*(60*a*\log(\text{abs}(\sin(d*x + c))) + 60*a*\sin(d*x + c) - (147*a*\sin(d*x + c)^6 - 180*a*\sin(d*x + c)^5 - 90*a*\sin(d*x + c)^4 + 60*a*\sin(d*x + c)^3 + 45*a*\sin(d*x + c)^2 - 12*a*\sin(d*x + c) - 10*a)/\sin(d*x + c)^6)/d$$

3.8 $\int (a + a \sin(c + dx)) \tan^6(c + dx) dx$

Optimal. Leaf size=101

$$\frac{a \cos(c + dx)}{d} + \frac{a \tan^5(c + dx)}{5d} - \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec^5(c + dx)}{5d} - \frac{a \sec^3(c + dx)}{d} + \frac{3a \sec(c + dx)}{d} - a$$

[Out] $-(a*x) + (a*\text{Cos}[c + d*x])/d + (3*a*\text{Sec}[c + d*x])/d - (a*\text{Sec}[c + d*x]^3)/d + (a*\text{Sec}[c + d*x]^5)/(5*d) + (a*\text{Tan}[c + d*x])/d - (a*\text{Tan}[c + d*x]^3)/(3*d) + (a*\text{Tan}[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.0922464, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2710, 3473, 8, 2590, 270}

$$\frac{a \cos(c + dx)}{d} + \frac{a \tan^5(c + dx)}{5d} - \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec^5(c + dx)}{5d} - \frac{a \sec^3(c + dx)}{d} + \frac{3a \sec(c + dx)}{d} - a$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])*\text{Tan}[c + d*x]^6, x]$

[Out] $-(a*x) + (a*\text{Cos}[c + d*x])/d + (3*a*\text{Sec}[c + d*x])/d - (a*\text{Sec}[c + d*x]^3)/d + (a*\text{Sec}[c + d*x]^5)/(5*d) + (a*\text{Tan}[c + d*x])/d - (a*\text{Tan}[c + d*x]^3)/(3*d) + (a*\text{Tan}[c + d*x]^5)/(5*d)$

Rule 2710

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * (g*\text{tan}[e + f*x] + h)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g*\text{Tan}[e + f*x])^p, (a + b*\text{Sin}[e + f*x])^m, x], x] /;$ FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

$\text{Int}[(b*\text{tan}[c + d*x])^n, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{n-1})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

$\text{Int}[a*x, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rule 2590

$\text{Int}[\text{sin}[e + f*x]^m * \text{tan}[e + f*x]^n, x_Symbol] \rightarrow -\text{Dist}[f^{-1}, \text{Subst}[\text{Int}[(1 - x^2)^{(m+n-1)/2}/x^n, x], x, \text{Cos}[e + f*x]], x] /;$ FreeQ[{e, f}, x] && IntegersQ[m, n, (m+n-1)/2]

Rule 270

$\text{Int}[(c*x)^m * (a + b*x^n)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m * (a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx)) \tan^6(c + dx) dx &= \int (a \tan^6(c + dx) + a \sin(c + dx) \tan^6(c + dx)) dx \\
&= a \int \tan^6(c + dx) dx + a \int \sin(c + dx) \tan^6(c + dx) dx \\
&= \frac{a \tan^5(c + dx)}{5d} - a \int \tan^4(c + dx) dx - \frac{a \operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^6} dx, x, \cos(c + dx)\right)}{d} \\
&= -\frac{a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d} + a \int \tan^2(c + dx) dx - \frac{a \operatorname{Subst}\left(\int (-1 + \right.}{d} \\
&= \frac{a \cos(c + dx)}{d} + \frac{3a \sec(c + dx)}{d} - \frac{a \sec^3(c + dx)}{d} + \frac{a \sec^5(c + dx)}{5d} + \frac{a \tan(c + dx)}{d} \\
&= -ax + \frac{a \cos(c + dx)}{d} + \frac{3a \sec(c + dx)}{d} - \frac{a \sec^3(c + dx)}{d} + \frac{a \sec^5(c + dx)}{5d} + \frac{a \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0654162, size = 110, normalized size = 1.09

$$\frac{a \cos(c + dx)}{d} + \frac{a \tan^5(c + dx)}{5d} - \frac{a \tan^3(c + dx)}{3d} - \frac{a \tan^{-1}(\tan(c + dx))}{d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec^5(c + dx)}{5d} - \frac{a \sec^3(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]

[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d + (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)

Maple [A] time = 0.071, size = 135, normalized size = 1.3

$$\frac{1}{d} \left(a \left(\frac{(\sin(dx + c))^8}{5 (\cos(dx + c))^5} - \frac{(\sin(dx + c))^8}{5 (\cos(dx + c))^3} + \frac{(\sin(dx + c))^8}{\cos(dx + c)} + \left(\frac{16}{5} + (\sin(dx + c))^6 + \frac{6 (\sin(dx + c))^4}{5} + \frac{8 (\sin(dx + c))^2}{5} \right) \cos(dx + c) \right) + a \left(\frac{1}{5} \tan(dx + c)^5 - \frac{1}{3} \tan(dx + c)^3 + \tan(dx + c) - dx - c \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))*tan(d*x+c)^6,x)

[Out] 1/d*(a*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c)

Maxima [A] time = 1.65939, size = 117, normalized size = 1.16

$$\frac{(3 \tan(dx + c)^5 - 5 \tan(dx + c)^3 - 15 dx - 15 c + 15 \tan(dx + c))a + 3 a \left(\frac{15 \cos(dx+c)^4 - 5 \cos(dx+c)^2 + 1}{\cos(dx+c)^5} + 5 \cos(dx + c) \right)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")

[Out] $\frac{1}{15} * ((3 * \tan(dx + c)^5 - 5 * \tan(dx + c)^3 - 15 * dx - 15 * c + 15 * \tan(dx + c)) * a + 3 * a * ((15 * \cos(dx + c)^4 - 5 * \cos(dx + c)^2 + 1) / \cos(dx + c)^5 + 5 * \cos(dx + c))) / d$

Fricas [A] time = 1.4669, size = 300, normalized size = 2.97

$$\frac{15 \, a \, dx \, \cos(dx + c)^3 - 38 \, a \, \cos(dx + c)^4 - 11 \, a \, \cos(dx + c)^2 - (15 \, a \, dx \, \cos(dx + c)^3 - 15 \, a \, \cos(dx + c)^4 - 22 \, a \, \cos(dx + c)^2 + 4 \, a) * \sin(dx + c) + a}{15 (d \, \cos(dx + c)^3 \, \sin(dx + c) - d \, \cos(dx + c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(dx+c))*tan(dx+c)^6,x, algorithm="fricas")`

[Out] $\frac{1}{15} * (15 * a * dx * \cos(dx + c)^3 - 38 * a * \cos(dx + c)^4 - 11 * a * \cos(dx + c)^2 - (15 * a * dx * \cos(dx + c)^3 - 15 * a * \cos(dx + c)^4 - 22 * a * \cos(dx + c)^2 + 4 * a) * \sin(dx + c) + a) / (d * \cos(dx + c)^3 * \sin(dx + c) - d * \cos(dx + c)^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(dx+c))*tan(dx+c)**6,x)`

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(dx+c))*tan(dx+c)^6,x, algorithm="giac")`

[Out] Timed out

3.9 $\int (a + a \sin(c + dx)) \tan^4(c + dx) dx$

Optimal. Leaf size=72

$$-\frac{a \cos(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + \frac{a \sec^3(c + dx)}{3d} - \frac{2a \sec(c + dx)}{d} + ax$$

[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0743084, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2710, 3473, 8, 2590, 270}

$$-\frac{a \cos(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + \frac{a \sec^3(c + dx)}{3d} - \frac{2a \sec(c + dx)}{d} + ax$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]

[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2590

Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx)) \tan^4(c + dx) dx &= \int (a \tan^4(c + dx) + a \sin(c + dx) \tan^4(c + dx)) dx \\
&= a \int \tan^4(c + dx) dx + a \int \sin(c + dx) \tan^4(c + dx) dx \\
&= \frac{a \tan^3(c + dx)}{3d} - a \int \tan^2(c + dx) dx - \frac{a \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4} dx, x, \cos(c + dx)\right)}{d} \\
&= -\frac{a \tan(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d} + a \int 1 dx - \frac{a \operatorname{Subst}\left(\int \left(1 + \frac{1}{x^4} - \frac{2}{x^2}\right) dx, x, \cos(c + dx)\right)}{d} \\
&= ax - \frac{a \cos(c + dx)}{d} - \frac{2a \sec(c + dx)}{d} + \frac{a \sec^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0483714, size = 81, normalized size = 1.12

$$-\frac{a \cos(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan^{-1}(\tan(c + dx))}{d} - \frac{a \tan(c + dx)}{d} + \frac{a \sec^3(c + dx)}{3d} - \frac{2a \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]

[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Maple [A] time = 0.057, size = 98, normalized size = 1.4

$$\frac{1}{d} \left(a \left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^6}{\cos(dx+c)} - \left(\frac{8}{3} + (\sin(dx+c))^4 + \frac{4(\sin(dx+c))^2}{3} \right) \cos(dx+c) \right) + a \left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))*tan(d*x+c)^4,x)

[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))

Maxima [A] time = 1.82471, size = 88, normalized size = 1.22

$$\frac{(\tan(dx+c)^3 + 3dx + 3c - 3 \tan(dx+c))a - a \left(\frac{6 \cos(dx+c)^2 - 1}{\cos(dx+c)^3} + 3 \cos(dx+c) \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")

[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d

Fricas [A] time = 1.4761, size = 224, normalized size = 3.11

$$\frac{3 a d x \cos (d x+c)-7 a \cos (d x+c)^2-\left(3 a d x \cos (d x+c)-3 a \cos (d x+c)^2-2 a\right) \sin (d x+c)-a}{3(d \cos (d x+c) \sin (d x+c)-d \cos (d x+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")

[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin (c+d x) \tan ^4(c+d x) d x+\int \tan ^4(c+d x) d x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**4,x)

[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x))

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

3.10 $\int (a + a \sin(c + dx)) \tan^2(c + dx) dx$

Optimal. Leaf size=39

$$\frac{a \cos(c + dx)}{d} + \frac{a \cos(c + dx)}{d(1 - \sin(c + dx))} - ax$$

[Out] $-(a*x) + (a*\text{Cos}[c + d*x])/d + (a*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x]))$

Rubi [A] time = 0.104128, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2708, 2746, 12, 2735, 2648}

$$\frac{a \cos(c + dx)}{d} + \frac{a \cos(c + dx)}{d(1 - \sin(c + dx))} - ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])* \text{Tan}[c + d*x]^2, x]$

[Out] $-(a*x) + (a*\text{Cos}[c + d*x])/d + (a*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x]))$

Rule 2708

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * \text{tan}[e + f*x]^p, x] \text{ :> } \text{Dist}[a^p, \text{Int}[\text{Sin}[e + f*x]^p / (a - b*\text{Sin}[e + f*x])^m, x], x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[p, 2*m]

Rule 2746

$\text{Int}[(a + b*\text{sin}[e + f*x])^2 / (c + d*\text{sin}[e + f*x]), x] \text{ :> } -\text{Simp}[b^2*\text{Cos}[e + f*x] / (d*f), x] + \text{Dist}[1/d, \text{Int}[\text{Simp}[a^2*d - b*(b*c - 2*a*d)*\text{Sin}[e + f*x], x] / (c + d*\text{Sin}[e + f*x]), x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 12

$\text{Int}[a*(u), x] \text{ :> } \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b)*(v)] /; FreeQ[b, x]

Rule 2735

$\text{Int}[(a + b*\text{sin}[e + f*x]) / (c + d*\text{sin}[e + f*x]), x] \text{ :> } \text{Simp}[(b*x)/d, x] - \text{Dist}[(b*c - a*d)/d, \text{Int}[1/(c + d*\text{Sin}[e + f*x]), x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2648

$\text{Int}[(a + b*\text{sin}[c + d*x])^{-1}, x] \text{ :> } -\text{Simp}[\text{Cos}[c + d*x] / (d*(b + a*\text{Sin}[c + d*x])), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx)) \tan^2(c + dx) dx &= a^2 \int \frac{\sin^2(c + dx)}{a - a \sin(c + dx)} dx \\
&= \frac{a \cos(c + dx)}{d} + a \int \frac{a \sin(c + dx)}{a - a \sin(c + dx)} dx \\
&= \frac{a \cos(c + dx)}{d} + a^2 \int \frac{\sin(c + dx)}{a - a \sin(c + dx)} dx \\
&= -ax + \frac{a \cos(c + dx)}{d} + a^2 \int \frac{1}{a - a \sin(c + dx)} dx \\
&= -ax + \frac{a \cos(c + dx)}{d} + \frac{a^2 \cos(c + dx)}{d(a - a \sin(c + dx))}
\end{aligned}$$

Mathematica [A] time = 0.0452538, size = 47, normalized size = 1.21

$$\frac{a \cos(c + dx)}{d} - \frac{a \tan^{-1}(\tan(c + dx))}{d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]

[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d

Maple [A] time = 0.039, size = 59, normalized size = 1.5

$$\frac{1}{d} \left(a \left(\frac{(\sin(dx + c))^4}{\cos(dx + c)} + (2 + (\sin(dx + c))^2) \cos(dx + c) \right) + a(\tan(dx + c) - dx - c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))*tan(d*x+c)^2,x)

[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-d*x-c))

Maxima [A] time = 1.68792, size = 53, normalized size = 1.36

$$-\frac{(dx + c - \tan(dx + c))a - a\left(\frac{1}{\cos(dx+c)} + \cos(dx + c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d

Fricas [B] time = 1.42136, size = 194, normalized size = 4.97

$$\frac{adx - a \cos(dx + c)^2 + (adx - 2a) \cos(dx + c) - (adx - a \cos(dx + c) + a) \sin(dx + c) - a}{d \cos(dx + c) - d \sin(dx + c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")

[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \tan^2(c + dx) dx + \int \tan^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**2,x)

[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))

Giac [B] time = 3.47197, size = 1361, normalized size = 34.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")

[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)^3 - 8*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*a*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*a*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a*tan(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*a*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*tan(1/2*d*x)^3*tan(1/2*c) + 24*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*tan(1/2*d*x)*tan(1/2*c)^3 + 2*a*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)

$$\begin{aligned}
& c) \tan(c) - a \cdot d \cdot x - 8 \cdot a \cdot \tan(1/2 \cdot d \cdot x) \cdot \tan(1/2 \cdot c) - 2 \cdot a \cdot \tan(d \cdot x) \cdot \tan(c) + a \cdot \tan(d \cdot x) + a \cdot \tan(c) + 2 \cdot a / (d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x)^4 \cdot \tan(1/2 \cdot c)^4 \cdot \tan(c) - \\
& d \cdot \tan(1/2 \cdot d \cdot x)^4 \cdot \tan(1/2 \cdot c)^4 - 4 \cdot d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x)^3 \cdot \tan(1/2 \cdot c)^3 \cdot \tan(c) + 4 \cdot d \cdot \tan(1/2 \cdot d \cdot x)^3 \cdot \tan(1/2 \cdot c)^3 - d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x)^4 \cdot \tan(c) - \\
& 4 \cdot d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x)^3 \cdot \tan(1/2 \cdot c) \cdot \tan(c) - 4 \cdot d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x) \cdot \tan(1/2 \cdot c)^3 \cdot \tan(c) - d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot c)^4 \cdot \tan(c) + d \cdot \tan(1/2 \cdot d \cdot x)^4 + 4 \\
& \cdot d \cdot \tan(1/2 \cdot d \cdot x)^3 \cdot \tan(1/2 \cdot c) + 4 \cdot d \cdot \tan(1/2 \cdot d \cdot x) \cdot \tan(1/2 \cdot c)^3 + d \cdot \tan(1/2 \cdot c)^4 - 4 \cdot d \cdot \tan(d \cdot x) \cdot \tan(1/2 \cdot d \cdot x) \cdot \tan(1/2 \cdot c) \cdot \tan(c) + 4 \cdot d \cdot \tan(1/2 \cdot d \cdot x) \cdot \tan(1/2 \\
& \cdot c) + d \cdot \tan(d \cdot x) \cdot \tan(c) - d)
\end{aligned}$$

3.11 $\int \cot^2(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=41

$$\frac{a \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} - \frac{a \tanh^{-1}(\cos(c + dx))}{d} - ax$$

[Out] $-(a*x) - (a*\text{ArcTanh}[\text{Cos}[c + d*x]])/d + (a*\text{Cos}[c + d*x])/d - (a*\text{Cot}[c + d*x])/d$

Rubi [A] time = 0.051557, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2710, 2592, 321, 206, 3473, 8}

$$\frac{a \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} - \frac{a \tanh^{-1}(\cos(c + dx))}{d} - ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^2*(a + a*\text{Sin}[c + d*x]), x]$

[Out] $-(a*x) - (a*\text{ArcTanh}[\text{Cos}[c + d*x]])/d + (a*\text{Cos}[c + d*x])/d - (a*\text{Cot}[c + d*x])/d$

Rule 2710

$\text{Int}[(a + (b \sin(e + f x))^m) \tan(e + f x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g \tan(e + f x))^p, (a + b \sin(e + f x))^m, x], x] /;$ FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

$\text{Int}[(a \sin(e + f x))^m \tan(e + f x)^n, x] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(ff x)^{m+n}/(a^2 - ff^2 x^2)^{(n+1)/2}, x], x, (a \sin(e + f x))/ff], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

$\text{Int}[(c + (a + b x^n)^p)^m, x] \rightarrow \text{Simp}[(c^{n-1} (c x)^{m-n+1} (a + b x^n)^{p+1}) / (b(m + n p + 1)), x] - \text{Dist}[(a c^{n-1} (m - n + 1)) / (b(m + n p + 1)), \text{Int}[(c x)^{m-n} (a + b x^n)^p, x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

$\text{Int}[(a + b x^2)^{-1}, x] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

$\text{Int}[(b \tan(c + d x))^n, x] \rightarrow \text{Simp}[(b * (b \tan(c + d x))^{n-1}) / (d * (n - 1)), x] - \text{Dist}[b^2, \text{Int}[(b \tan(c + d x))^{n-2}, x],$

$x] /; \text{FreeQ}\{b, c, d\}, x\} \&\& \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + a \sin(c + dx)) dx &= \int (a \cos(c + dx) \cot(c + dx) + a \cot^2(c + dx)) dx \\ &= a \int \cos(c + dx) \cot(c + dx) dx + a \int \cot^2(c + dx) dx \\ &= -\frac{a \cot(c + dx)}{d} - a \int 1 dx - \frac{a \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -ax + \frac{a \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} - \frac{a \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -ax - \frac{a \tanh^{-1}(\cos(c + dx))}{d} + \frac{a \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} \end{aligned}$$

Mathematica [C] time = 0.0480103, size = 75, normalized size = 1.83

$$-\frac{a \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right)}{d} + \frac{a \cos(c + dx)}{d} + \frac{a \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} - \frac{a \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]

[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d

Maple [A] time = 0.029, size = 57, normalized size = 1.4

$$-ax + \frac{\cos(dx + c) a}{d} - \frac{a \cot(dx + c)}{d} + \frac{a \ln(\csc(dx + c) - \cot(dx + c))}{d} - \frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c)),x)

[Out] -a*x+a*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a

Maxima [A] time = 2.30272, size = 73, normalized size = 1.78

$$\frac{2\left(dx + c + \frac{1}{\tan(dx+c)}\right)a - a(2 \cos(dx + c) - \log(\cos(dx + c) + 1) + \log(\cos(dx + c) - 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/2*(2*(d*x + c + 1/\tan(d*x + c))*a - a*(2*\cos(d*x + c) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)))/d$

Fricas [B] time = 1.43844, size = 236, normalized size = 5.76

$$\frac{a \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - a \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + 2a \cos(dx + c) + 2(adx - a \cos(dx + c))}{2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")`

[Out] $-1/2*(a*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - a*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 2*a*\cos(d*x + c) + 2*(a*d*x - a*\cos(d*x + c))*\sin(d*x + c))/(d*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \sin(c + dx) \cot^2(c + dx) dx + \int \cot^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+a*sin(d*x+c)),x)`

[Out] `a*(Integral(sin(c + d*x)*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))`

Giac [B] time = 1.25007, size = 146, normalized size = 3.56

$$\frac{6(dx + c)a - 6a \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - 3a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 10a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3a}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")`

[Out] $-1/6*(6*(d*x + c)*a - 6*a*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - 3*a*\tan(1/2*d*x + 1/2*c) + (2*a*\tan(1/2*d*x + 1/2*c)^3 + 3*a*\tan(1/2*d*x + 1/2*c)^2 - 10*a*\tan(1/2*d*x + 1/2*c) + 3*a)/(\tan(1/2*d*x + 1/2*c)^3 + \tan(1/2*d*x + 1/2*c)))/d$

3.12 $\int \cot^4(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=82

$$-\frac{3a \cos(c + dx)}{2d} - \frac{a \cot^3(c + dx)}{3d} + \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{3a \tanh^{-1}(\cos(c + dx))}{2d} + ax$$

[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0797018, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$-\frac{3a \cos(c + dx)}{2d} - \frac{a \cot^3(c + dx)}{3d} + \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{3a \tanh^{-1}(\cos(c + dx))}{2d} + ax$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]

[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + a \sin(c + dx)) dx &= \int (a \cos(c + dx) \cot^3(c + dx) + a \cot^4(c + dx)) dx \\
&= a \int \cos(c + dx) \cot^3(c + dx) dx + a \int \cot^4(c + dx) dx \\
&= -\frac{a \cot^3(c + dx)}{3d} - a \int \cot^2(c + dx) dx - \frac{a \operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2} dx, x, \cos(c + dx)\right)}{d} \\
&= \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^2(c + dx)}{2d} - \frac{a \cot^3(c + dx)}{3d} + a \int 1 dx + \frac{(3a) \operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2} dx, x, \cos(c + dx)\right)}{d} \\
&= ax - \frac{3a \cos(c + dx)}{2d} + \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^2(c + dx)}{2d} - \frac{a \cot^3(c + dx)}{3d} \\
&= ax + \frac{3a \tanh^{-1}(\cos(c + dx))}{2d} - \frac{3a \cos(c + dx)}{2d} + \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^2(c + dx)}{2d}
\end{aligned}$$

Mathematica [C] time = 0.0531241, size = 125, normalized size = 1.52

$$-\frac{a \cot^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c + dx)\right)}{3d} - \frac{a \cos(c + dx)}{d} - \frac{a \csc^2\left(\frac{1}{2}(c + dx)\right)}{8d} + \frac{a \sec^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{3a \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c + d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.034, size = 106, normalized size = 1.3

$$-\frac{a (\cos(dx + c))^5}{2d (\sin(dx + c))^2} - \frac{a (\cos(dx + c))^3}{2d} - \frac{3 \cos(dx + c) a}{2d} - \frac{3 a \ln(\csc(dx + c) - \cot(dx + c))}{2d} - \frac{a (\cot(dx + c))^3}{3d} + \frac{a \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c)),x)
```


[Out] $-1/2/d*a/\sin(d*x+c)^2*\cos(d*x+c)^5-1/2/d*\cos(d*x+c)^3*a-3/2*a*\cos(d*x+c)/d-3/2/d*a*\ln(\csc(d*x+c)-\cot(d*x+c))-1/3*a*\cot(d*x+c)^3/d+a*\cot(d*x+c)/d+a*x+1/d*c*a$

Maxima [A] time = 1.69055, size = 124, normalized size = 1.51

$$\frac{4\left(3dx + 3c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a + 3a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - 4\cos(dx+c) + 3\log(\cos(dx+c)+1) - 3\log(\cos(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] $1/12*(4*(3*d*x + 3*c + (3*\tan(d*x + c)^2 - 1)/\tan(d*x + c)^3)*a + 3*a*(2*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) - 4*\cos(d*x + c) + 3*\log(\cos(d*x + c) + 1) - 3*\log(\cos(d*x + c) - 1)))/d$

Fricas [B] time = 1.50448, size = 425, normalized size = 5.18

$$\frac{16a\cos(dx+c)^3 + 9(a\cos(dx+c)^2 - a)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 9(a\cos(dx+c)^2 - a)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 12a\cos(dx+c) + 6*(2*a*d*x*\cos(dx+c)^2 - 2*a*\cos(dx+c)^3 - 2*a*d*x + 3*a*\cos(dx+c))*\sin(dx+c)}{12(d\cos(dx+c) + \sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $1/12*(16*a*\cos(d*x + c)^3 + 9*(a*\cos(d*x + c)^2 - a)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 9*(a*\cos(d*x + c)^2 - a)*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 12*a*\cos(d*x + c) + 6*(2*a*d*x*\cos(d*x + c)^2 - 2*a*\cos(d*x + c)^3 - 2*a*d*x + 3*a*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c)^2 - d)*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin(c + dx) \cot^4(c + dx) dx + \int \cot^4(c + dx) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c)),x)

[Out] $a*(\text{Integral}(\sin(c + d*x)*\cot(c + d*x)**4, x) + \text{Integral}(\cot(c + d*x)**4, x))$

Giac [A] time = 1.27813, size = 190, normalized size = 2.32

$$a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24(dx+c)a - 36a \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - 15a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \frac{1}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.13 $\int \cot^6(c + dx)(a + a \sin(c + dx)) dx$

Optimal. Leaf size=122

$$\frac{15a \cos(c + dx)}{8d} - \frac{a \cot^5(c + dx)}{5d} + \frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{5a \cos(c + dx) \cot^2(c + dx)}{8d}$$

[Out] $-(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.0962739, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$\frac{15a \cos(c + dx)}{8d} - \frac{a \cot^5(c + dx)}{5d} + \frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{a \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{5a \cos(c + dx) \cot^2(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]

[Out] $-(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d)$

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p, 0]

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \cot^6(c + dx)(a + a \sin(c + dx)) dx &= \int (a \cos(c + dx) \cot^5(c + dx) + a \cot^6(c + dx)) dx \\
 &= a \int \cos(c + dx) \cot^5(c + dx) dx + a \int \cot^6(c + dx) dx \\
 &= -\frac{a \cot^5(c + dx)}{5d} - a \int \cot^4(c + dx) dx - \frac{a \operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^3} dx, x, \cos(c + dx)\right)}{d} \\
 &= \frac{a \cot^3(c + dx)}{3d} - \frac{a \cos(c + dx) \cot^4(c + dx)}{4d} - \frac{a \cot^5(c + dx)}{5d} + a \int \cot^2(c + dx) dx \\
 &= -\frac{a \cot(c + dx)}{d} + \frac{5a \cos(c + dx) \cot^2(c + dx)}{8d} + \frac{a \cot^3(c + dx)}{3d} - \frac{a \cos(c + dx) \cot^4(c + dx)}{4d} \\
 &= -ax + \frac{15a \cos(c + dx)}{8d} - \frac{a \cot(c + dx)}{d} + \frac{5a \cos(c + dx) \cot^2(c + dx)}{8d} + \frac{a \cot^3(c + dx)}{3d} \\
 &= -ax - \frac{15a \tanh^{-1}(\cos(c + dx))}{8d} + \frac{15a \cos(c + dx)}{8d} - \frac{a \cot(c + dx)}{d} + \frac{5a \cos(c + dx) \cot^2(c + dx)}{8d}
 \end{aligned}$$

Mathematica [C] time = 0.0710632, size = 164, normalized size = 1.34

$$-\frac{a \cot^5(c + dx) {}_2F_1\left(-\frac{5}{2}, 1; -\frac{3}{2}; -\tan^2(c + dx)\right)}{5d} + \frac{a \cos(c + dx)}{d} - \frac{a \csc^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{9a \csc^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{a \sec^4\left(\frac{1}{2}(c + dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]

[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)

Maple [A] time = 0.038, size = 159, normalized size = 1.3

$$\frac{a(\cos(dx+c))^7}{4d(\sin(dx+c))^4} + \frac{3a(\cos(dx+c))^7}{8d(\sin(dx+c))^2} + \frac{3(\cos(dx+c))^5 a}{8d} + \frac{5a(\cos(dx+c))^3}{8d} + \frac{15\cos(dx+c)a}{8d} + \frac{15a\ln(\csc(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c)),x)

[Out]
$$-1/4/d*a/\sin(d*x+c)^4*\cos(d*x+c)^7+3/8/d*a/\sin(d*x+c)^2*\cos(d*x+c)^7+3/8/d*\cos(d*x+c)^5*a+5/8/d*\cos(d*x+c)^3*a+15/8*a*\cos(d*x+c)/d+15/8/d*a*\ln(\csc(d*x+c)-\cot(d*x+c))-1/5*a*\cot(d*x+c)^5/d+1/3*a*\cot(d*x+c)^3/d-a*\cot(d*x+c)/d-a*x-1/d*c*a$$

Maxima [A] time = 1.63037, size = 169, normalized size = 1.39

$$\frac{16\left(15dx + 15c + \frac{15\tan(dx+c)^4 - 5\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a + 15a\left(\frac{2(9\cos(dx+c)^3 - 7\cos(dx+c))}{\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1} - 16\cos(dx+c) + 15\log(\cos(dx+c) + 1) - 15\log(\cos(dx+c) - 1)\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out]
$$-1/240*(16*(15*d*x + 15*c + (15*\tan(d*x + c)^4 - 5*\tan(d*x + c)^2 + 3)/\tan(d*x + c)^5)*a + 15*a*(2*(9*\cos(d*x + c)^3 - 7*\cos(d*x + c))/(\cos(d*x + c)^4 - 2*\cos(d*x + c)^2 + 1) - 16*\cos(d*x + c) + 15*\log(\cos(d*x + c) + 1) - 15*\log(\cos(d*x + c) - 1)))/d$$

Fricas [B] time = 1.53839, size = 620, normalized size = 5.08

$$\frac{368a\cos(dx+c)^5 - 560a\cos(dx+c)^3 + 225\left(a\cos(dx+c)^4 - 2a\cos(dx+c)^2 + a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out]
$$-1/240*(368*a*\cos(d*x + c)^5 - 560*a*\cos(d*x + c)^3 + 225*(a*\cos(d*x + c)^4 - 2*a*\cos(d*x + c)^2 + a)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 225*(a*\cos(d*x + c)^4 - 2*a*\cos(d*x + c)^2 + a)*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 240*a*\cos(d*x + c) + 30*(8*a*d*x*\cos(d*x + c)^4 - 8*a*\cos(d*x + c)^5 - 16*a*d*x*\cos(d*x + c)^2 + 25*a*\cos(d*x + c)^3 + 8*a*d*x - 15*a*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c)^4 - 2*d*\cos(d*x + c)^2 + d)*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29258, size = 269, normalized size = 2.2

$$6 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 15 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 70 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 240 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 960 (dx + c)a + 1800 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.14 $\int (a + a \sin(c + dx))^2 \tan^5(c + dx) dx$

Optimal. Leaf size=119

$$-\frac{a^2 \sin^2(c + dx)}{2d} + \frac{a^4}{4d(a - a \sin(c + dx))^2} - \frac{9a^3}{4d(a - a \sin(c + dx))} - \frac{2a^2 \sin(c + dx)}{d} - \frac{31a^2 \log(1 - \sin(c + dx))}{8d} - \frac{a^2}{8d}$$

[Out] $(-31*a^2*\text{Log}[1 - \text{Sin}[c + d*x]])/(8*d) - (a^2*\text{Log}[1 + \text{Sin}[c + d*x]])/(8*d) - (2*a^2*\text{Sin}[c + d*x])/d - (a^2*\text{Sin}[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*\text{Sin}[c + d*x])^2) - (9*a^3)/(4*d*(a - a*\text{Sin}[c + d*x]))$

Rubi [A] time = 0.0818173, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{a^2 \sin^2(c + dx)}{2d} + \frac{a^4}{4d(a - a \sin(c + dx))^2} - \frac{9a^3}{4d(a - a \sin(c + dx))} - \frac{2a^2 \sin(c + dx)}{d} - \frac{31a^2 \log(1 - \sin(c + dx))}{8d} - \frac{a^2}{8d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^2*\text{Tan}[c + d*x]^5, x]$

[Out] $(-31*a^2*\text{Log}[1 - \text{Sin}[c + d*x]])/(8*d) - (a^2*\text{Log}[1 + \text{Sin}[c + d*x]])/(8*d) - (2*a^2*\text{Sin}[c + d*x])/d - (a^2*\text{Sin}[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*\text{Sin}[c + d*x])^2) - (9*a^3)/(4*d*(a - a*\text{Sin}[c + d*x]))$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m*\tan(e + f*x)^p, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

$\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^2 \tan^5(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a + \frac{a^4}{2(a-x)^3} - \frac{9a^3}{4(a-x)^2} + \frac{31a^2}{8(a-x)} - x - \frac{a^2}{8(a+x)}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{31a^2 \log(1 - \sin(c + dx))}{8d} - \frac{a^2 \log(1 + \sin(c + dx))}{8d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{a^2}{8d} \end{aligned}$$

Mathematica [A] time = 0.244541, size = 75, normalized size = 0.63

$$\frac{a^2 \left(4 \sin^2(c + dx) + 16 \sin(c + dx) - \frac{18}{\sin(c + dx) - 1} - \frac{2}{(\sin(c + dx) - 1)^2} + 31 \log(1 - \sin(c + dx)) + \log(\sin(c + dx) + 1) \right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]

[Out] -(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/(8*d)

Maple [B] time = 0.085, size = 261, normalized size = 2.2

$$\frac{a^2 (\sin(dx+c))^8}{4d (\cos(dx+c))^4} - \frac{a^2 (\sin(dx+c))^8}{2d (\cos(dx+c))^2} - \frac{a^2 (\sin(dx+c))^6}{2d} - \frac{3a^2 (\sin(dx+c))^4}{4d} - \frac{3a^2 (\sin(dx+c))^2}{2d} - 4 \frac{a^2 \ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x)

[Out] 1/4/d*a^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^2*sin(d*x+c)^8/cos(d*x+c)^2-1/2/d*a^2*sin(d*x+c)^6-3/4/d*a^2*sin(d*x+c)^4-3/2*a^2*sin(d*x+c)^2/d-4/d*a^2*ln(cos(d*x+c))+1/2/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2-3/4/d*a^2*sin(d*x+c)^5-5/4/d*a^2*sin(d*x+c)^3-15/4*a^2*sin(d*x+c)/d+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2

Maxima [A] time = 1.12912, size = 130, normalized size = 1.09

$$\frac{4a^2 \sin(dx+c)^2 + a^2 \log(\sin(dx+c)+1) + 31a^2 \log(\sin(dx+c)-1) + 16a^2 \sin(dx+c) - \frac{2(9a^2 \sin(dx+c)-8a^2)}{\sin(dx+c)^2 - 2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")

[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d

Fricas [A] time = 1.55703, size = 412, normalized size = 3.46

$$\frac{4a^2 \cos(dx+c)^4 + 22a^2 \cos(dx+c)^2 - 12a^2 - (a^2 \cos(dx+c)^2 + 2a^2 \sin(dx+c) - 2a^2) \log(\sin(dx+c)+1) - 31(a^2 \cos(dx+c)^2 + 2a^2 \sin(dx+c) - 2a^2) \log(-\sin(dx+c)+1) - 2(4a^2 \cos(dx+c)^2 - 5a^2) \sin(dx+c)}{8(d \cos(dx+c))^2 + 2d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")

[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2

*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**5,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")

[Out] Timed out

3.15 $\int (a + a \sin(c + dx))^2 \tan^3(c + dx) dx$

Optimal. Leaf size=72

$$\frac{a^2 \sin^2(c + dx)}{2d} + \frac{a^3}{d(a - a \sin(c + dx))} + \frac{2a^2 \sin(c + dx)}{d} + \frac{3a^2 \log(1 - \sin(c + dx))}{d}$$

[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))

Rubi [A] time = 0.0621593, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 43}

$$\frac{a^2 \sin^2(c + dx)}{2d} + \frac{a^3}{d(a - a \sin(c + dx))} + \frac{2a^2 \sin(c + dx)}{d} + \frac{3a^2 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]

[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^2 \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(2a + \frac{a^3}{(a-x)^2} - \frac{3a^2}{a-x} + x\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{3a^2 \log(1 - \sin(c + dx))}{d} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \sin^2(c + dx)}{2d} + \frac{a^3}{d(a - a \sin(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.102539, size = 54, normalized size = 0.75

$$\frac{a^2 \left(\sin^2(c + dx) + 4 \sin(c + dx) + \frac{2}{1 - \sin(c + dx)} + 6 \log(1 - \sin(c + dx)) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]

[Out] (a^2*(6*Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]) + 4*Sin[c + d*x] + Sin[c + d*x]^2))/(2*d)

Maple [B] time = 0.071, size = 162, normalized size = 2.3

$$\frac{a^2 (\sin(dx + c))^6}{2d (\cos(dx + c))^2} + \frac{a^2 (\sin(dx + c))^4}{2d} + \frac{a^2 (\sin(dx + c))^2}{d} + 3 \frac{a^2 \ln(\cos(dx + c))}{d} + \frac{a^2 (\sin(dx + c))^5}{d (\cos(dx + c))^2} + \frac{a^2 (\sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x)

[Out] 1/2/d*a^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*a^2*sin(d*x+c)^4+a^2*sin(d*x+c)^2/d+3/d*a^2*ln(cos(d*x+c))+1/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2+1/d*a^2*sin(d*x+c)^3+3*a^2*sin(d*x+c)/d-3/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*tan(d*x+c)^2

Maxima [A] time = 1.11192, size = 78, normalized size = 1.08

$$\frac{a^2 \sin(dx + c)^2 + 6 a^2 \log(\sin(dx + c) - 1) + 4 a^2 \sin(dx + c) - \frac{2 a^2}{\sin(dx + c) - 1}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/2*(a^2*sin(d*x + c)^2 + 6*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c) - 2*a^2/(sin(d*x + c) - 1))/d

Fricas [A] time = 1.48194, size = 212, normalized size = 2.94

$$\frac{6 a^2 \cos(dx + c)^2 - 3 a^2 - 12 (a^2 \sin(dx + c) - a^2) \log(-\sin(dx + c) + 1) + (2 a^2 \cos(dx + c)^2 + 7 a^2) \sin(dx + c)}{4 (d \sin(dx + c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")

[Out] -1/4*(6*a^2*cos(d*x + c)^2 - 3*a^2 - 12*(a^2*sin(d*x + c) - a^2)*log(-sin(d*x + c) + 1) + (2*a^2*cos(d*x + c)^2 + 7*a^2)*sin(d*x + c))/(d*sin(d*x + c) - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c + dx) \tan^3(c + dx) dx + \int \sin^2(c + dx) \tan^3(c + dx) dx + \int \tan^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**2*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.16 $\int (a + a \sin(c + dx))^2 \tan(c + dx) dx$

Optimal. Leaf size=52

$$-\frac{a^2 \sin^2(c + dx)}{2d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{2a^2 \log(1 - \sin(c + dx))}{d}$$

[Out] $(-2*a^2*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (2*a^2*\text{Sin}[c + d*x])/d - (a^2*\text{Sin}[c + d*x]^2)/(2*d)$

Rubi [A] time = 0.0375399, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 77}

$$-\frac{a^2 \sin^2(c + dx)}{2d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{2a^2 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^2*\text{Tan}[c + d*x], x]$

[Out] $(-2*a^2*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (2*a^2*\text{Sin}[c + d*x])/d - (a^2*\text{Sin}[c + d*x]^2)/(2*d)$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m*\tan(e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ $\text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[(p + 1)/2]$

Rule 77

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ ((\text{ILtQ}[n, 0] \ \&\& \ \text{ILtQ}[p, 0]) \ || \ \text{EqQ}[p, 1] \ || \ (\text{IGtQ}[p, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ \text{LeQ}[9*p + 5*(n + 2), 0] \ || \ \text{GeQ}[n + p + 1, 0] \ || \ (\text{GeQ}[n + p + 2, 0] \ \&\& \ \text{RationalQ}[a, b, c, d, e, f])))$

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^2 \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{a+x}}{a-x} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a + \frac{2a^2}{a-x} - x\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{2a^2 \log(1 - \sin(c + dx))}{d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{a^2 \sin^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.038228, size = 40, normalized size = 0.77

$$\frac{a^2 (\sin^2(c + dx) + 4 \sin(c + dx) + 4 \log(1 - \sin(c + dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]

[Out] $-(a^2*(4*\text{Log}[1 - \text{Sin}[c + d*x]] + 4*\text{Sin}[c + d*x] + \text{Sin}[c + d*x]^2))/(2*d)$

Maple [A] time = 0.042, size = 69, normalized size = 1.3

$$-\frac{a^2 (\sin(dx + c))^2}{2d} - 2 \frac{a^2 \ln(\cos(dx + c))}{d} - 2 \frac{a^2 \sin(dx + c)}{d} + 2 \frac{a^2 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^2*tan(d*x+c),x)

[Out] $-1/2*a^2*\sin(d*x+c)^2/d-2/d*a^2*\ln(\cos(d*x+c))-2*a^2*\sin(d*x+c)/d+2/d*a^2*\ln(\sec(d*x+c)+\tan(d*x+c))$

Maxima [A] time = 1.12482, size = 58, normalized size = 1.12

$$\frac{a^2 \sin(dx + c)^2 + 4 a^2 \log(\sin(dx + c) - 1) + 4 a^2 \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")

[Out] $-1/2*(a^2*\sin(d*x + c)^2 + 4*a^2*\log(\sin(d*x + c) - 1) + 4*a^2*\sin(d*x + c))/d$

Fricas [A] time = 1.41834, size = 108, normalized size = 2.08

$$\frac{a^2 \cos(dx + c)^2 - 4 a^2 \log(-\sin(dx + c) + 1) - 4 a^2 \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")

[Out] $1/2*(a^2*\cos(d*x + c)^2 - 4*a^2*\log(-\sin(d*x + c) + 1) - 4*a^2*\sin(d*x + c))/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c + dx) \tan(c + dx) dx + \int \sin^2(c + dx) \tan(c + dx) dx + \int \tan(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x)

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x), x) + Integral(sin(c + d*x)**2*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] time = 4.57216, size = 9038, normalized size = 173.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*c)
```


$$\begin{aligned}
& *d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 \\
& + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(c)^2 - 4*a^2*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(c)^2 + 4*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(c)^2 + 16*a^2*\tan(1/2*d*x)*\tan(c)^2 + 16*a^2*\tan(1/2*c)*\tan(c)^2 + a^2*\tan(d*x)^2 - a^2*\tan(1/2*d*x)^2 - a^2*\tan(1/2*c)^2 + 4*a^2*\tan(d*x)*\tan(c) + a^2*\tan(c)^2 + 4*a^2*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) - 4*a^2*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) + 4*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 16*a^2*\tan(1/2*d*x) + 16*a^2*\tan(1/2*c) - a^2)/(d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(c)^2 + d*\tan(d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2 + d*\tan(d*x)^2*\tan(1/2*c)^2 + d*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(d*x)^2*\tan(c)^2 + d*\tan(1/2*d*x)^2*\tan(c)^2 + d*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2 + d*\tan(1/2*d*x)^2 + d*\tan(1/2*c)^2 + d*\tan(c)^2 + d)
\end{aligned}$$

3.17 $\int \cot^3(c + dx)(a + a \sin(c + dx))^2 dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c + dx)(a \sin(c + dx) + a)^4}{2a^2d}$$

[Out] $-(\text{Csc}[c + d*x]^2*(a + a*\text{Sin}[c + d*x])^4)/(2*a^2*d)$

Rubi [A] time = 0.0392531, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 74}

$$-\frac{\csc^2(c + dx)(a \sin(c + dx) + a)^4}{2a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^3*(a + a*\text{Sin}[c + d*x])^2, x]$

[Out] $-(\text{Csc}[c + d*x]^2*(a + a*\text{Sin}[c + d*x])^4)/(2*a^2*d)$

Rule 2707

$\text{Int}[(a + b*\sin[e + f*x])^m*\tan[e + f*x]^p, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[(p + 1)/2]$

Rule 74

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x_Symbol] := \text{Simp}[(b*(c + d*x)^{n + 1}*(e + f*x)^{p + 1})/(d*f*(n + p + 2)), x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x \ \&\& \ \text{NeQ}[n + p + 2, 0] \ \&\& \ \text{EqQ}[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]$

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + a \sin(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^3}{x^3} dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{\csc^2(c + dx)(a + a \sin(c + dx))^4}{2a^2d} \end{aligned}$$

Mathematica [A] time = 0.0412311, size = 28, normalized size = 0.93

$$-\frac{a^2(\sin(c + dx) + 1)^4 \csc^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cot}[c + d*x]^3*(a + a*\text{Sin}[c + d*x])^2, x]$

[Out] $-(a^2*\text{Csc}[c + d*x]^2*(1 + \text{Sin}[c + d*x])^4)/(2*d)$

Maple [B] time = 0.049, size = 94, normalized size = 3.1

$$\frac{a^2 (\cos(dx+c))^2}{2d} - 2 \frac{a^2 (\cos(dx+c))^4}{d \sin(dx+c)} - 2 \frac{a^2 (\cos(dx+c))^2 \sin(dx+c)}{d} - 4 \frac{a^2 \sin(dx+c)}{d} - \frac{a^2 (\cot(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x)

[Out] 1/2/d*a^2*cos(d*x+c)^2-2/d*a^2/sin(d*x+c)*cos(d*x+c)^4-2/d*a^2*cos(d*x+c)^2*sin(d*x+c)-4*a^2*sin(d*x+c)/d-1/2/d*a^2*cot(d*x+c)^2

Maxima [A] time = 1.11304, size = 72, normalized size = 2.4

$$\frac{a^2 \sin(dx+c)^2 + 4a^2 \sin(dx+c) + \frac{4a^2 \sin(dx+c) + a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*(a^2*sin(d*x+c)^2 + 4*a^2*sin(d*x+c) + (4*a^2*sin(d*x+c) + a^2)/sin(d*x+c)^2)/d

Fricas [B] time = 1.44029, size = 173, normalized size = 5.77

$$\frac{2a^2 \cos(dx+c)^4 - 3a^2 \cos(dx+c)^2 + 3a^2 - 8(a^2 \cos(dx+c)^2 - 2a^2) \sin(dx+c)}{4(d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/4*(2*a^2*cos(d*x+c)^4 - 3*a^2*cos(d*x+c)^2 + 3*a^2 - 8*(a^2*cos(d*x+c)^2 - 2*a^2)*sin(d*x+c))/(d*cos(d*x+c)^2 - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c+dx) \cot^3(c+dx) dx + \int \sin^2(c+dx) \cot^3(c+dx) dx + \int \cot^3(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**2,x)

[Out] a**2*(Integral(2*sin(c+d*x)*cot(c+d*x)**3, x) + Integral(sin(c+d*x)**2*cot(c+d*x)**3, x) + Integral(cot(c+d*x)**3, x))

Giac [A] time = 1.37063, size = 63, normalized size = 2.1

$$\frac{a^2 \left(\frac{1}{\sin(dx+c)} + \sin(dx+c) \right)^2 + 4a^2 \left(\frac{1}{\sin(dx+c)} + \sin(dx+c) \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*(a^2*(1/sin(d*x + c) + sin(d*x + c))^2 + 4*a^2*(1/sin(d*x + c) + sin(d*x + c)))/d

3.18 $\int \cot^7(c + dx)(a + a \sin(c + dx))^2 dx$

Optimal. Leaf size=132

$$-\frac{a^2 \sin^2(c + dx)}{2d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{a^2 \csc^6(c + dx)}{6d} - \frac{2a^2 \csc^5(c + dx)}{5d} + \frac{a^2 \csc^4(c + dx)}{2d} + \frac{2a^2 \csc^3(c + dx)}{d} - \frac{6a^2 \csc(c + dx)}{d}$$

[Out] $(-6a^2 \text{Csc}[c + dx])/d + (2a^2 \text{Csc}[c + dx]^3)/d + (a^2 \text{Csc}[c + dx]^4)/(2d) - (2a^2 \text{Csc}[c + dx]^5)/(5d) - (a^2 \text{Csc}[c + dx]^6)/(6d) + (2a^2 \text{Log}[\text{Sin}[c + dx]])/d - (2a^2 \text{Sin}[c + dx])/d - (a^2 \text{Sin}[c + dx]^2)/(2d)$

Rubi [A] time = 0.0754306, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{a^2 \sin^2(c + dx)}{2d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{a^2 \csc^6(c + dx)}{6d} - \frac{2a^2 \csc^5(c + dx)}{5d} + \frac{a^2 \csc^4(c + dx)}{2d} + \frac{2a^2 \csc^3(c + dx)}{d} - \frac{6a^2 \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + dx]^7(a + a \text{Sin}[c + dx])^2, x]$

[Out] $(-6a^2 \text{Csc}[c + dx])/d + (2a^2 \text{Csc}[c + dx]^3)/d + (a^2 \text{Csc}[c + dx]^4)/(2d) - (2a^2 \text{Csc}[c + dx]^5)/(5d) - (a^2 \text{Csc}[c + dx]^6)/(6d) + (2a^2 \text{Log}[\text{Sin}[c + dx]])/d - (2a^2 \text{Sin}[c + dx])/d - (a^2 \text{Sin}[c + dx]^2)/(2d)$

Rule 2707

$\text{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.) (x_.)]^{(m_.)} \tan[(e_.) + (f_.) (x_.)]^{(p_.)}, x_Symbol] :> \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p (a + x)^{(m - (p + 1)/2)})/(a - x)^{(p + 1)/2}, x], x, b \text{Sin}[e + f x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

$\text{Int}[(a_.) + (b_.) (x_.)^{(m_.)} ((c_.) + (d_.) (x_.)^{(n_.)}) ((e_.) + (f_.) (x_.)^{(p_.)}), x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b x)^m (c + d x)^n (e + f x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \cot^7(c + dx)(a + a \sin(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^5}{x^7} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a + \frac{a^8}{x^7} + \frac{2a^7}{x^6} - \frac{2a^6}{x^5} - \frac{6a^5}{x^4} + \frac{6a^3}{x^2} + \frac{2a^2}{x} - x\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{6a^2 \csc(c + dx)}{d} + \frac{2a^2 \csc^3(c + dx)}{d} + \frac{a^2 \csc^4(c + dx)}{2d} - \frac{2a^2 \csc^5(c + dx)}{5d} - \frac{a^2 \csc^6(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 0.215678, size = 86, normalized size = 0.65

$$\frac{a^2(15 \sin^2(c + dx) + 60 \sin(c + dx) + 5 \csc^6(c + dx) + 12 \csc^5(c + dx) - 15 \csc^4(c + dx) - 60 \csc^3(c + dx) + 180 \csc(c + dx))}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]

[Out] $-(a^2*(180*\text{Csc}[c + d*x] - 60*\text{Csc}[c + d*x]^3 - 15*\text{Csc}[c + d*x]^4 + 12*\text{Csc}[c + d*x]^5 + 5*\text{Csc}[c + d*x]^6 - 60*\text{Log}[\text{Sin}[c + d*x]] + 60*\text{Sin}[c + d*x] + 15*\text{Sin}[c + d*x]^2))/(30*d)$

Maple [B] time = 0.049, size = 313, normalized size = 2.4

$$-\frac{a^2 (\cos(dx + c))^8}{4d (\sin(dx + c))^4} + \frac{a^2 (\cos(dx + c))^8}{2d (\sin(dx + c))^2} + \frac{a^2 (\cos(dx + c))^6}{2d} + \frac{3a^2 (\cos(dx + c))^4}{4d} + \frac{3a^2 (\cos(dx + c))^2}{2d} + 2 \frac{a^2 \ln(\sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x)

[Out] $-1/4/d*a^2/\sin(d*x+c)^4*\cos(d*x+c)^8+1/2/d*a^2/\sin(d*x+c)^2*\cos(d*x+c)^8+1/2/d*a^2*\cos(d*x+c)^6+3/4/d*a^2*\cos(d*x+c)^4+3/2/d*a^2*\cos(d*x+c)^2+2*a^2*\ln(\sin(d*x+c))/d-2/5/d*a^2/\sin(d*x+c)^5*\cos(d*x+c)^8+2/5/d*a^2/\sin(d*x+c)^3*\cos(d*x+c)^8-2/d*a^2/\sin(d*x+c)*\cos(d*x+c)^8-32/5*a^2*\sin(d*x+c)/d-2/d*a^2*\sin(d*x+c)*\cos(d*x+c)^6-12/5/d*a^2*\sin(d*x+c)*\cos(d*x+c)^4-16/5/d*a^2*\cos(d*x+c)^2*\sin(d*x+c)-1/6/d*a^2*\cot(d*x+c)^6+1/4/d*a^2*\cot(d*x+c)^4-1/2/d*a^2*\cot(d*x+c)^2$

Maxima [A] time = 1.08562, size = 144, normalized size = 1.09

$$\frac{15a^2 \sin(dx + c)^2 - 60a^2 \log(\sin(dx + c)) + 60a^2 \sin(dx + c) + \frac{180a^2 \sin(dx + c)^5 - 60a^2 \sin(dx + c)^3 - 15a^2 \sin(dx + c)^2 + 12a^2 \sin(dx + c)}{\sin(dx + c)^6}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/30*(15*a^2*\sin(d*x + c)^2 - 60*a^2*\log(\sin(d*x + c)) + 60*a^2*\sin(d*x + c) + (180*a^2*\sin(d*x + c)^5 - 60*a^2*\sin(d*x + c)^3 - 15*a^2*\sin(d*x + c)^2 + 12*a^2*\sin(d*x + c) + 5*a^2)/\sin(d*x + c)^6)/d$

Fricas [A] time = 1.76805, size = 508, normalized size = 3.85

$$\frac{30a^2 \cos(dx + c)^8 - 105a^2 \cos(dx + c)^6 + 135a^2 \cos(dx + c)^4 - 45a^2 \cos(dx + c)^2 - 5a^2 + 120(a^2 \cos(dx + c)^6 - 3a^2 \cos(dx + c)^4 + 3a^2 \cos(dx + c)^2 - a^2) \log(1/2 \sin(dx + c)) - 24(5a^2 \cos(dx + c)^6 - 15a^2 \cos(dx + c)^4 + 15a^2 \cos(dx + c)^2 - a^2)}{60(d \cos(dx + c) + \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $1/60*(30*a^2*\cos(d*x + c)^8 - 105*a^2*\cos(d*x + c)^6 + 135*a^2*\cos(d*x + c)^4 - 45*a^2*\cos(d*x + c)^2 - 5*a^2 + 120*(a^2*\cos(d*x + c)^6 - 3*a^2*\cos(d*x + c)^4 + 3*a^2*\cos(d*x + c)^2 - a^2)*\log(1/2*\sin(d*x + c)) - 24*(5*a^2*\cos(dx + c)^6 - 15*a^2*\cos(dx + c)^4 + 15*a^2*\cos(dx + c)^2 - a^2)$

$$\frac{\sin(dx + c)^6 - 30a^2 \cos(dx + c)^4 + 40a^2 \cos(dx + c)^2 - 16a^2 \sin(dx + c)}{d \cos(dx + c)^6 - 3d \cos(dx + c)^4 + 3d \cos(dx + c)^2 - d}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**7*(a+a*sin(dx+c))**2,x)

[Out] Timed out

Giac [A] time = 1.49559, size = 163, normalized size = 1.23

$$\frac{15a^2 \sin(dx + c)^2 - 60a^2 \log(|\sin(dx + c)|) + 60a^2 \sin(dx + c) + \frac{147a^2 \sin(dx+c)^6 + 180a^2 \sin(dx+c)^5 - 60a^2 \sin(dx+c)^3 - 15a^2 \sin(dx+c)}{\sin(dx+c)^6}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^7*(a+a*sin(dx+c))^2,x, algorithm="giac")

[Out] -1/30*(15*a^2*sin(dx + c)^2 - 60*a^2*log(abs(sin(dx + c))) + 60*a^2*sin(dx + c) + (147*a^2*sin(dx + c)^6 + 180*a^2*sin(dx + c)^5 - 60*a^2*sin(dx + c)^3 - 15*a^2*sin(dx + c)^2 + 12*a^2*sin(dx + c) + 5*a^2)/sin(dx + c)^6)/d

3.19 $\int (a + a \sin(c + dx))^2 \tan^6(c + dx) dx$

Optimal. Leaf size=149

$$\frac{2a^2 \cos(c + dx)}{d} + \frac{9a^2 \tan^5(c + dx)}{10d} - \frac{3a^2 \tan^3(c + dx)}{2d} + \frac{9a^2 \tan(c + dx)}{2d} + \frac{2a^2 \sec^5(c + dx)}{5d} - \frac{2a^2 \sec^3(c + dx)}{d} + \frac{6a^2 \sec(c + dx)}{d} - \frac{2a^2 \sin^3(c + dx)}{d} + \frac{2a^2 \sin(c + dx)}{d}$$

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d) - (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rubi [A] time = 0.165139, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2710, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{2a^2 \cos(c + dx)}{d} + \frac{9a^2 \tan^5(c + dx)}{10d} - \frac{3a^2 \tan^3(c + dx)}{2d} + \frac{9a^2 \tan(c + dx)}{2d} + \frac{2a^2 \sec^5(c + dx)}{5d} - \frac{2a^2 \sec^3(c + dx)}{d} + \frac{6a^2 \sec(c + dx)}{d} - \frac{2a^2 \sin^3(c + dx)}{d} + \frac{2a^2 \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d) - (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rule 2710

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx))^2 \tan^6(c + dx) dx &= \int (a^2 \tan^6(c + dx) + 2a^2 \sin(c + dx) \tan^6(c + dx) + a^2 \sin^2(c + dx) \tan^6(c + dx)) dx \\
&= a^2 \int \tan^6(c + dx) dx + a^2 \int \sin^2(c + dx) \tan^6(c + dx) dx + (2a^2) \int \sin(c + dx) \tan^6(c + dx) dx \\
&= \frac{a^2 \tan^5(c + dx)}{5d} - a^2 \int \tan^4(c + dx) dx + \frac{a^2 \operatorname{Subst}\left(\int \frac{x^8}{(1+x^2)^2} dx, x, \tan(c + dx)\right)}{d} \\
&= -\frac{a^2 \tan^3(c + dx)}{3d} + \frac{a^2 \tan^5(c + dx)}{5d} - \frac{a^2 \sin^2(c + dx) \tan^5(c + dx)}{2d} + a^2 \int \tan^2(c + dx) dx \\
&= \frac{2a^2 \cos(c + dx)}{d} + \frac{6a^2 \sec(c + dx)}{d} - \frac{2a^2 \sec^3(c + dx)}{d} + \frac{2a^2 \sec^5(c + dx)}{5d} + \frac{a^2 \tan^2(c + dx)}{d} \\
&= -a^2 x + \frac{2a^2 \cos(c + dx)}{d} + \frac{6a^2 \sec(c + dx)}{d} - \frac{2a^2 \sec^3(c + dx)}{d} + \frac{2a^2 \sec^5(c + dx)}{5d} \\
&= -\frac{9a^2 x}{2} + \frac{2a^2 \cos(c + dx)}{d} + \frac{6a^2 \sec(c + dx)}{d} - \frac{2a^2 \sec^3(c + dx)}{d} + \frac{2a^2 \sec^5(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.837593, size = 174, normalized size = 1.17

$$\frac{a^2 \sec^5(c + dx) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^4 (250 \sin(c + dx) - 720c \sin(2(c + dx)) - 720dx \sin(2(c + dx)) - 82}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*Sec[c + d*x]^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-500 + 10*(10
3 + 90*c + 90*d*x)*Cos[c + d*x] - 544*Cos[2*(c + d*x)] - 206*Cos[3*(c + d*x
)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] + 20*Cos[4*(c + d*x)
] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] - 720*c*Sin[2*(c + d*x)] - 720*
d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)]))/(160*d)
```

Maple [A] time = 0.078, size = 251, normalized size = 1.7

$$\frac{1}{d} \left(a^2 \left(\frac{(\sin(dx+c))^9}{5(\cos(dx+c))^5} - \frac{4(\sin(dx+c))^9}{15(\cos(dx+c))^3} + \frac{8(\sin(dx+c))^9}{5\cos(dx+c)} + \frac{8\cos(dx+c)}{5} \left((\sin(dx+c))^7 + \frac{7(\sin(dx+c))^5}{6} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*
sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)
^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)+2*a^2*(1/5*sin(d*x+c)^8/cos(
d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*
x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/5*tan(d*x+c)^5
-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.60183, size = 205, normalized size = 1.38

$$\frac{\left(6 \tan(dx+c)^5 - 20 \tan(dx+c)^3 - 105 dx - 105 c + \frac{15 \tan(dx+c)}{\tan(dx+c)^2+1} + 90 \tan(dx+c) \right) a^2 + 2 \left(3 \tan(dx+c)^5 - 5 \tan(dx+c)^3 - 15 dx - 15 c + 15 \tan(dx+c) \right) a^2 + 12 a^2 \left(\frac{15 \cos(dx+c)^4 - 5 \cos(dx+c)^2 + 1}{\cos(dx+c)^5 + 5 \cos(dx+c)} \right) / d}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x
+ c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 - 5*
tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 + 12*a^2*((15*cos(d*x
+ c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d
```

Fricas [A] time = 1.70126, size = 382, normalized size = 2.56

$$\frac{45 a^2 dx \cos(dx+c)^3 - 10 a^2 \cos(dx+c)^4 - 90 a^2 dx \cos(dx+c) + 78 a^2 \cos(dx+c)^2 - 4 a^2 - (5 a^2 \cos(dx+c)^4 - 90 a^2 dx \cos(dx+c) + 84 a^2 \cos(dx+c)^2 - 6 a^2) \sin(dx+c)}{10 (d \cos(dx+c)^3 + 2 d \cos(dx+c) \sin(dx+c) - 2 d \cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/10*(45*a^2*d*x*cos(d*x + c)^3 - 10*a^2*cos(d*x + c)^4 - 90*a^2*d*x*cos(d
*x + c) + 78*a^2*cos(d*x + c)^2 - 4*a^2 - (5*a^2*cos(d*x + c)^4 - 90*a^2*d*
x*cos(d*x + c) + 84*a^2*cos(d*x + c)^2 - 6*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.20 $\int (a + a \sin(c + dx))^2 \tan^4(c + dx) dx$

Optimal. Leaf size=120

$$\frac{16a^2 \cos(c + dx)}{3d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{3d(a - a \sin(c + dx))^2} - \frac{8a^2 \sin^2(c + dx) \cos(c + dx)}{3d(1 - \sin(c + dx))} - \frac{7a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{7a^2 x}{2}$$

[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/ (2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)

Rubi [A] time = 0.203143, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2708, 2765, 2977, 2734}

$$\frac{16a^2 \cos(c + dx)}{3d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{3d(a - a \sin(c + dx))^2} - \frac{8a^2 \sin^2(c + dx) \cos(c + dx)}{3d(1 - \sin(c + dx))} - \frac{7a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{7a^2 x}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]

[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/ (2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)

Rule 2708

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[p, 2*m]

Rule 2765

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*
(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^2 \tan^4(c + dx) dx &= a^4 \int \frac{\sin^4(c + dx)}{(a - a \sin(c + dx))^2} dx \\ &= \frac{a^4 \cos(c + dx) \sin^3(c + dx)}{3d(a - a \sin(c + dx))^2} + \frac{1}{3} a^2 \int \frac{\sin^2(c + dx)(-3a - 5a \sin(c + dx))}{a - a \sin(c + dx)} dx \\ &= -\frac{8a^2 \cos(c + dx) \sin^2(c + dx)}{3d(1 - \sin(c + dx))} + \frac{a^4 \cos(c + dx) \sin^3(c + dx)}{3d(a - a \sin(c + dx))^2} - \frac{1}{3} \int \sin(c + dx) dx \\ &= \frac{7a^2 x}{2} - \frac{16a^2 \cos(c + dx)}{3d} - \frac{7a^2 \cos(c + dx) \sin(c + dx)}{2d} - \frac{8a^2 \cos(c + dx) \sin^2(c + dx)}{3d(1 - \sin(c + dx))} \end{aligned}$$

Mathematica [A] time = 1.3057, size = 159, normalized size = 1.32

$$\frac{a^2 \left(-21(12c + 12dx + 7) \cos\left(\frac{1}{2}(c + dx)\right) + (84c + 84dx + 239) \cos\left(\frac{3}{2}(c + dx)\right) + 3 \left(-5 \cos\left(\frac{5}{2}(c + dx)\right) + \cos\left(\frac{7}{2}(c + dx)\right) \right) \right)}{48d \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right) \right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]

[Out] -(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)

Maple [A] time = 0.071, size = 186, normalized size = 1.6

$$\frac{1}{d} \left(a^2 \left(\frac{(\sin(dx+c))^7}{3(\cos(dx+c))^3} - \frac{4(\sin(dx+c))^7}{3\cos(dx+c)} - \frac{4\cos(dx+c)}{3} \left((\sin(dx+c))^5 + \frac{5(\sin(dx+c))^3}{4} + \frac{15\sin(dx+c)}{8} \right) + \frac{5d}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x)

[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))

Maxima [A] time = 1.625, size = 162, normalized size = 1.35

$$\frac{\left(2 \tan(dx+c)^3 + 15 dx + 15 c - \frac{3 \tan(dx+c)}{\tan(dx+c)^2+1} - 12 \tan(dx+c) \right) a^2 + 2 \left(\tan(dx+c)^3 + 3 dx + 3 c - 3 \tan(dx+c) \right) a^2 - 4 dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")

[Out] $\frac{1}{6} * ((2 * \tan(d * x + c) ^ 3 + 15 * d * x + 15 * c - 3 * \tan(d * x + c) / (\tan(d * x + c) ^ 2 + 1) - 12 * \tan(d * x + c)) * a ^ 2 + 2 * (\tan(d * x + c) ^ 3 + 3 * d * x + 3 * c - 3 * \tan(d * x + c)) * a ^ 2 - 4 * a ^ 2 * ((6 * \cos(d * x + c) ^ 2 - 1) / \cos(d * x + c) ^ 3 + 3 * \cos(d * x + c))) / d$

Fricas [A] time = 1.47114, size = 468, normalized size = 3.9

$$\frac{3 a^2 \cos(dx + c)^4 - 6 a^2 \cos(dx + c)^3 - 42 a^2 dx + (21 a^2 dx + 31 a^2) \cos(dx + c)^2 - 2 a^2 - (21 a^2 dx - 38 a^2) \cos(dx + c)}{6 (d \cos(dx + c)^2 - d \cos(dx + c) + (d \cos(dx + c) + 2 * d) * \sin(dx + c) - 2 * d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")

[Out] $\frac{1}{6} * (3 * a ^ 2 * \cos(d * x + c) ^ 4 - 6 * a ^ 2 * \cos(d * x + c) ^ 3 - 42 * a ^ 2 * d * x + (21 * a ^ 2 * d * x + 31 * a ^ 2) * \cos(d * x + c) ^ 2 - 2 * a ^ 2 - (21 * a ^ 2 * d * x - 38 * a ^ 2) * \cos(d * x + c) - (3 * a ^ 2 * \cos(d * x + c) ^ 3 - 42 * a ^ 2 * d * x + 9 * a ^ 2 * \cos(d * x + c) ^ 2 + 2 * a ^ 2 - (21 * a ^ 2 * d * x - 40 * a ^ 2) * \cos(d * x + c)) * \sin(d * x + c) / (d * \cos(d * x + c) ^ 2 - d * \cos(d * x + c) + (d * \cos(d * x + c) + 2 * d) * \sin(d * x + c) - 2 * d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**4,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

3.21 $\int (a + a \sin(c + dx))^2 \tan^2(c + dx) dx$

Optimal. Leaf size=71

$$\frac{2a^2 \cos(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{2a^2 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{5a^2 x}{2}$$

[Out] $(-5*a^2*x)/2 + (2*a^2*\text{Cos}[c + d*x])/d + (2*a^2*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (a^2*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rubi [A] time = 0.0889338, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2709, 2648, 2638, 2635, 8}

$$\frac{2a^2 \cos(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{2a^2 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{5a^2 x}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^2*\text{Tan}[c + d*x]^2, x]$

[Out] $(-5*a^2*x)/2 + (2*a^2*\text{Cos}[c + d*x])/d + (2*a^2*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (a^2*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2709

$\text{Int}[(a + b*\text{sin}[e + f*x])^m*\text{tan}[e + f*x]^p, x_Symbol] \rightarrow \text{Dist}[a^p, \text{Int}[\text{ExpandIntegrand}[(\text{Sin}[e + f*x])^{p*(a + b*\text{Sin}[e + f*x])^{m - p/2}}]/(a - b*\text{Sin}[e + f*x])^{p/2}, x], x] /;$ $\text{FreeQ}\{a, b, e, f, x\}$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{IntegersQ}[m, p/2]$ && $(\text{LtQ}[p, 0] \mid \mid \text{GtQ}[m - p/2, 0])$

Rule 2648

$\text{Int}[(a + b*\text{sin}[c + d*x])^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /;$ $\text{FreeQ}\{a, b, c, d, x\}$ && $\text{EqQ}[a^2 - b^2, 0]$

Rule 2638

$\text{Int}[\text{sin}[c + d*x], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rule 2635

$\text{Int}[(b*\text{sin}[c + d*x])^n, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{n-1}/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{n-2}, x], x] /;$ $\text{FreeQ}\{b, c, d, x\}$ && $\text{GtQ}[n, 1]$ && $\text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ $\text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx))^2 \tan^2(c + dx) dx &= a^2 \int \left(-2 - \frac{2}{-1 + \sin(c + dx)} - 2 \sin(c + dx) - \sin^2(c + dx) \right) dx \\
&= -2a^2x - a^2 \int \sin^2(c + dx) dx - (2a^2) \int \frac{1}{-1 + \sin(c + dx)} dx - (2a^2) \int \sin(c + dx) dx \\
&= -2a^2x + \frac{2a^2 \cos(c + dx)}{d} + \frac{2a^2 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d} \\
&= -\frac{5a^2x}{2} + \frac{2a^2 \cos(c + dx)}{d} + \frac{2a^2 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d}
\end{aligned}$$

Mathematica [B] time = 0.437947, size = 145, normalized size = 2.04

$$\frac{a^2(\sin(c + dx) + 1)^2 \left(\cos\left(\frac{1}{2}(c + dx)\right) (10(c + dx) - \sin(2(c + dx)) - 8 \cos(c + dx)) + \sin\left(\frac{1}{2}(c + dx)\right) (-2(5c + 5dx + 4d \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^4 \right)}{4d \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]

[Out] -(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c + d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)])))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)

Maple [A] time = 0.049, size = 117, normalized size = 1.7

$$\frac{1}{d} \left(a^2 \left(\frac{(\sin(dx + c))^5}{\cos(dx + c)} + \left((\sin(dx + c))^3 + \frac{3 \sin(dx + c)}{2} \right) \cos(dx + c) - \frac{3 dx}{2} - \frac{3c}{2} \right) + 2a^2 \left(\frac{(\sin(dx + c))^4}{\cos(dx + c)} + (2 + \sin(dx + c)) \cos(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x)

[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^2*(tan(d*x+c)-d*x-c))

Maxima [A] time = 1.61458, size = 113, normalized size = 1.59

$$\frac{\left(3 dx + 3c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2 \tan(dx+c) \right) a^2 + 2(dx+c - \tan(dx+c)) a^2 - 4a^2 \left(\frac{1}{\cos(dx+c)} + \cos(dx+c) \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -1/2*((3*d*x + 3*c - tan(d*x + c))/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c))/d

Fricas [A] time = 1.53691, size = 296, normalized size = 4.17

$$\frac{a^2 \cos(dx + c)^3 - 5a^2 dx + 4a^2 \cos(dx + c)^2 + 4a^2 - (5a^2 dx - 7a^2) \cos(dx + c) + (5a^2 dx + a^2 \cos(dx + c))^2 - 3a^2 \cos(dx + c)}{2(d \cos(dx + c) - d \sin(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")

[Out] $\frac{1}{2} * (a^2 * \cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*\cos(d*x + c)^2 + 4*a^2 - (5*a^2*d*x - 7*a^2)*\cos(d*x + c) + (5*a^2*d*x + a^2*\cos(d*x + c))^2 - 3*a^2*\cos(d*x + c) + 4*a^2)*\sin(d*x + c) / (d*\cos(d*x + c) - d*\sin(d*x + c) + d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c + dx) \tan^2(c + dx) dx + \int \sin^2(c + dx) \tan^2(c + dx) dx + \int \tan^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**2,x)

[Out] $a**2*(Integral(2*\sin(c + d*x)*\tan(c + d*x)**2, x) + Integral(\sin(c + d*x)**2*\tan(c + d*x)**2, x) + Integral(\tan(c + d*x)**2, x))$

Giac [B] time = 64.9824, size = 7250, normalized size = 102.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")

[Out] $-1/2*(5*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 + 5*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) - 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 + 5*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 + 5*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 + 5*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 - 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) + 5*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 - 5*a^2*d*x*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 + 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 5*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 + 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - 5*a^2*d*x*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^3 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 + 4*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c)^4 + 2*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) - 20*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 2*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 20*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)$

$$\begin{aligned}
&)^3 + 4*a^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3 - 5*a^2*d*x*\tan(1/2*d*x)^4*\tan(1/2*c)^4 + 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 5*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c) - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) + 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) - 5*a^2*d*x*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c) - 8*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) + 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^2 + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 20*a^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 - 32*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^2 + 8*a^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 5*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c)^3 - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 96*a^2*\tan(d*x)^3*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 + 32*a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - 5*a^2*d*x*\tan(d*x)*\tan(1/2*c)^4*\tan(c)^3 - 8*a^2*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^3 - 16*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3 + 5*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) + 5*a^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) - 5*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c)^2 - 20*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 - 20*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 - 5*a^2*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^2 - 5*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^3 - 20*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 20*a^2*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 16*a^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - 5*a^2*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^3 + 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^4 + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c) + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3 + 20*a^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)^3 - 32*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3 + 5*a^2*d*x*\tan(d*x)^2*\tan(1/2*c)^4 + 8*a^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 5*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c) - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c) - 20*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 96*a^2*\tan(d*x)^3*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c) - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) + 32*a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) - 5*a^2*d*x*\tan(d*x)*\tan(1/2*c)^4*\tan(c) - 8*a^2*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c) + 5*a^2*d*x*\tan(1/2*d*x)^4*\tan(c)^2 + 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^2 + 20*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 20*a^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 32*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 96*a^2*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + 20*a^2*d*x*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 32*a^2*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 - 32*a^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 5*a^2*d*x*\tan(1/2*c)^4*\tan(c)^2 + 8*a^2*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^2 + 5*a^2*d*x*\tan(d*x)^3*\tan(c)^3 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c)^3 - 20*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 + 32*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 32*a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 96*a^2*\tan(d*x)*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^3 - 32*a^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 8*a^2*\tan(d*x)*\tan(1/2*c)^4*\tan(c)^3 - 4*a^2*\tan(d*x)^3*\tan(1/2*d*x)^4 - 16*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c) - 16*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3 - 20*a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3 - 4*a^2*\tan(d*x)^3*\tan(1/2*c)^4 - 2*a^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c) - 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - 20*a^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) - 2*a^2*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c) - 2*a^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c)^2 - 20*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)
\end{aligned}$$

$$\begin{aligned}
&^2 - 8a^2 \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^2 - 8a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c)^3 \tan(c)^2 - 2a^2 \tan(dx) \tan(1/2 c)^4 \tan(c)^2 - 4a^2 \tan(1/2 dx)^4 \tan(c)^3 - 20a^2 \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c) \tan(c)^3 - 16a^2 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^3 - 16a^2 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c)^3 - 4a^2 \tan(1/2 c)^4 \tan(c)^3 + 5a^2 dx \tan(1/2 dx)^4 + 8a^2 \tan(dx)^2 \tan(1/2 dx)^4 + 20a^2 dx \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c) + 20a^2 dx \tan(1/2 dx)^3 \tan(1/2 c) + 32a^2 \tan(dx)^2 \tan(1/2 dx)^3 \tan(1/2 c) + 96a^2 \tan(dx)^2 \tan(1/2 dx)^2 \tan(1/2 c)^2 + 20a^2 dx \tan(1/2 dx) \tan(1/2 c)^3 + 32a^2 \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c)^3 - 32a^2 \tan(1/2 dx)^3 \tan(1/2 c)^3 + 5a^2 dx \tan(1/2 c)^4 + 8a^2 \tan(dx)^2 \tan(1/2 c)^4 + 5a^2 dx \tan(dx)^3 \tan(c) - 8a^2 \tan(dx) \tan(1/2 dx)^4 \tan(c) - 20a^2 dx \tan(dx) \tan(1/2 dx) \tan(1/2 c) \tan(c) + 32a^2 \tan(dx)^3 \tan(1/2 dx) \tan(1/2 c) \tan(c) - 32a^2 \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c) \tan(c) - 96a^2 \tan(dx) \tan(1/2 dx)^2 \tan(1/2 c)^2 \tan(c) - 32a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c)^3 \tan(c) - 8a^2 \tan(dx) \tan(1/2 c)^4 \tan(c) - 5a^2 dx \tan(dx)^2 \tan(c)^2 + 8a^2 \tan(1/2 dx)^4 \tan(c)^2 + 20a^2 dx \tan(1/2 dx) \tan(1/2 c) \tan(c)^2 - 32a^2 \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c) \tan(c)^2 + 32a^2 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^2 + 96a^2 \tan(1/2 dx)^2 \tan(1/2 c)^2 \tan(c)^2 + 32a^2 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c)^2 + 8a^2 \tan(1/2 c)^4 \tan(c)^2 + 5a^2 dx \tan(dx) \tan(c)^3 - 8a^2 \tan(dx)^3 \tan(c)^3 + 32a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c) \tan(c)^3 - 5a^2 \tan(dx) \tan(1/2 dx)^4 - 16a^2 \tan(dx)^3 \tan(1/2 dx) \tan(1/2 c) - 20a^2 \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c) - 20a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c)^3 - 5a^2 \tan(dx) \tan(1/2 c)^4 - 5a^2 \tan(1/2 dx)^4 \tan(c) - 8a^2 \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c) \tan(c) - 20a^2 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c) - 20a^2 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c) - 5a^2 \tan(1/2 c)^4 \tan(c) + 5a^2 \tan(dx)^3 \tan(c)^2 - 8a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c) \tan(c)^2 + 5a^2 \tan(dx)^2 \tan(c)^3 - 16a^2 \tan(1/2 dx) \tan(1/2 c) \tan(c)^3 - 5a^2 dx \tan(dx)^2 + 8a^2 \tan(1/2 dx)^4 + 20a^2 dx \tan(1/2 dx) \tan(1/2 c) - 32a^2 \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c) + 32a^2 \tan(1/2 dx)^3 \tan(1/2 c) + 96a^2 \tan(1/2 dx)^2 \tan(1/2 c)^2 + 32a^2 \tan(1/2 dx) \tan(1/2 c)^3 + 8a^2 \tan(1/2 c)^4 + 5a^2 dx \tan(dx) \tan(c) - 8a^2 \tan(dx)^3 \tan(c) + 32a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c) \tan(c) - 5a^2 dx \tan(c)^2 + 8a^2 \tan(dx)^2 \tan(c)^2 - 32a^2 \tan(1/2 dx) \tan(1/2 c) \tan(c)^2 - 8a^2 \tan(dx) \tan(c)^3 + 4a^2 \tan(dx)^3 - 20a^2 \tan(dx) \tan(1/2 dx) \tan(1/2 c) + 2a^2 \tan(dx)^2 \tan(c) - 20a^2 \tan(1/2 dx) \tan(1/2 c) \tan(c) + 2a^2 \tan(dx) \tan(c)^2 + 4a^2 \tan(c)^3 - 5a^2 dx + 8a^2 \tan(dx)^2 - 32a^2 \tan(1/2 dx) \tan(1/2 c) - 8a^2 \tan(dx) \tan(c) + 8a^2 \tan(c)^2 + 5a^2 \tan(dx) + 5a^2 \tan(c) + 8a^2) / (d \tan(dx)^3 \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c)^3 + d \tan(dx)^3 \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c) - d \tan(dx)^2 \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c)^2 - 4d \tan(dx)^3 \tan(1/2 dx)^3 \tan(1/2 c)^3 \tan(c)^3 + d \tan(dx) \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c)^3 - d \tan(dx)^2 \tan(1/2 dx)^4 \tan(1/2 c)^4 - 4d \tan(dx)^3 \tan(1/2 dx)^3 \tan(1/2 c)^3 \tan(c) + d \tan(dx) \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c) + 4d \tan(dx)^2 \tan(1/2 dx)^3 \tan(1/2 c)^3 \tan(c)^2 - d \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c)^2 - d \tan(dx)^3 \tan(1/2 dx)^4 \tan(c)^3 - 4d \tan(dx)^3 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^3 - 4d \tan(dx)^3 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c)^3 - d \tan(dx)^3 \tan(1/2 c)^4 \tan(c)^3 + 4d \tan(dx)^2 \tan(1/2 dx)^3 \tan(1/2 c)^3 - d \tan(1/2 dx)^4 \tan(1/2 c)^4 - d \tan(dx)^3 \tan(1/2 dx)^4 \tan(c) - 4d \tan(dx)^3 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c) - 4d \tan(dx)^3 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c) - d \tan(dx)^3 \tan(1/2 c)^4 \tan(c) + d \tan(dx)^2 \tan(1/2 dx)^4 \tan(c)^2 + 4d \tan(dx)^2 \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^2 + 4d \tan(dx)^2 \tan(1/2 dx) \tan(1/2 c)^3 \tan(c)^2 + 4d \tan(1/2 dx)^3 \tan(1/2 c)^3 \tan(c)^2 + d \tan(dx)^2 \tan(1/2 c)^4 \tan(c)^2 - d \tan(dx) \tan(1/2 dx)^4 \tan(c)^3 - 4d \tan(dx)^3 \tan(1/2 dx) \tan(1/2 c) \tan(c)^3 - 4d \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c) \tan(c)^3 - d \tan(dx) \tan(1/2 c)^4 \tan(c)^3 + d \tan(dx)^2 \tan(1/2 dx)^4 + 4d \tan(dx)^2 \tan(1/2 dx)
\end{aligned}$$

$$\begin{aligned}
&)^3 \tan(1/2*c) + 4*d*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3 + 4*d*\tan(1/2*d*x) \\
&)^3*\tan(1/2*c)^3 + d*\tan(d*x)^2*\tan(1/2*c)^4 - d*\tan(d*x)*\tan(1/2*d*x)^4*\tan \\
&n(c) - 4*d*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 4*d*\tan(d*x)*\tan(1/2 \\
&*d*x)^3*\tan(1/2*c)*\tan(c) - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - \\
&d*\tan(d*x)*\tan(1/2*c)^4*\tan(c) + d*\tan(1/2*d*x)^4*\tan(c)^2 + 4*d*\tan(d*x)^ \\
&2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 4*d*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 \\
&+ 4*d*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + d*\tan(1/2*c)^4*\tan(c)^2 + d*\tan \\
&(d*x)^3*\tan(c)^3 - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 + d*\tan(1/ \\
&2*d*x)^4 + 4*d*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c) + 4*d*\tan(1/2*d*x)^3*\tan(\\
&1/2*c) + 4*d*\tan(1/2*d*x)*\tan(1/2*c)^3 + d*\tan(1/2*c)^4 + d*\tan(d*x)^3*\tan(\\
&c) - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - d*\tan(d*x)^2*\tan(c)^2 + \\
&4*d*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + d*\tan(d*x)*\tan(c)^3 - d*\tan(d*x)^2 + \\
&4*d*\tan(1/2*d*x)*\tan(1/2*c) + d*\tan(d*x)*\tan(c) - d*\tan(c)^2 - d)
\end{aligned}$$

3.22 $\int (a + a \sin(c + dx))^2 dx$

Optimal. Leaf size=45

$$-\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3a^2 x}{2}$$

[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0136893, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2644}

$$-\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3a^2 x}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^2,x]

[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 2644

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]

Rubi steps

$$\int (a + a \sin(c + dx))^2 dx = \frac{3a^2 x}{2} - \frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] time = 0.19256, size = 34, normalized size = 0.76

$$\frac{a^2(-6(c + dx) + \sin(2(c + dx)) + 8 \cos(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^2,x]

[Out] -(a^2*(-6*(c + d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)]))/(4*d)

Maple [A] time = 0.023, size = 52, normalized size = 1.2

$$\frac{1}{d} \left(a^2 \left(-\frac{\cos(dx + c) \sin(dx + c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) - 2 \cos(dx + c) a^2 + a^2(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(d*x+c))^2,x)`

[Out] `1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*cos(d*x+c)*a^2+a^2*(d*x+c))`

Maxima [A] time = 1.08459, size = 63, normalized size = 1.4

$$a^2x + \frac{(2dx + 2c - \sin(2dx + 2c))a^2}{4d} - \frac{2a^2 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^2,x, algorithm="maxima")`

[Out] `a^2*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - 2*a^2*cos(d*x + c)/d`

Fricas [A] time = 1.55815, size = 97, normalized size = 2.16

$$\frac{3a^2dx - a^2 \cos(dx + c) \sin(dx + c) - 4a^2 \cos(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^2,x, algorithm="fricas")`

[Out] `1/2*(3*a^2*d*x - a^2*cos(d*x + c)*sin(d*x + c) - 4*a^2*cos(d*x + c))/d`

Sympy [A] time = 0.697112, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x \sin^2(c+dx)}{2} + \frac{a^2x \cos^2(c+dx)}{2} + a^2x - \frac{a^2 \sin(c+dx) \cos(c+dx)}{2d} - \frac{2a^2 \cos(c+dx)}{d} & \text{for } d \neq 0 \\ x(a \sin(c) + a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))**2,x)`

[Out] `Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x - a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**2, True))`

Giac [A] time = 1.85333, size = 51, normalized size = 1.13

$$\frac{3}{2}a^2x - \frac{2a^2 \cos(dx + c)}{d} - \frac{a^2 \sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^2,x, algorithm="giac")`

[Out] `3/2*a^2*x - 2*a^2*cos(d*x + c)/d - 1/4*a^2*sin(2*d*x + 2*c)/d`

3.23 $\int \cot^2(c + dx)(a + a \sin(c + dx))^2 dx$

Optimal. Leaf size=74

$$\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} - \frac{2a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^2 x}{2}$$

[Out] $-(a^2 x)/2 - (2a^2 \operatorname{ArcTanh}[\cos(c + dx)])/d + (2a^2 \cos(c + dx))/d - (a^2 \cot(c + dx))/d + (a^2 \cos(c + dx) \sin(c + dx))/(2d)$

Rubi [A] time = 0.102471, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} - \frac{2a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^2 x}{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\cot(c + dx)^2 (a + a \sin(c + dx))^2, x]$

[Out] $-(a^2 x)/2 - (2a^2 \operatorname{ArcTanh}[\cos(c + dx)])/d + (2a^2 \cos(c + dx))/d - (a^2 \cot(c + dx))/d + (a^2 \cos(c + dx) \sin(c + dx))/(2d)$

Rule 2709

$\operatorname{Int}[(a + (b \sin(e + f x))^m) \tan(e + f x)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[a^p, \operatorname{Int}[\operatorname{ExpandIntegrand}[(\sin(e + f x))^p (a + b \sin(e + f x))^{m - p/2}) / (a - b \sin(e + f x))^{p/2}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{IntegersQ}[m, p/2] \ \&\& \ (\operatorname{LtQ}[p, 0] \ || \ \operatorname{GtQ}[m - p/2, 0])$

Rule 3770

$\operatorname{Int}[\operatorname{csc}(c + dx) (a + b \sin(c + dx))^n, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos(c + dx)]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 3767

$\operatorname{Int}[\operatorname{csc}(c + dx) (a + b \sin(c + dx))^n, x_{\text{Symbol}}] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \cot(c + dx)], x] /;$ $\operatorname{FreeQ}\{c, d, x\} \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 8

$\operatorname{Int}[a x, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[a x, x] /;$ $\operatorname{FreeQ}[a, x]$

Rule 2638

$\operatorname{Int}[\sin(c + dx) (a + b \sin(c + dx))^n, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\cos(c + dx)/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 2635

$\operatorname{Int}[(b \sin(c + dx))^n, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(b \cos(c + dx)) (b \sin(c + dx))^{n - 1} / (d n), x] + \operatorname{Dist}[(b^2 (n - 1)) / n, \operatorname{Int}[(b \sin(c + dx))^{n - 2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, x\} \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{IntegerQ}[2n]$

]

Rubi steps

$$\begin{aligned}
\int \cot^2(c + dx)(a + a \sin(c + dx))^2 dx &= \frac{\int (2a^4 \csc(c + dx) + a^4 \csc^2(c + dx) - 2a^4 \sin(c + dx) - a^4 \sin^2(c + dx)) dx}{a^2} \\
&= a^2 \int \csc^2(c + dx) dx - a^2 \int \sin^2(c + dx) dx + (2a^2) \int \csc(c + dx) dx - (2a^2) \int \sin(c + dx) dx \\
&= -\frac{2a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2a^2 \cos(c + dx)}{d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d} - \frac{2a^2 \sin(c + dx)}{2d} \\
&= -\frac{a^2 x}{2} - \frac{2a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx)}{d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.58033, size = 94, normalized size = 1.27

$$\frac{a^2 \csc\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{1}{2}(c + dx)\right) \left(7 \cos(c + dx) + \cos(3(c + dx)) + 4 \sin(c + dx)\right) \left(-4 \cos(c + dx) - 4 \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]

[Out] $-(a^2 \text{Csc}[(c + d*x)/2] \text{Sec}[(c + d*x)/2] (7 \text{Cos}[c + d*x] + \text{Cos}[3(c + d*x)] + 4(c + d*x - 4 \text{Cos}[c + d*x] + 4 \text{Log}[\text{Cos}[(c + d*x)/2]] - 4 \text{Log}[\text{Sin}[(c + d*x)/2]]) \text{Sin}[c + d*x]))/(16*d)$

Maple [A] time = 0.036, size = 89, normalized size = 1.2

$$\frac{\cos(dx + c) a^2 \sin(dx + c)}{2d} - \frac{a^2 x}{2} - \frac{a^2 c}{2d} + 2 \frac{\cos(dx + c) a^2}{d} + 2 \frac{a^2 \ln(\csc(dx + c) - \cot(dx + c))}{d} - \frac{a^2 \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x)

[Out] $1/2*a^2*\cos(d*x+c)*\sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+2*a^2*\cos(d*x+c)/d+2/d*a^2*\ln(\csc(d*x+c)-\cot(d*x+c))-a^2*\cot(d*x+c)/d$

Maxima [A] time = 1.64023, size = 107, normalized size = 1.45

$$\frac{(2dx + 2c + \sin(2dx + 2c))a^2 - 4\left(dx + c + \frac{1}{\tan(dx + c)}\right)a^2 + 4a^2(2 \cos(dx + c) - \log(\cos(dx + c) + 1) + \log(\cos(dx + c) - 1))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $1/4*((2*d*x + 2*c + \sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/\tan(d*x + c))*a^2 + 4*a^2*(2*\cos(d*x + c) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)))$

/d

Fricas [A] time = 1.52358, size = 281, normalized size = 3.8

$$\frac{a^2 \cos(dx + c)^3 + 2a^2 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 2a^2 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + a^2 \cos(dx + c)}{2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c + dx) \cot^2(c + dx) dx + \int \sin^2(c + dx) \cot^2(c + dx) dx + \int \cot^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**2,x)

[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**2*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))

Giac [B] time = 1.73197, size = 193, normalized size = 2.61

$$\frac{(dx + c)a^2 - 4a^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{4a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a^2}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)} + \frac{2\left(a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^3 - 4a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 + 1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d

3.24 $\int \cot^4(c + dx)(a + a \sin(c + dx))^2 dx$

Optimal. Leaf size=98

$$\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} - \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{d}$$

[Out] $-(a^2 x)/2 + (3a^2 \operatorname{ArcTanh}[\cos(c + dx)])/d - (2a^2 \cos(c + dx))/d - (a^2 \cot^3(c + dx))/(3d) - (a^2 \cot(c + dx) \csc(c + dx))/d - (a^2 \cos(c + dx) \sin(c + dx))/(2d)$

Rubi [A] time = 0.161405, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} - \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\int \cot^4(c + dx)(a + a \sin(c + dx))^2 dx$

[Out] $-(a^2 x)/2 + (3a^2 \operatorname{ArcTanh}[\cos(c + dx)])/d - (2a^2 \cos(c + dx))/d - (a^2 \cot^3(c + dx))/(3d) - (a^2 \cot(c + dx) \csc(c + dx))/d - (a^2 \cos(c + dx) \sin(c + dx))/(2d)$

Rule 2709

$\operatorname{Int}[(a + b \sin(e + f x))^m \tan(e + f x)^p, x] \rightarrow \operatorname{Dist}[a^p, \operatorname{Int}[\operatorname{ExpandIntegrand}[(\sin(e + f x))^p (a + b \sin(e + f x))^{m-p/2}]/(a - b \sin(e + f x))^{p/2}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\}$ && $\operatorname{EqQ}[a^2 - b^2, 0]$ && $\operatorname{IntegersQ}[m, p/2]$ && $(\operatorname{LtQ}[p, 0] \mid \mid \operatorname{GtQ}[m - p/2, 0])$

Rule 3770

$\operatorname{Int}[\csc(c + dx), x] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos(c + dx)]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 3767

$\operatorname{Int}[\csc(c + dx)^n, x] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2-1}, x], x], \cot(c + dx)], x] /;$ $\operatorname{FreeQ}\{c, d, x\}$ && $\operatorname{IGtQ}[n/2, 0]$

Rule 8

$\operatorname{Int}[a x, x] \rightarrow \operatorname{Simp}[a x, x] /;$ $\operatorname{FreeQ}[a, x]$

Rule 3768

$\operatorname{Int}[(\csc(c + dx) + d \cot(c + dx))^n, x] \rightarrow -\operatorname{Simp}[(b \cos(c + dx))^n (b \csc(c + dx))^{n-1}/(d(n-1)), x] + \operatorname{Dist}[(b^2(n-2))/(n-1), \operatorname{Int}[(b \csc(c + dx))^{n-2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, x\}$ && $\operatorname{GtQ}[n, 1]$ && $\operatorname{IntegerQ}[2n]$

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^4(c + dx)(a + a \sin(c + dx))^2 dx = \frac{\int (-a^6 - 4a^6 \csc(c + dx) - a^6 \csc^2(c + dx) + 2a^6 \csc^3(c + dx) + a^6 \csc^4(c + dx) - a^6 \csc^5(c + dx)) dx}{a^4}$$

$$= -a^2x - a^2 \int \csc^2(c + dx) dx + a^2 \int \csc^4(c + dx) dx + a^2 \int \sin^2(c + dx) dx + (2a^2 \int \sin^4(c + dx) dx)$$

$$= -a^2x + \frac{4a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{d}$$

$$= -\frac{a^2x}{2} + \frac{3a^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{2a^2 \cos(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} - \frac{a^2 \cot(c + dx)}{d}$$

Mathematica [A] time = 5.58143, size = 191, normalized size = 1.95

$$\frac{a^2(\sin(c + dx) + 1)^2 \left(-12(c + dx) - 6 \sin(2(c + dx)) - 48 \cos(c + dx) - 4 \tan\left(\frac{1}{2}(c + dx)\right) + 4 \cot\left(\frac{1}{2}(c + dx)\right) - 6 \csc^2\left(\frac{1}{2}(c + dx)\right) \right)}{24d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x)
]/2) - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2])/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [B] time = 0.044, size = 190, normalized size = 1.9

$$\frac{a^2 (\cos(dx + c))^5}{d \sin(dx + c)} - \frac{a^2 \sin(dx + c) (\cos(dx + c))^3}{d} - \frac{3 \cos(dx + c) a^2 \sin(dx + c)}{2d} - \frac{a^2 x}{2} - \frac{a^2 c}{2d} - \frac{a^2 (\cos(dx + c))^5}{d (\sin(dx + c))^2} - \frac{a^2 \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-1/d*a^2*sin(d*x+c)*cos(d*x+c)^3-3/2*a^2*co
s(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)
^5-1/d*a^2*cos(d*x+c)^3-3*a^2*cos(d*x+c)/d-3/d*a^2*ln(csc(d*x+c)-cot(d*x+c)
)-1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.64741, size = 188, normalized size = 1.92

$$\frac{3 \left(3 dx + 3 c + \frac{3 \tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)} \right) a^2 - 2 \left(3 dx + 3 c + \frac{3 \tan(dx+c)^2-1}{\tan(dx+c)^3} \right) a^2 - 3 a^2 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - 4 \cos(dx+c) + 3 \log(\cos(dx+c) + 1) - 3 \log(\cos(dx+c) - 1) \right)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c))) * a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3) * a^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d

Fricas [B] time = 1.70258, size = 474, normalized size = 4.84

$$3 a^2 \cos(dx+c)^5 - 4 a^2 \cos(dx+c)^3 + 3 a^2 \cos(dx+c) + 9 \left(a^2 \cos(dx+c)^2 - a^2 \right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2 \sin(c + dx) \cot^4(c + dx) dx + \int \sin^2(c + dx) \cot^4(c + dx) dx + \int \cot^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**2,x)

[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(sin(c + d*x)**2*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x))

Giac [B] time = 1.76195, size = 282, normalized size = 2.88

$$a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 6 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 12(dx+c)a^2 - 72 a^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 3 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c)))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.25 $\int (a + a \sin(c + dx))^3 \tan^7(c + dx) dx$

Optimal. Leaf size=160

$$\frac{a^3 \sin^3(c + dx)}{3d} + \frac{3a^3 \sin^2(c + dx)}{2d} + \frac{a^6}{6d(a - a \sin(c + dx))^3} - \frac{13a^5}{8d(a - a \sin(c + dx))^2} + \frac{71a^4}{8d(a - a \sin(c + dx))} + \frac{7a^3 \sin^3(c + dx)}{3d}$$

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
+ (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
+ a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c + d*x])^2)
+ (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.109078, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$\frac{a^3 \sin^3(c + dx)}{3d} + \frac{3a^3 \sin^2(c + dx)}{2d} + \frac{a^6}{6d(a - a \sin(c + dx))^3} - \frac{13a^5}{8d(a - a \sin(c + dx))^2} + \frac{71a^4}{8d(a - a \sin(c + dx))} + \frac{7a^3 \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
+ (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
+ a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c + d*x])^2)
+ (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol]
:= Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)^(p_)), x_Symbol]
:= Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^3 \tan^7(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(7a^2 + \frac{a^6}{2(a-x)^4} - \frac{13a^5}{4(a-x)^3} + \frac{71a^4}{8(a-x)^2} - \frac{209a^3}{16(a-x)} + 3ax + x^2 - \frac{a^3}{16(a+x)}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{209a^3 \log(1 - \sin(c + dx))}{16d} - \frac{a^3 \log(1 + \sin(c + dx))}{16d} + \frac{7a^3 \sin(c + dx)}{d} + \frac{7a^3 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.563704, size = 99, normalized size = 0.62

$$\frac{a^3 \left(16 \sin^3(c + dx) + 72 \sin^2(c + dx) + 336 \sin(c + dx) - \frac{426}{\sin(c+dx)-1} - \frac{78}{(\sin(c+dx)-1)^2} - \frac{8}{(\sin(c+dx)-1)^3} + 627 \log(1 - \sin(c + dx)) \right)}{48d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]

[Out] (a^3*(627*Log[1 - Sin[c + d*x]] - 3*Log[1 + Sin[c + d*x]] - 8/(-1 + Sin[c + d*x])^3 - 78/(-1 + Sin[c + d*x])^2 - 426/(-1 + Sin[c + d*x]) + 336*Sin[c + d*x] + 72*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(48*d)

Maple [B] time = 0.105, size = 445, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x)

[Out] 35/48/d*a^3*sin(d*x+c)^9+3/2/d*a^3*sin(d*x+c)^8+15/8/d*a^3*sin(d*x+c)^7+21/8/d*a^3*sin(d*x+c)^5+35/8*a^3*sin(d*x+c)^3/d+105/8*a^3*sin(d*x+c)/d-105/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^3*sin(d*x+c)^6+3/d*a^3*sin(d*x+c)^4+6*a^3*sin(d*x+c)^2/d+13/d*a^3*ln(cos(d*x+c))+1/6/d*a^3*sin(d*x+c)^11/cos(d*x+c)^6-5/24/d*a^3*sin(d*x+c)^11/cos(d*x+c)^4+35/48/d*a^3*sin(d*x+c)^11/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^6-1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^4+3/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^9/cos(d*x+c)^6-3/8/d*a^3*sin(d*x+c)^9/cos(d*x+c)^4+15/16/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2+1/6/d*a^3*tan(d*x+c)^6-1/4/d*a^3*tan(d*x+c)^4+1/2/d*a^3*tan(d*x+c)^2

Maxima [A] time = 1.07702, size = 180, normalized size = 1.12

$$\frac{16 a^3 \sin(dx + c)^3 + 72 a^3 \sin(dx + c)^2 - 3 a^3 \log(\sin(dx + c) + 1) + 627 a^3 \log(\sin(dx + c) - 1) + 336 a^3 \sin(dx + c) - 2(213 a^3 \sin(dx + c)^2 - 387 a^3 \sin(dx + c) + 178 a^3)}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")

[Out] 1/48*(16*a^3*sin(d*x + c)^3 + 72*a^3*sin(d*x + c)^2 - 3*a^3*log(sin(d*x + c) + 1) + 627*a^3*log(sin(d*x + c) - 1) + 336*a^3*sin(d*x + c) - 2*(213*a^3*sin(d*x + c)^2 - 387*a^3*sin(d*x + c) + 178*a^3)/(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c) - 1))/d

Fricas [A] time = 1.57514, size = 594, normalized size = 3.71

$$\frac{16 a^3 \cos(dx + c)^6 - 216 a^3 \cos(dx + c)^4 + 1002 a^3 \cos(dx + c)^2 - 482 a^3 + 3(3 a^3 \cos(dx + c)^2 - 4 a^3 - (a^3 \cos(dx + c) - 1))}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")

[Out]
$$-1/48*(16*a^3*\cos(d*x + c)^6 - 216*a^3*\cos(d*x + c)^4 + 1002*a^3*\cos(d*x + c)^2 - 482*a^3 + 3*(3*a^3*\cos(d*x + c)^2 - 4*a^3 - (a^3*\cos(d*x + c)^2 - 4*a^3)*\sin(d*x + c))*\log(\sin(d*x + c) + 1) - 627*(3*a^3*\cos(d*x + c)^2 - 4*a^3 - (a^3*\cos(d*x + c)^2 - 4*a^3)*\sin(d*x + c))*\log(-\sin(d*x + c) + 1) - 2*(12*a^3*\cos(d*x + c)^4 + 398*a^3*\cos(d*x + c)^2 - 245*a^3)*\sin(d*x + c))/(3*d*\cos(d*x + c)^2 - (d*\cos(d*x + c)^2 - 4*d)*\sin(d*x + c) - 4*d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")

[Out] Timed out

3.26 $\int (a + a \sin(c + dx))^3 \tan^3(c + dx) dx$

Optimal. Leaf size=91

$$\frac{a^3 \sin^3(c + dx)}{3d} + \frac{3a^3 \sin^2(c + dx)}{2d} + \frac{2a^4}{d(a - a \sin(c + dx))} + \frac{5a^3 \sin(c + dx)}{d} + \frac{7a^3 \log(1 - \sin(c + dx))}{d}$$

[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x]))

Rubi [A] time = 0.0707841, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 77}

$$\frac{a^3 \sin^3(c + dx)}{3d} + \frac{3a^3 \sin^2(c + dx)}{2d} + \frac{2a^4}{d(a - a \sin(c + dx))} + \frac{5a^3 \sin(c + dx)}{d} + \frac{7a^3 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]

[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^3 \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(a-x)^2} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(5a^2 + \frac{2a^4}{(a-x)^2} - \frac{7a^3}{a-x} + 3ax + x^2\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{7a^3 \log(1 - \sin(c + dx))}{d} + \frac{5a^3 \sin(c + dx)}{d} + \frac{3a^3 \sin^2(c + dx)}{2d} + \frac{a^3 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.156734, size = 66, normalized size = 0.73

$$\frac{a^3 \left(2 \sin^3(c + dx) + 9 \sin^2(c + dx) + 30 \sin(c + dx) + \frac{12}{1 - \sin(c + dx)} + 42 \log(1 - \sin(c + dx)) \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]

[Out] (a^3*(42*Log[1 - Sin[c + d*x]] + 12/(1 - Sin[c + d*x]) + 30*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)

Maple [B] time = 0.078, size = 205, normalized size = 2.3

$$\frac{a^3 (\sin(dx + c))^7}{2d (\cos(dx + c))^2} + \frac{a^3 (\sin(dx + c))^5}{2d} + \frac{7a^3 (\sin(dx + c))^3}{3d} + 7 \frac{a^3 \sin(dx + c)}{d} - 7 \frac{a^3 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x)

[Out] 1/2/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^5+7/3*a^3*sin(d*x+c)^3/d+7*a^3*sin(d*x+c)/d-7/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*sin(d*x+c)^6/cos(d*x+c)^2+3/2/d*a^3*sin(d*x+c)^4+3*a^3*sin(d*x+c)^2/d+7/d*a^3*ln(cos(d*x+c))+3/2/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^3*tan(d*x+c)^2

Maxima [A] time = 1.07083, size = 97, normalized size = 1.07

$$\frac{2a^3 \sin(dx + c)^3 + 9a^3 \sin(dx + c)^2 + 42a^3 \log(\sin(dx + c) - 1) + 30a^3 \sin(dx + c) - \frac{12a^3}{\sin(dx + c) - 1}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 42*a^3*log(sin(d*x + c) - 1) + 30*a^3*sin(d*x + c) - 12*a^3/(sin(d*x + c) - 1))/d

Fricas [A] time = 1.59931, size = 248, normalized size = 2.73

$$\frac{4a^3 \cos(dx + c)^4 - 50a^3 \cos(dx + c)^2 + 31a^3 + 84(a^3 \sin(dx + c) - a^3) \log(-\sin(dx + c) + 1) - (14a^3 \cos(dx + c))^2}{12(d \sin(dx + c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")

[Out] 1/12*(4*a^3*cos(d*x + c)^4 - 50*a^3*cos(d*x + c)^2 + 31*a^3 + 84*(a^3*sin(d*x + c) - a^3)*log(-sin(d*x + c) + 1) - (14*a^3*cos(d*x + c)^2 + 55*a^3)*sin(d*x + c))/(d*sin(d*x + c) - d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.27 $\int (a + a \sin(c + dx))^3 \tan(c + dx) dx$

Optimal. Leaf size=70

$$-\frac{a^3 \sin^3(c + dx)}{3d} - \frac{3a^3 \sin^2(c + dx)}{2d} - \frac{4a^3 \sin(c + dx)}{d} - \frac{4a^3 \log(1 - \sin(c + dx))}{d}$$

[Out] $(-4*a^3*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (4*a^3*\text{Sin}[c + d*x])/d - (3*a^3*\text{Sin}[c + d*x]^2)/(2*d) - (a^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0449772, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 77}

$$-\frac{a^3 \sin^3(c + dx)}{3d} - \frac{3a^3 \sin^2(c + dx)}{2d} - \frac{4a^3 \sin(c + dx)}{d} - \frac{4a^3 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^3*\text{Tan}[c + d*x], x]$

[Out] $(-4*a^3*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (4*a^3*\text{Sin}[c + d*x])/d - (3*a^3*\text{Sin}[c + d*x]^2)/(2*d) - (a^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m*\tan(e + f*x)^p, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}], x], x, b*\text{Sin}[e + f*x], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

$\text{Int}[(a + b*x)*(c + d*x)^n*((e + f*x))^p, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^3 \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+x)^2}}{a-x} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-4a^2 + \frac{4a^3}{a-x} - 3ax - x^2\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{4a^3 \log(1 - \sin(c + dx))}{d} - \frac{4a^3 \sin(c + dx)}{d} - \frac{3a^3 \sin^2(c + dx)}{2d} - \frac{a^3 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.045258, size = 52, normalized size = 0.74

$$-\frac{a^3 (2 \sin^3(c + dx) + 9 \sin^2(c + dx) + 24 \sin(c + dx) + 24 \log(1 - \sin(c + dx)))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]

[Out] $-(a^3*(24*\text{Log}[1 - \text{Sin}[c + d*x]] + 24*\text{Sin}[c + d*x] + 9*\text{Sin}[c + d*x]^2 + 2*\text{Sin}[c + d*x]^3))/(6*d)$

Maple [A] time = 0.051, size = 85, normalized size = 1.2

$$-\frac{a^3 (\sin(dx + c))^3}{3d} - 4 \frac{a^3 \sin(dx + c)}{d} + 4 \frac{a^3 \ln(\sec(dx + c) + \tan(dx + c))}{d} - \frac{3 a^3 (\sin(dx + c))^2}{2d} - 4 \frac{a^3 \ln(\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*tan(d*x+c),x)

[Out] $-1/3*a^3*\sin(d*x+c)^3/d-4*a^3*\sin(d*x+c)/d+4/d*a^3*\ln(\sec(d*x+c)+\tan(d*x+c))-3/2*a^3*\sin(d*x+c)^2/d-4/d*a^3*\ln(\cos(d*x+c))$

Maxima [A] time = 1.07111, size = 77, normalized size = 1.1

$$\frac{2 a^3 \sin(dx + c)^3 + 9 a^3 \sin(dx + c)^2 + 24 a^3 \log(\sin(dx + c) - 1) + 24 a^3 \sin(dx + c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")

[Out] $-1/6*(2*a^3*\sin(d*x + c)^3 + 9*a^3*\sin(d*x + c)^2 + 24*a^3*\log(\sin(d*x + c) - 1) + 24*a^3*\sin(d*x + c))/d$

Fricas [A] time = 1.51562, size = 147, normalized size = 2.1

$$\frac{9 a^3 \cos(dx + c)^2 - 24 a^3 \log(-\sin(dx + c) + 1) + 2 (a^3 \cos(dx + c)^2 - 13 a^3) \sin(dx + c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")

[Out] $1/6*(9*a^3*\cos(d*x + c)^2 - 24*a^3*\log(-\sin(d*x + c) + 1) + 2*(a^3*\cos(d*x + c)^2 - 13*a^3)*\sin(d*x + c))/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int 3 \sin(c + dx) \tan(c + dx) dx + \int 3 \sin^2(c + dx) \tan(c + dx) dx + \int \sin^3(c + dx) \tan(c + dx) dx + \int \tan(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x), x) + Integral(3*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(sin(c + d*x)**3*tan(c + d*x), x) + Integral(ta
n(c + d*x), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.28 $\int \cot^3(c + dx)(a + a \sin(c + dx))^3 dx$

Optimal. Leaf size=98

$$\frac{a^3 \sin^3(c + dx)}{3d} - \frac{3a^3 \sin^2(c + dx)}{2d} - \frac{2a^3 \sin(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} - \frac{3a^3 \csc(c + dx)}{d} + \frac{2a^3 \log(\sin(c + dx))}{d}$$

[Out] $(-3*a^3*\text{Csc}[c + d*x])/d - (a^3*\text{Csc}[c + d*x]^2)/(2*d) + (2*a^3*\text{Log}[\text{Sin}[c + d*x]])/d - (2*a^3*\text{Sin}[c + d*x])/d - (3*a^3*\text{Sin}[c + d*x]^2)/(2*d) - (a^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0648456, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 75}

$$\frac{a^3 \sin^3(c + dx)}{3d} - \frac{3a^3 \sin^2(c + dx)}{2d} - \frac{2a^3 \sin(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} - \frac{3a^3 \csc(c + dx)}{d} + \frac{2a^3 \log(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^3*(a + a*\text{Sin}[c + d*x])^3, x]$

[Out] $(-3*a^3*\text{Csc}[c + d*x])/d - (a^3*\text{Csc}[c + d*x]^2)/(2*d) + (2*a^3*\text{Log}[\text{Sin}[c + d*x]])/d - (2*a^3*\text{Sin}[c + d*x])/d - (3*a^3*\text{Sin}[c + d*x]^2)/(2*d) - (a^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 2707

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*\text{tan}[(e_.) + (f_.)*(x_.)]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 75

$\text{Int}[(d_.)*(x_.)^{(n_.)}*((a_.) + (b_.)*(x_.)*((e_.) + (f_.)*(x_.)^{(p_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p + 2, 0] && GtQ[n + 2*p, 0])

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + a \sin(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^4}{x^3} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a^2 + \frac{a^5}{x^3} + \frac{3a^4}{x^2} + \frac{2a^3}{x} - 3ax - x^2\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{3a^3 \csc(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} + \frac{2a^3 \log(\sin(c + dx))}{d} - \frac{2a^3 \sin(c + dx)}{d} - 3 \end{aligned}$$

Mathematica [A] time = 0.201659, size = 67, normalized size = 0.68

$$\frac{a^3 (2 \sin^3(c + dx) + 9 \sin^2(c + dx) + 12 \sin(c + dx) + 3 \csc^2(c + dx) + 18 \csc(c + dx) - 12 \log(\sin(c + dx)) + 30)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]

[Out] $-(a^3(30 + 18\text{Csc}[c + d*x] + 3\text{Csc}[c + d*x]^2 - 12\text{Log}[\text{Sin}[c + d*x]] + 12\text{Sin}[c + d*x] + 9\text{Sin}[c + d*x]^2 + 2\text{Sin}[c + d*x]^3))/(6*d)$

Maple [A] time = 0.053, size = 109, normalized size = 1.1

$$-\frac{8a^3 \sin(dx+c)(\cos(dx+c))^2}{3d} - \frac{16a^3 \sin(dx+c)}{3d} + \frac{3a^3(\cos(dx+c))^2}{2d} + 2\frac{a^3 \ln(\sin(dx+c))}{d} - 3\frac{a^3(\cos(dx+c))}{d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x)

[Out] $-8/3/d*a^3*\sin(d*x+c)*\cos(d*x+c)^2-16/3*a^3*\sin(d*x+c)/d+3/2/d*a^3*\cos(d*x+c)^2+2*a^3*\ln(\sin(d*x+c))/d-3/d*a^3/\sin(d*x+c)*\cos(d*x+c)^4-1/2/d*a^3*\cot(d*x+c)^2$

Maxima [A] time = 1.05985, size = 108, normalized size = 1.1

$$\frac{2a^3 \sin(dx+c)^3 + 9a^3 \sin(dx+c)^2 - 12a^3 \log(\sin(dx+c)) + 12a^3 \sin(dx+c) + \frac{3(6a^3 \sin(dx+c)+a^3)}{\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/6*(2*a^3*\sin(d*x + c)^3 + 9*a^3*\sin(d*x + c)^2 - 12*a^3*\log(\sin(d*x + c)) + 12*a^3*\sin(d*x + c) + 3*(6*a^3*\sin(d*x + c) + a^3)/\sin(d*x + c)^2)/d$

Fricas [A] time = 1.48336, size = 284, normalized size = 2.9

$$\frac{18a^3 \cos(dx+c)^4 - 27a^3 \cos(dx+c)^2 + 15a^3 + 24(a^3 \cos(dx+c)^2 - a^3) \log\left(\frac{1}{2} \sin(dx+c)\right) + 4(a^3 \cos(dx+c)^4 - 8a^3 \cos(dx+c)^2 + 16a^3) \sin(dx+c)}{12(d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $1/12*(18*a^3*\cos(d*x + c)^4 - 27*a^3*\cos(d*x + c)^2 + 15*a^3 + 24*(a^3*\cos(d*x + c)^2 - a^3)*\log(1/2*\sin(d*x + c)) + 4*(a^3*\cos(d*x + c)^4 - 8*a^3*\cos(d*x + c)^2 + 16*a^3)*\sin(d*x + c))/(d*\cos(d*x + c)^2 - d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int 3 \sin(c + dx) \cot^3(c + dx) dx + \int 3 \sin^2(c + dx) \cot^3(c + dx) dx + \int \sin^3(c + dx) \cot^3(c + dx) dx + \int \cot^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**3,x)

[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(3*sin(c + d*x)**2*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x))

Giac [A] time = 1.37156, size = 127, normalized size = 1.3

$$\frac{2a^3 \sin(dx+c)^3 + 9a^3 \sin(dx+c)^2 - 12a^3 \log(|\sin(dx+c)|) + 12a^3 \sin(dx+c) + \frac{3(6a^3 \sin(dx+c)^2 + 6a^3 \sin(dx+c) + a^3)}{\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(abs(sin(d*x + c))) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d

3.29 $\int (a + a \sin(c + dx))^3 \tan^6(c + dx) dx$

Optimal. Leaf size=180

$$\frac{136a^3 \cos^3(c + dx)}{15d} + \frac{136a^3 \cos(c + dx)}{5d} + \frac{23a^6 \sin^3(c + dx) \cos(c + dx)}{3d(a^3 - a^3 \sin(c + dx))} + \frac{a^6 \sin^5(c + dx) \cos(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \sin^4(c + dx)}{15d(a - a \sin(c + dx))}$$

```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c + d*x]^5)/(5*d*(a - a*SIN[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)/(15*d*(a - a*SIN[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a^3 - a^3*SIN[c + d*x]))
```

Rubi [A] time = 0.356664, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2708, 2765, 2977, 2748, 2635, 8, 2633}

$$\frac{136a^3 \cos^3(c + dx)}{15d} + \frac{136a^3 \cos(c + dx)}{5d} + \frac{23a^6 \sin^3(c + dx) \cos(c + dx)}{3d(a^3 - a^3 \sin(c + dx))} + \frac{a^6 \sin^5(c + dx) \cos(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \sin^4(c + dx)}{15d(a - a \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*SIN[c + d*x])^3*TAN[c + d*x]^6,x]
```

```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c + d*x]^5)/(5*d*(a - a*SIN[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)/(15*d*(a - a*SIN[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a^3 - a^3*SIN[c + d*x]))
```

Rule 2708

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[SIN[e + f*x]^p/(a - b*SIN[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[p, 2*m]
```

Rule 2765

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*(c + d*SIN[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*SIN[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*(c + d*SIN[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*SIN[e + f*x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
 \int (a + a \sin(c + dx))^3 \tan^6(c + dx) dx &= a^6 \int \frac{\sin^6(c + dx)}{(a - a \sin(c + dx))^3} dx \\
 &= \frac{a^6 \cos(c + dx) \sin^5(c + dx)}{5d(a - a \sin(c + dx))^3} + \frac{1}{5} a^4 \int \frac{\sin^4(c + dx)(-5a - 8a \sin(c + dx))}{(a - a \sin(c + dx))^2} dx \\
 &= \frac{a^6 \cos(c + dx) \sin^5(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \cos(c + dx) \sin^4(c + dx)}{15d(a - a \sin(c + dx))^2} - \frac{1}{15} a^2 \int \frac{\sin^3(c + dx)}{a - a \sin(c + dx)} dx \\
 &= \frac{a^6 \cos(c + dx) \sin^5(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \cos(c + dx) \sin^4(c + dx)}{15d(a - a \sin(c + dx))^2} + \frac{23a^4 \cos(c + dx) \sin^3(c + dx)}{3d(a - a \sin(c + dx))} \\
 &= \frac{a^6 \cos(c + dx) \sin^5(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \cos(c + dx) \sin^4(c + dx)}{15d(a - a \sin(c + dx))^2} + \frac{23a^4 \cos(c + dx) \sin^3(c + dx)}{3d(a - a \sin(c + dx))} \\
 &= \frac{23a^3 \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^6 \cos(c + dx) \sin^5(c + dx)}{5d(a - a \sin(c + dx))^3} - \frac{13a^5 \cos(c + dx) \sin^4(c + dx)}{15d(a - a \sin(c + dx))^2} \\
 &= -\frac{23a^3 x}{2} + \frac{136a^3 \cos(c + dx)}{5d} - \frac{136a^3 \cos^3(c + dx)}{15d} + \frac{23a^3 \cos(c + dx) \sin(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] time = 5.61378, size = 243, normalized size = 1.35

$$\frac{(a \sin(c + dx) + a)^3 \left(-690(c + dx) + 45 \sin(2(c + dx)) + 405 \cos(c + dx) - 5 \cos(3(c + dx)) \right) + \frac{1576 \sin\left(\frac{1}{2}(c + dx)\right)}{\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)} - \frac{1576 \sin\left(\frac{1}{2}(c + dx)\right)}{\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)}}{60d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]

[Out] ((a + a*Sin[c + d*x])^3*(-690*(c + d*x) + 405*Cos[c + d*x] - 5*Cos[3*(c + d*x)] + 12/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 - 112/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (24*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 - (224*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (1576*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 45*Sin[2*(c + d*x)]))/(60*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)

Maple [B] time = 0.131, size = 359, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x)

[Out] 1/d*(a^3*(1/5*sin(d*x+c)^10/cos(d*x+c)^5-1/3*sin(d*x+c)^10/cos(d*x+c)^3+7/3*sin(d*x+c)^10/cos(d*x+c)+7/3*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*in(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)+3*a^3*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^3*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))

Maxima [A] time = 1.648, size = 282, normalized size = 1.57

$$\frac{3 \left(6 \tan(dx + c)^5 - 20 \tan(dx + c)^3 - 105 dx - 105 c + \frac{15 \tan(dx + c)}{\tan(dx + c)^2 + 1} + 90 \tan(dx + c) \right) a^3 + 2 \left(3 \tan(dx + c)^5 - 5 \tan(dx + c)^3 - 15 dx - 15 c + 15 \tan(dx + c) \right) a^3 - 2 \left(5 \cos(dx + c)^3 - (90 \cos(dx + c)^4 - 20 \cos(dx + c)^2 + 3) / \cos(dx + c)^5 - 60 \cos(dx + c) \right) a^3 + 18 a^3 \left((15 \cos(dx + c)^4 - 5 \cos(dx + c)^2 + 1) / \cos(dx + c)^5 + 5 \cos(dx + c) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")

[Out] 1/30*(3*(6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x + c))/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^3 + 2*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 - 2*(5*cos(d*x + c)^3 - (90*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 3)/cos(d*x + c)^5 - 60*cos(d*x + c))*a^3 + 18*a^3*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d

Fricas [A] time = 1.59798, size = 734, normalized size = 4.08

$$\frac{10 a^3 \cos(dx + c)^6 - 15 a^3 \cos(dx + c)^5 - 140 a^3 \cos(dx + c)^4 - 1380 a^3 dx + (345 a^3 dx - 839 a^3) \cos(dx + c)^3 + 6 a^3 \cos(dx + c)^2 - 10 a^3 dx - 10 a^3 c + 15 a^3 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")

```
[Out] -1/30*(10*a^3*cos(d*x + c)^6 - 15*a^3*cos(d*x + c)^5 - 140*a^3*cos(d*x + c)^4 - 1380*a^3*d*x + (345*a^3*d*x - 839*a^3)*cos(d*x + c)^3 + 6*a^3 + (1035*a^3*d*x + 668*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 233*a^3)*cos(d*x + c) - (10*a^3*cos(d*x + c)^5 + 25*a^3*cos(d*x + c)^4 - 115*a^3*cos(d*x + c)^3 - 1380*a^3*d*x - 6*a^3 + (345*a^3*d*x + 724*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 232*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.30 $\int (a + a \sin(c + dx))^3 \tan^4(c + dx) dx$

Optimal. Leaf size=119

$$\frac{a^3 \cos^3(c + dx)}{3d} - \frac{6a^3 \cos(c + dx)}{d} - \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} - \frac{25a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))} + \frac{2a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} + \frac{17a^3}{2}$$

[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.194183, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{a^3 \cos^3(c + dx)}{3d} - \frac{6a^3 \cos(c + dx)}{d} - \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} - \frac{25a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))} + \frac{2a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} + \frac{17a^3}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]

[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^3 \tan^4(c + dx) dx &= a^4 \int \left(\frac{7}{a} + \frac{2}{a(-1 + \sin(c + dx))^2} + \frac{9}{a(-1 + \sin(c + dx))} + \frac{5 \sin(c + dx)}{a} + \frac{3 \sin^2(c + dx)}{a} \right) dx \\ &= 7a^3 x + a^3 \int \sin^3(c + dx) dx + (2a^3) \int \frac{1}{(-1 + \sin(c + dx))^2} dx + (3a^3) \int \sin^2(c + dx) dx \\ &= 7a^3 x - \frac{5a^3 \cos(c + dx)}{d} + \frac{2a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} - \frac{9a^3 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{3a^3 \cos(c + dx)}{d} \\ &= \frac{17a^3 x}{2} - \frac{6a^3 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d} + \frac{2a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} - \frac{25a^3 \cos(c + dx)}{3d(1 - \sin(c + dx))} \end{aligned}$$

Mathematica [A] time = 2.25007, size = 177, normalized size = 1.49

$$\frac{(a \sin(c + dx) + a)^3 \left(102(c + dx) - 9 \sin(2(c + dx)) - 69 \cos(c + dx) + \cos(3(c + dx)) - \frac{200 \sin\left(\frac{1}{2}(c + dx)\right)}{\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)} + \frac{16}{\cos\left(\frac{1}{2}(c + dx)\right)} \right)}{12d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^6}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]`

[Out] `((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)] + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)`

Maple [B] time = 0.085, size = 266, normalized size = 2.2

$$\frac{1}{d} \left(a^3 \left(\frac{(\sin(dx + c))^8}{3(\cos(dx + c))^3} - \frac{5(\sin(dx + c))^8}{3\cos(dx + c)} - \frac{5\cos(dx + c)}{3} \left(\frac{16}{5} + (\sin(dx + c))^6 + \frac{6(\sin(dx + c))^4}{5} + \frac{8(\sin(dx + c))^2}{5} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x)`

[Out] `1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s`

$$\frac{\sin(dx+c)^7/\cos(dx+c)^3 - 4/3 \sin(dx+c)^7/\cos(dx+c) - 4/3 (\sin(dx+c)^5 + 5/4 \sin(dx+c)^3 + 15/8 \sin(dx+c)) \cos(dx+c) + 5/2 dx + 5/2 c + 3a^3 (1/3 \sin(dx+c)^6/\cos(dx+c)^3 - \sin(dx+c)^6/\cos(dx+c) - (8/3 + \sin(dx+c)^4 + 4/3 \sin(dx+c)^2) \cos(dx+c)) + a^3 (1/3 \tan(dx+c)^3 - \tan(dx+c) + dx+c)}{6d}$$

Maxima [A] time = 1.61188, size = 223, normalized size = 1.87

$$\frac{2 \left(\cos(dx+c)^3 - \frac{9 \cos(dx+c)^2 - 1}{\cos(dx+c)^3} - 9 \cos(dx+c) \right) a^3 + 3 \left(2 \tan(dx+c)^3 + 15 dx + 15 c - \frac{3 \tan(dx+c)}{\tan(dx+c)^2 + 1} - 12 \tan(dx+c) \right) a^3}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(dx+c))^3*tan(dx+c)^4,x, algorithm="maxima")

[Out] $\frac{1}{6} * (2 * (\cos(dx+c)^3 - (9 * \cos(dx+c)^2 - 1) / \cos(dx+c)^3 - 9 * \cos(dx+c))) * a^3 + 3 * (2 * \tan(dx+c)^3 + 15 * dx + 15 * c - 3 * \tan(dx+c) / (\tan(dx+c)^2 + 1) - 12 * \tan(dx+c)) * a^3 + 2 * (\tan(dx+c)^3 + 3 * dx + 3 * c - 3 * \tan(dx+c)) * a^3 - 6 * a^3 * ((6 * \cos(dx+c)^2 - 1) / \cos(dx+c)^3 + 3 * \cos(dx+c))) / d$

Fricas [B] time = 1.5826, size = 539, normalized size = 4.53

$$\frac{2 a^3 \cos(dx+c)^5 + 7 a^3 \cos(dx+c)^4 - 22 a^3 \cos(dx+c)^3 - 102 a^3 dx - 4 a^3 + (51 a^3 dx + 77 a^3) \cos(dx+c)^2 - (51 a^3 dx + 77 a^3) \cos(dx+c)}{6 (d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(dx+c))^3*tan(dx+c)^4,x, algorithm="fricas")

[Out] $\frac{1}{6} * (2 * a^3 * \cos(dx+c)^5 + 7 * a^3 * \cos(dx+c)^4 - 22 * a^3 * \cos(dx+c)^3 - 102 * a^3 * dx - 4 * a^3 + (51 * a^3 * dx + 77 * a^3) * \cos(dx+c)^2 - (51 * a^3 * dx - 100 * a^3) * \cos(dx+c) + (2 * a^3 * \cos(dx+c)^4 - 5 * a^3 * \cos(dx+c)^3 + 102 * a^3 * dx - 27 * a^3 * \cos(dx+c)^2 - 4 * a^3 + (51 * a^3 * dx - 104 * a^3) * \cos(dx+c)) * \sin(dx+c)) / (d * \cos(dx+c)^2 - d * \cos(dx+c) + (d * \cos(dx+c) + 2 * d) * \sin(dx+c) - 2 * d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(dx+c))**3*tan(dx+c)**4,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.31 $\int (a + a \sin(c + dx))^3 \tan^2(c + dx) dx$

Optimal. Leaf size=89

$$-\frac{a^3 \cos^3(c + dx)}{3d} + \frac{5a^3 \cos(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{4a^3 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{11a^3 x}{2}$$

[Out] $(-11*a^3*x)/2 + (5*a^3*\text{Cos}[c + d*x])/d - (a^3*\text{Cos}[c + d*x]^3)/(3*d) + (4*a^3*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (3*a^3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rubi [A] time = 0.125111, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3 \cos^3(c + dx)}{3d} + \frac{5a^3 \cos(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{4a^3 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{11a^3 x}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^3*\text{Tan}[c + d*x]^2, x]$

[Out] $(-11*a^3*x)/2 + (5*a^3*\text{Cos}[c + d*x])/d - (a^3*\text{Cos}[c + d*x]^3)/(3*d) + (4*a^3*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (3*a^3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2709

$\text{Int}[(a + (b*\text{sin}[(e + f*x)])^m)*\text{tan}[(e + f*x)]^p, x_Symbol] \rightarrow \text{Dist}[a^p, \text{Int}[\text{ExpandIntegrand}[(\text{Sin}[e + f*x])^{p*(a + b*\text{Sin}[e + f*x])^{m - p/2}}]/(a - b*\text{Sin}[e + f*x])^{p/2}, x], x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 2648

$\text{Int}[(a + (b*\text{sin}[(c + d*x)])^{-1}), x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2638

$\text{Int}[\text{sin}[(c + d*x)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 2635

$\text{Int}[(b*\text{sin}[(c + d*x)])^n, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{n - 1}]/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{n - 2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a*x, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^3 \tan^2(c + dx) dx &= a^2 \int \left(-4a - \frac{4a}{-1 + \sin(c + dx)} - 4a \sin(c + dx) - 3a \sin^2(c + dx) - a \sin^3(c + dx) \right) dx \\ &= -4a^3 x - a^3 \int \sin^3(c + dx) dx - (3a^3) \int \sin^2(c + dx) dx - (4a^3) \int \frac{1}{-1 + \sin(c + dx)} dx \\ &= -4a^3 x + \frac{4a^3 \cos(c + dx)}{d} + \frac{4a^3 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} - \frac{1}{2} \int \frac{1}{-1 + \sin(c + dx)} dx \\ &= -\frac{11a^3 x}{2} + \frac{5a^3 \cos(c + dx)}{d} - \frac{a^3 \cos^3(c + dx)}{3d} + \frac{4a^3 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{3a^3 \cos(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.490405, size = 115, normalized size = 1.29

$$\frac{(a \sin(c + dx) + a)^3 \left(-66(c + dx) + 9 \sin(2(c + dx)) + 57 \cos(c + dx) - \cos(3(c + dx)) + \frac{96 \sin\left(\frac{1}{2}(c + dx)\right)}{\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)} \right)}{12d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]

[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)] + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)

Maple [A] time = 0.053, size = 167, normalized size = 1.9

$$\frac{1}{d} \left(a^3 \left(\frac{(\sin(dx + c))^6}{\cos(dx + c)} + \left(\frac{8}{3} + (\sin(dx + c))^4 + \frac{4(\sin(dx + c))^2}{3} \right) \cos(dx + c) \right) + 3a^3 \left(\frac{(\sin(dx + c))^5}{\cos(dx + c)} + ((\sin(dx + c))^3 + \sin(dx + c)) \cos(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x)

[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*(tan(d*x+c)-d*x-c))

Maxima [A] time = 1.57021, size = 158, normalized size = 1.78

$$2 \left(\cos(dx + c)^3 - \frac{3}{\cos(dx + c)} - 6 \cos(dx + c) \right) a^3 + 9 \left(3 dx + 3c - \frac{\tan(dx + c)}{\tan(dx + c)^2 + 1} - 2 \tan(dx + c) \right) a^3 + 6(dx + c - \tan(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")

[Out]
$$-1/6*(2*(\cos(dx + c)^3 - 3/\cos(dx + c) - 6*\cos(dx + c))*a^3 + 9*(3*d*x + 3*c - \tan(dx + c)/(\tan(dx + c)^2 + 1) - 2*\tan(dx + c))*a^3 + 6*(d*x + c - \tan(dx + c))*a^3 - 18*a^3*(1/\cos(dx + c) + \cos(dx + c)))/d$$

Fricas [A] time = 1.42002, size = 378, normalized size = 4.25

$$\frac{2a^3 \cos(dx + c)^4 - 7a^3 \cos(dx + c)^3 + 33a^3 dx - 30a^3 \cos(dx + c)^2 - 24a^3 + 3(11a^3 dx - 15a^3) \cos(dx + c) - (2a^3 \cos(dx + c)^3 + 33a^3 dx + 9a^3 \cos(dx + c)^2 - 21a^3 \cos(dx + c) + 24a^3) \sin(dx + c)}{6(d \cos(dx + c) - d \sin(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")

[Out]
$$-1/6*(2*a^3*\cos(dx + c)^4 - 7*a^3*\cos(dx + c)^3 + 33*a^3*d*x - 30*a^3*\cos(dx + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*\cos(dx + c) - (2*a^3*\cos(dx + c)^3 + 33*a^3*d*x + 9*a^3*\cos(dx + c)^2 - 21*a^3*\cos(dx + c) + 24*a^3)*\sin(dx + c))/(d*\cos(dx + c) - d*\sin(dx + c) + d)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int 3 \sin(c + dx) \tan^2(c + dx) dx + \int 3 \sin^2(c + dx) \tan^2(c + dx) dx + \int \sin^3(c + dx) \tan^2(c + dx) dx + \int \tan^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**2,x)

[Out]
$$a**3*(\text{Integral}(3*\sin(c + d*x)*\tan(c + d*x)**2, x) + \text{Integral}(3*\sin(c + d*x)**2*\tan(c + d*x)**2, x) + \text{Integral}(\sin(c + d*x)**3*\tan(c + d*x)**2, x) + \text{Integral}(\tan(c + d*x)**2, x))$$

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")

[Out] Timed out

3.32 $\int (a + a \sin(c + dx))^3 dx$

Optimal. Leaf size=63

$$\frac{a^3 \cos^3(c + dx)}{3d} - \frac{4a^3 \cos(c + dx)}{d} - \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{5a^3 x}{2}$$

[Out] $(5*a^3*x)/2 - (4*a^3*\text{Cos}[c + d*x])/d + (a^3*\text{Cos}[c + d*x]^3)/(3*d) - (3*a^3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rubi [A] time = 0.0544642, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{a^3 \cos^3(c + dx)}{3d} - \frac{4a^3 \cos(c + dx)}{d} - \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{5a^3 x}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^3,x]

[Out] $(5*a^3*x)/2 - (4*a^3*\text{Cos}[c + d*x])/d + (a^3*\text{Cos}[c + d*x]^3)/(3*d) - (3*a^3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2645

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx))^3 dx &= \int (a^3 + 3a^3 \sin(c + dx) + 3a^3 \sin^2(c + dx) + a^3 \sin^3(c + dx)) dx \\
&= a^3 x + a^3 \int \sin^3(c + dx) dx + (3a^3) \int \sin(c + dx) dx + (3a^3) \int \sin^2(c + dx) dx \\
&= a^3 x - \frac{3a^3 \cos(c + dx)}{d} - \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} + \frac{1}{2} (3a^3) \int 1 dx - \frac{a^3 \text{Subst} \left(\int (1 - \sin^2(u)) du \right)}{d} \\
&= \frac{5a^3 x}{2} - \frac{4a^3 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d} - \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.326201, size = 44, normalized size = 0.7

$$\frac{a^3(-9 \sin(2(c + dx)) - 45 \cos(c + dx) + \cos(3(c + dx)) + 30c + 30dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3,x]

[Out] (a^3*(30*c + 30*d*x - 45*Cos[c + d*x] + Cos[3*(c + d*x)] - 9*Sin[2*(c + d*x)])/(12*d)

Maple [A] time = 0.027, size = 74, normalized size = 1.2

$$\frac{1}{d} \left(-\frac{a^3 (2 + (\sin(dx + c))^2) \cos(dx + c)}{3} + 3a^3 (-1/2 \cos(dx + c) \sin(dx + c) + 1/2 dx + c/2) - 3a^3 \cos(dx + c) + a^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3,x)

[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-3*a^3*cos(d*x+c)+a^3*(d*x+c))

Maxima [A] time = 1.07352, size = 97, normalized size = 1.54

$$a^3 x + \frac{(\cos(dx + c)^3 - 3 \cos(dx + c))a^3}{3d} + \frac{3(2dx + 2c - \sin(2dx + 2c))a^3}{4d} - \frac{3a^3 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] a^3*x + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^3/d - 3*a^3*cos(d*x + c)/d

Fricas [A] time = 1.45802, size = 134, normalized size = 2.13

$$\frac{2a^3 \cos(dx + c)^3 + 15a^3 dx - 9a^3 \cos(dx + c) \sin(dx + c) - 24a^3 \cos(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*a^3*\cos(d*x + c)^3 + 15*a^3*d*x - 9*a^3*\cos(d*x + c)*\sin(d*x + c) - 24*a^3*\cos(d*x + c))/d$

Sympy [A] time = 0.655117, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^2(c+dx)}{2} + a^3x - \frac{a^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{2a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos(c+dx)}{d} \\ x(a\sin(c) + a)^3 \end{cases} \text{ for } d \neq 0 \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**3,x)

[Out] Piecewise(((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x - a**3*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**3*cos(c + d*x)**3/(3*d) - 3*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**3, True))

Giac [A] time = 1.64017, size = 74, normalized size = 1.17

$$\frac{5}{2}a^3x + \frac{a^3\cos(3dx + 3c)}{12d} - \frac{15a^3\cos(dx + c)}{4d} - \frac{3a^3\sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{5}{2}*a^3*x + \frac{1}{12}*a^3*\cos(3*d*x + 3*c)/d - \frac{15}{4}*a^3*\cos(d*x + c)/d - \frac{3}{4}*a^3*\sin(2*d*x + 2*c)/d$

3.33 $\int \cot^2(c + dx)(a + a \sin(c + dx))^3 dx$

Optimal. Leaf size=92

$$-\frac{a^3 \cos^3(c + dx)}{3d} + \frac{3a^3 \cos(c + dx)}{d} - \frac{a^3 \cot(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{a^3 x}{2}$$

[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.136857, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3 \cos^3(c + dx)}{3d} + \frac{3a^3 \cos(c + dx)}{d} - \frac{a^3 \cot(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{a^3 x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]

[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)^(n_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + a \sin(c + dx))^3 dx &= \frac{\int (2a^5 + 3a^5 \csc(c + dx) + a^5 \csc^2(c + dx) - 2a^5 \sin(c + dx) - 3a^5 \sin^2(c + dx) - \dots)}{a^2} \\ &= 2a^3x + a^3 \int \csc^2(c + dx) dx - a^3 \int \sin^3(c + dx) dx - (2a^3) \int \sin(c + dx) dx + \dots \\ &= 2a^3x - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2a^3 \cos(c + dx)}{d} + \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} \\ &= \frac{a^3x}{2} - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{3a^3 \cos(c + dx)}{d} - \frac{a^3 \cos^3(c + dx)}{3d} - \frac{a^3 \cot(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 1.06475, size = 106, normalized size = 1.15

$$\frac{a^3 \csc\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{1}{2}(c + dx)\right) \left((15 - 66 \sin(c + dx)) \cos(c + dx) + (2 \sin(c + dx) + 9) \cos(3(c + dx)) - 12 \sin(c + dx) \right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c + d*x]
) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*Sin[c
+ d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/(48*d)
```

Maple [A] time = 0.043, size = 105, normalized size = 1.1

$$-\frac{a^3 (\cos(dx + c))^3}{3d} + \frac{3a^3 \cos(dx + c) \sin(dx + c)}{2d} + \frac{a^3x}{2} + \frac{a^3c}{2d} + 3 \frac{a^3 \ln(\csc(dx + c) - \cot(dx + c))}{d} + 3 \frac{a^3 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3
*c+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^3*cos(d*x+c)/d-a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.63734, size = 126, normalized size = 1.37

$$\frac{4a^3 \cos(dx + c)^3 - 9(2dx + 2c + \sin(2dx + 2c))a^3 + 12\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^3 - 18a^3(2 \cos(dx + c) - \log(\cos(dx + c)))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/12*(4*a^3*\cos(d*x + c)^3 - 9*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^3 + 12*(d*x + c + 1/\tan(d*x + c))*a^3 - 18*a^3*(2*\cos(d*x + c) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)))/d$

Fricas [A] time = 1.56613, size = 321, normalized size = 3.49

$$\frac{9a^3 \cos(dx + c)^3 + 9a^3 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 9a^3 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 3a^3 \cos(dx + c)}{6d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/6*(9*a^3*\cos(d*x + c)^3 + 9*a^3*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 9*a^3*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 3*a^3*\cos(d*x + c) + (2*a^3*\cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*\cos(d*x + c))*\sin(d*x + c))/(d*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int 3 \sin(c + dx) \cot^2(c + dx) dx + \int 3 \sin^2(c + dx) \cot^2(c + dx) dx + \int \sin^3(c + dx) \cot^2(c + dx) dx + \int \cot^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**3,x)

[Out] $a**3*(Integral(3*\sin(c + d*x)*\cot(c + d*x)**2, x) + Integral(3*\sin(c + d*x)**2*\cot(c + d*x)**2, x) + Integral(\sin(c + d*x)**3*\cot(c + d*x)**2, x) + Integral(\cot(c + d*x)**2, x))$

Giac [A] time = 2.05943, size = 219, normalized size = 2.38

$$\frac{3(dx + c)a^3 + 18a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + 3a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \frac{3\left(6a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a^3\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)} - \frac{2\left(9a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^5 - 12a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)} - 12a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] $1/6*(3*(d*x + c)*a^3 + 18*a^3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + 3*a^3*\tan(1/2*d*x + 1/2*c) - 3*(6*a^3*\tan(1/2*d*x + 1/2*c) + a^3)/\tan(1/2*d*x + 1/2*c) - 2*(9*a^3*\tan(1/2*d*x + 1/2*c)^5 - 12*a^3*\tan(1/2*d*x + 1/2*c)^4 - 36*a^3*\tan(1/2*d*x + 1/2*c)^2 - 9*a^3*\tan(1/2*d*x + 1/2*c) - 16*a^3)/(\tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d$

3.34 $\int (a + a \sin(c + dx))^4 \tan^5(c + dx) dx$

Optimal. Leaf size=129

$$\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{9a^4 \sin^2(c + dx)}{2d} + \frac{a^6}{d(a - a \sin(c + dx))^2} - \frac{11a^5}{d(a - a \sin(c + dx))} - \frac{16a^4 \sin(c + dx)}{d}$$

[Out] $(-25*a^4*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (16*a^4*\text{Sin}[c + d*x])/d - (9*a^4*\text{Sin}[c + d*x]^2)/(2*d) - (4*a^4*\text{Sin}[c + d*x]^3)/(3*d) - (a^4*\text{Sin}[c + d*x]^4)/(4*d) + a^6/(d*(a - a*\text{Sin}[c + d*x])^2) - (11*a^5)/(d*(a - a*\text{Sin}[c + d*x]))$

Rubi [A] time = 0.0947062, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 77}

$$\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{9a^4 \sin^2(c + dx)}{2d} + \frac{a^6}{d(a - a \sin(c + dx))^2} - \frac{11a^5}{d(a - a \sin(c + dx))} - \frac{16a^4 \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^4*\text{Tan}[c + d*x]^5, x]$

[Out] $(-25*a^4*\text{Log}[1 - \text{Sin}[c + d*x]])/d - (16*a^4*\text{Sin}[c + d*x])/d - (9*a^4*\text{Sin}[c + d*x]^2)/(2*d) - (4*a^4*\text{Sin}[c + d*x]^3)/(3*d) - (a^4*\text{Sin}[c + d*x]^4)/(4*d) + a^6/(d*(a - a*\text{Sin}[c + d*x])^2) - (11*a^5)/(d*(a - a*\text{Sin}[c + d*x]))$

Rule 2707

$\text{Int}[(a + b*\text{sin}[(e + f)*(x)])^m*\text{tan}[(e + f)*(x)]^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

$\text{Int}[(a + b*(x))*(c + d*(x))^n*((e + f)*(x))^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^4 \tan^5(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{5(a+x)}}{(a-x)^3} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-16a^3 + \frac{2a^6}{(a-x)^3} - \frac{11a^5}{(a-x)^2} + \frac{25a^4}{a-x} - 9a^2x - 4ax^2 - x^3\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{25a^4 \log(1 - \sin(c + dx))}{d} - \frac{16a^4 \sin(c + dx)}{d} - \frac{9a^4 \sin^2(c + dx)}{2d} - \frac{4a^4 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.469245, size = 83, normalized size = 0.64

$$\frac{a^4 \left(3 \sin^4(c + dx) + 16 \sin^3(c + dx) + 54 \sin^2(c + dx) + 192 \sin(c + dx) + \frac{120 - 132 \sin(c + dx)}{(\sin(c + dx) - 1)^2} + 300 \log(1 - \sin(c + dx)) \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]

[Out] -(a^4*(300*Log[1 - Sin[c + d*x]] + (120 - 132*Sin[c + d*x])/(-1 + Sin[c + d*x])^2 + 192*Sin[c + d*x] + 54*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)

Maple [B] time = 0.1, size = 387, normalized size = 3.

$$\frac{5 a^4 (\sin(dx + c))^7}{2d} - 4 \frac{a^4 (\sin(dx + c))^6}{d} - 5 \frac{a^4 (\sin(dx + c))^5}{d} - \frac{3 a^4 (\sin(dx + c))^8}{4d} + \frac{a^4 (\sin(dx + c))^{10}}{4d (\cos(dx + c))^4} - \frac{3 a^4 (\sin(dx + c))^{10}}{4d (\cos(dx + c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x)

[Out] -5/2/d*a^4*sin(d*x+c)^7-4/d*a^4*sin(d*x+c)^6-5/d*a^4*sin(d*x+c)^5-3/4/d*a^4*sin(d*x+c)^8+1/4/d*a^4*sin(d*x+c)^10/cos(d*x+c)^4-3/4/d*a^4*sin(d*x+c)^10/cos(d*x+c)^2+1/d*a^4*sin(d*x+c)^9/cos(d*x+c)^4-5/2/d*a^4*sin(d*x+c)^9/cos(d*x+c)^2+3/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^4-3/d*a^4*sin(d*x+c)^8/cos(d*x+c)^2+1/d*a^4*sin(d*x+c)^7/cos(d*x+c)^4-3/2/d*a^4*sin(d*x+c)^7/cos(d*x+c)^2+1/4/d*a^4*tan(d*x+c)^4-1/2/d*a^4*tan(d*x+c)^2-25/3*a^4*sin(d*x+c)^3/d-25*a^4*sin(d*x+c)/d+25/d*a^4*ln(sec(d*x+c)+tan(d*x+c))-6*a^4*sin(d*x+c)^4/d-12*a^4*sin(d*x+c)^2/d-25/d*a^4*ln(cos(d*x+c))

Maxima [A] time = 1.10668, size = 147, normalized size = 1.14

$$\frac{3 a^4 \sin(dx + c)^4 + 16 a^4 \sin(dx + c)^3 + 54 a^4 \sin(dx + c)^2 + 300 a^4 \log(\sin(dx + c) - 1) + 192 a^4 \sin(dx + c) - \frac{12 (\sin(dx + c) - 1)^2}{\sin(dx + c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="maxima")

[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 54*a^4*sin(d*x + c)^2 + 300*a^4*log(sin(d*x + c) - 1) + 192*a^4*sin(d*x + c) - 12*(11*a^4*sin(d*x + c) - 10*a^4)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d

Fricas [A] time = 1.6719, size = 394, normalized size = 3.05

$$\frac{24 a^4 \cos(dx + c)^6 - 272 a^4 \cos(dx + c)^4 - 2393 a^4 \cos(dx + c)^2 + 1906 a^4 + 2400 (a^4 \cos(dx + c)^2 + 2 a^4 \sin(dx + c) \cos(dx + c) - a^4 \sin^2(dx + c))}{96 (d \cos(dx + c)^2 + 2 d \sin(dx + c) \cos(dx + c) - d \sin^2(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 272*a^4*cos(d*x + c)^4 - 2393*a^4*cos(d*x +
c)^2 + 1906*a^4 + 2400*(a^4*cos(d*x + c)^2 + 2*a^4*sin(d*x + c) - 2*a^4)*lo
g(-sin(d*x + c) + 1) - 10*(8*a^4*cos(d*x + c)^4 - 96*a^4*cos(d*x + c)^2 + 1
81*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.35 $\int (a + a \sin(c + dx))^4 \tan^3(c + dx) dx$

Optimal. Leaf size=107

$$\frac{a^4 \sin^4(c + dx)}{4d} + \frac{4a^4 \sin^3(c + dx)}{3d} + \frac{4a^4 \sin^2(c + dx)}{d} + \frac{4a^5}{d(a - a \sin(c + dx))} + \frac{12a^4 \sin(c + dx)}{d} + \frac{16a^4 \log(1 - \sin(c + dx))}{d}$$

[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^5)/(d*(a - a*Sin[c + d*x]))

Rubi [A] time = 0.0786493, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$\frac{a^4 \sin^4(c + dx)}{4d} + \frac{4a^4 \sin^3(c + dx)}{3d} + \frac{4a^4 \sin^2(c + dx)}{d} + \frac{4a^5}{d(a - a \sin(c + dx))} + \frac{12a^4 \sin(c + dx)}{d} + \frac{16a^4 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]

[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^5)/(d*(a - a*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^4 \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(a-x)^2} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(12a^3 + \frac{4a^5}{(a-x)^2} - \frac{16a^4}{a-x} + 8a^2x + 4ax^2 + x^3\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{16a^4 \log(1 - \sin(c + dx))}{d} + \frac{12a^4 \sin(c + dx)}{d} + \frac{4a^4 \sin^2(c + dx)}{d} + \frac{4a^4 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.155479, size = 76, normalized size = 0.71

$$\frac{a^4 \left(3 \sin^4(c + dx) + 16 \sin^3(c + dx) + 48 \sin^2(c + dx) + 144 \sin(c + dx) + \frac{48}{1 - \sin(c + dx)} + 192 \log(1 - \sin(c + dx)) \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]

[Out] (a^4*(192*Log[1 - Sin[c + d*x]] + 48/(1 - Sin[c + d*x]) + 144*Sin[c + d*x] + 48*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)

Maple [B] time = 0.084, size = 245, normalized size = 2.3

$$\frac{a^4 (\sin(dx + c))^8}{2d (\cos(dx + c))^2} + \frac{a^4 (\sin(dx + c))^6}{2d} + \frac{15 a^4 (\sin(dx + c))^4}{4d} + \frac{15 a^4 (\sin(dx + c))^2}{2d} + 16 \frac{a^4 \ln(\cos(dx + c))}{d} + 2 \frac{a^4 (\sin(dx + c))}{d (\cos(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x)

[Out] 1/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^2+1/2/d*a^4*sin(d*x+c)^6+15/4*a^4*sin(d*x+c)^4/d+15/2*a^4*sin(d*x+c)^2/d+16/d*a^4*ln(cos(d*x+c))+2/d*a^4*sin(d*x+c)^7/cos(d*x+c)^2+2/d*a^4*sin(d*x+c)^5+16/3*a^4*sin(d*x+c)^3/d+16*a^4*sin(d*x+c)/d-16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^4*sin(d*x+c)^6/cos(d*x+c)^2+2/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^4*tan(d*x+c)^2

Maxima [A] time = 1.06484, size = 115, normalized size = 1.07

$$\frac{3 a^4 \sin(dx + c)^4 + 16 a^4 \sin(dx + c)^3 + 48 a^4 \sin(dx + c)^2 + 192 a^4 \log(\sin(dx + c) - 1) + 144 a^4 \sin(dx + c) - \frac{48 a^4}{\sin(dx + c)}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 48*a^4*sin(d*x + c)^2 + 192*a^4*log(sin(d*x + c) - 1) + 144*a^4*sin(d*x + c) - 48*a^4/(sin(d*x + c) - 1))/d

Fricas [A] time = 1.5324, size = 293, normalized size = 2.74

$$\frac{104 a^4 \cos(dx + c)^4 - 976 a^4 \cos(dx + c)^2 + 689 a^4 + 1536 (a^4 \sin(dx + c) - a^4) \log(-\sin(dx + c) + 1) + (24 a^4 \cos(dx + c) - 304 a^4) \sin(dx + c)}{96 (d \sin(dx + c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="fricas")

[Out] 1/96*(104*a^4*cos(d*x + c)^4 - 976*a^4*cos(d*x + c)^2 + 689*a^4 + 1536*(a^4*sin(d*x + c) - a^4)*log(-sin(d*x + c) + 1) + (24*a^4*cos(d*x + c)^4 - 304*a^4*cos(d*x + c)^2 - 1073*a^4)*sin(d*x + c))/(d*sin(d*x + c) - d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.36 $\int (a + a \sin(c + dx))^4 \tan(c + dx) dx$

Optimal. Leaf size=88

$$-\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{7a^4 \sin^2(c + dx)}{2d} - \frac{8a^4 \sin(c + dx)}{d} - \frac{8a^4 \log(1 - \sin(c + dx))}{d}$$

[Out] $(-8a^4 \text{Log}[1 - \text{Sin}[c + d*x]])/d - (8a^4 \text{Sin}[c + d*x])/d - (7a^4 \text{Sin}[c + d*x]^2)/(2*d) - (4a^4 \text{Sin}[c + d*x]^3)/(3*d) - (a^4 \text{Sin}[c + d*x]^4)/(4*d)$

Rubi [A] time = 0.0526452, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 77}

$$-\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{7a^4 \sin^2(c + dx)}{2d} - \frac{8a^4 \sin(c + dx)}{d} - \frac{8a^4 \log(1 - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^4*\text{Tan}[c + d*x],x]$

[Out] $(-8a^4 \text{Log}[1 - \text{Sin}[c + d*x]])/d - (8a^4 \text{Sin}[c + d*x])/d - (7a^4 \text{Sin}[c + d*x]^2)/(2*d) - (4a^4 \text{Sin}[c + d*x]^3)/(3*d) - (a^4 \text{Sin}[c + d*x]^4)/(4*d)$

Rule 2707

$\text{Int}[(a + b*\text{sin}[(e + f)*(x)])^{m}*\text{tan}[(e + f)*(x)]^{p}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{((p + 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /;$ $\text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[(p + 1)/2]$

Rule 77

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \mid \mid \text{EqQ}[p, 1] \mid \mid (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \mid \mid \text{LeQ}[9*p + 5*(n + 2), 0] \mid \mid \text{GeQ}[n + p + 1, 0] \mid \mid (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^4 \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+x)^3}}{a-x} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-8a^3 + \frac{8a^4}{a-x} - 7a^2x - 4ax^2 - x^3\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{8a^4 \log(1 - \sin(c + dx))}{d} - \frac{8a^4 \sin(c + dx)}{d} - \frac{7a^4 \sin^2(c + dx)}{2d} - \frac{4a^4 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0699065, size = 62, normalized size = 0.7

$$\frac{a^4 (3 \sin^4(c + dx) + 16 \sin^3(c + dx) + 42 \sin^2(c + dx) + 96 \sin(c + dx) + 96 \log(1 - \sin(c + dx)))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]

[Out] $-(a^4(96\text{Log}[1 - \text{Sin}[c + d*x]] + 96\text{Sin}[c + d*x] + 42\text{Sin}[c + d*x]^2 + 16\text{Sin}[c + d*x]^3 + 3\text{Sin}[c + d*x]^4))/(12*d)$

Maple [A] time = 0.052, size = 101, normalized size = 1.2

$$\frac{a^4 (\sin(dx + c))^4}{4d} - \frac{7a^4 (\sin(dx + c))^2}{2d} - 8 \frac{a^4 \ln(\cos(dx + c))}{d} - \frac{4a^4 (\sin(dx + c))^3}{3d} - 8 \frac{a^4 \sin(dx + c)}{d} + 8 \frac{a^4 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4*tan(d*x+c),x)

[Out] $-1/4*a^4*\sin(d*x+c)^4/d-7/2*a^4*\sin(d*x+c)^2/d-8/d*a^4*\ln(\cos(d*x+c))-4/3*a^4*\sin(d*x+c)^3/d-8*a^4*\sin(d*x+c)/d+8/d*a^4*\ln(\sec(d*x+c)+\tan(d*x+c))$

Maxima [A] time = 1.07582, size = 95, normalized size = 1.08

$$\frac{3a^4 \sin(dx + c)^4 + 16a^4 \sin(dx + c)^3 + 42a^4 \sin(dx + c)^2 + 96a^4 \log(\sin(dx + c) - 1) + 96a^4 \sin(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="maxima")

[Out] $-1/12*(3*a^4*\sin(d*x + c)^4 + 16*a^4*\sin(d*x + c)^3 + 42*a^4*\sin(d*x + c)^2 + 96*a^4*\log(\sin(d*x + c) - 1) + 96*a^4*\sin(d*x + c))/d$

Fricas [A] time = 1.53773, size = 182, normalized size = 2.07

$$\frac{3a^4 \cos(dx + c)^4 - 48a^4 \cos(dx + c)^2 + 96a^4 \log(-\sin(dx + c) + 1) - 16(a^4 \cos(dx + c)^2 - 7a^4) \sin(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="fricas")

[Out] $-1/12*(3*a^4*\cos(d*x + c)^4 - 48*a^4*\cos(d*x + c)^2 + 96*a^4*\log(-\sin(d*x + c) + 1) - 16*(a^4*\cos(d*x + c)^2 - 7*a^4)*\sin(d*x + c))/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4 \left(\int 4 \sin(c + dx) \tan(c + dx) dx + \int 6 \sin^2(c + dx) \tan(c + dx) dx + \int 4 \sin^3(c + dx) \tan(c + dx) dx + \int \sin^4(c + dx) \tan(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c),x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x), x) + Integral(6*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(4*sin(c + d*x)**3*tan(c + d*x), x) + Integral(
sin(c + d*x)**4*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.37 $\int \cot^3(c + dx)(a + a \sin(c + dx))^4 dx$

Optimal. Leaf size=102

$$\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{5a^4 \sin^2(c + dx)}{2d} - \frac{a^4 \csc^2(c + dx)}{2d} - \frac{4a^4 \csc(c + dx)}{d} + \frac{5a^4 \log(\sin(c + dx))}{d}$$

[Out] $(-4*a^4*\text{Csc}[c + d*x])/d - (a^4*\text{Csc}[c + d*x]^2)/(2*d) + (5*a^4*\text{Log}[\text{Sin}[c + d*x]])/d - (5*a^4*\text{Sin}[c + d*x]^2)/(2*d) - (4*a^4*\text{Sin}[c + d*x]^3)/(3*d) - (a^4*\text{Sin}[c + d*x]^4)/(4*d)$

Rubi [A] time = 0.0667044, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 75}

$$\frac{a^4 \sin^4(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} - \frac{5a^4 \sin^2(c + dx)}{2d} - \frac{a^4 \csc^2(c + dx)}{2d} - \frac{4a^4 \csc(c + dx)}{d} + \frac{5a^4 \log(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^3*(a + a*\text{Sin}[c + d*x])^4, x]$

[Out] $(-4*a^4*\text{Csc}[c + d*x])/d - (a^4*\text{Csc}[c + d*x]^2)/(2*d) + (5*a^4*\text{Log}[\text{Sin}[c + d*x]])/d - (5*a^4*\text{Sin}[c + d*x]^2)/(2*d) - (4*a^4*\text{Sin}[c + d*x]^3)/(3*d) - (a^4*\text{Sin}[c + d*x]^4)/(4*d)$

Rule 2707

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))^{(m_)}*\tan[(e_ + (f_)*(x_))]^{(p_)}, x_Symbol] :> \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{((p + 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[(p + 1)/2]$

Rule 75

$\text{Int}[(d_)*(x_)]^{(n_)}*((a_ + (b_)*(x_))*((e_ + (f_)*(x_)))^{(p_)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b*e + a*f, 0] \&\& !(\text{ILtQ}[n + p + 2, 0] \&\& \text{GtQ}[n + 2*p, 0])$

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + a \sin(c + dx))^4 dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^5}{x^3} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^3} + \frac{4a^5}{x^2} + \frac{5a^4}{x} - 5a^2x - 4ax^2 - x^3\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= -\frac{4a^4 \csc(c + dx)}{d} - \frac{a^4 \csc^2(c + dx)}{2d} + \frac{5a^4 \log(\sin(c + dx))}{d} - \frac{5a^4 \sin^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.137296, size = 78, normalized size = 0.76

$$\frac{a^4 \sin^4(c + dx) (6 \csc^6(c + dx) + 48 \csc^5(c + dx) + 30 \csc^2(c + dx) + 16 \csc(c + dx) + \csc^4(c + dx)(90 - 60 \log(\sin(c + dx))))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]

[Out] $-(a^4(3 + 16\text{Csc}[c + d*x] + 30\text{Csc}[c + d*x]^2 + 48\text{Csc}[c + d*x]^5 + 6\text{Csc}[c + d*x]^6 + \text{Csc}[c + d*x]^4(90 - 60\text{Log}[\text{Sin}[c + d*x]]))\text{Sin}[c + d*x]^4)/(12*d)$

Maple [A] time = 0.054, size = 125, normalized size = 1.2

$$-\frac{a^4(\cos(dx+c))^4}{4d} - \frac{8a^4\sin(dx+c)(\cos(dx+c))^2}{3d} - \frac{16a^4\sin(dx+c)}{3d} + 3\frac{a^4(\cos(dx+c))^2}{d} + 5\frac{a^4\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x)

[Out] $-1/4/d*a^4*\cos(d*x+c)^4-8/3/d*a^4*\sin(d*x+c)*\cos(d*x+c)^2-16/3*a^4*\sin(d*x+c)/d+3/d*a^4*\cos(d*x+c)^2+5*a^4*\ln(\sin(d*x+c))/d-4/d*a^4/\sin(d*x+c)*\cos(d*x+c)^4-1/2/d*a^4*\cot(d*x+c)^2$

Maxima [A] time = 1.11802, size = 111, normalized size = 1.09

$$\frac{3a^4\sin(dx+c)^4 + 16a^4\sin(dx+c)^3 + 30a^4\sin(dx+c)^2 - 60a^4\log(\sin(dx+c)) + \frac{6(8a^4\sin(dx+c)+a^4)}{\sin(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] $-1/12*(3*a^4*\sin(d*x+c)^4 + 16*a^4*\sin(d*x+c)^3 + 30*a^4*\sin(d*x+c)^2 - 60*a^4*\log(\sin(d*x+c)) + 6*(8*a^4*\sin(d*x+c) + a^4)/\sin(d*x+c)^2)/d$

Fricas [A] time = 1.73429, size = 324, normalized size = 3.18

$$\frac{24a^4\cos(dx+c)^6 - 312a^4\cos(dx+c)^4 + 423a^4\cos(dx+c)^2 - 183a^4 - 480(a^4\cos(dx+c)^2 - a^4)\log\left(\frac{1}{2}\sin(dx+c)\right)}{96(d\cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] $-1/96*(24*a^4*\cos(d*x+c)^6 - 312*a^4*\cos(d*x+c)^4 + 423*a^4*\cos(d*x+c)^2 - 183*a^4 - 480*(a^4*\cos(d*x+c)^2 - a^4)*\log(1/2*\sin(d*x+c)) - 128*(a^4*\cos(d*x+c)^4 - 2*a^4*\cos(d*x+c)^2 + 4*a^4)*\sin(d*x+c))/(d*\cos(d*x+c)^2 - d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.39327, size = 130, normalized size = 1.27

$$\frac{3 a^4 \sin(dx+c)^4 + 16 a^4 \sin(dx+c)^3 + 30 a^4 \sin(dx+c)^2 - 60 a^4 \log(|\sin(dx+c)|) + \frac{6(15 a^4 \sin(dx+c)^2 + 8 a^4 \sin(dx+c) + a^4)}{\sin(dx+c)^2}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x + c)))) + 6*(15*a^4*sin(d*x + c)^2 + 8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2/d

3.38 $\int (a + a \sin(c + dx))^4 \tan^4(c + dx) dx$

Optimal. Leaf size=143

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{16a^4 \cos(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{35a^4 \sin(c + dx) \cos(c + dx)}{8d} - \frac{56a^4 \cos(c + dx)}{3d(1 - \sin(c + dx))}$$

[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rubi [A] time = 0.198685, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{16a^4 \cos(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{35a^4 \sin(c + dx) \cos(c + dx)}{8d} - \frac{56a^4 \cos(c + dx)}{3d(1 - \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]

[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635


```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^4 \tan^4(c + dx) dx &= a^4 \int \left(16 + \frac{4}{(-1 + \sin(c + dx))^2} + \frac{20}{-1 + \sin(c + dx)} + 12 \sin(c + dx) + 8 \sin^2 \right. \\ &= 16a^4 x + a^4 \int \sin^4(c + dx) dx + (4a^4) \int \frac{1}{(-1 + \sin(c + dx))^2} dx + (4a^4) \int \sin \\ &= 16a^4 x - \frac{12a^4 \cos(c + dx)}{d} + \frac{4a^4 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} - \frac{20a^4 \cos(c + dx)}{d(1 - \sin(c + dx))} - \frac{4a^4}{3d} \\ &= 20a^4 x - \frac{16a^4 \cos(c + dx)}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{4a^4 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} - \frac{56a^4}{3d} \\ &= \frac{163a^4 x}{8} - \frac{16a^4 \cos(c + dx)}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{4a^4 \cos(c + dx)}{3d(1 - \sin(c + dx))^2} - \frac{56}{3d} \end{aligned}$$

Mathematica [A] time = 1.65143, size = 252, normalized size = 1.76

$$a^4 \left(-11736c \sin\left(\frac{1}{2}(c + dx)\right) - 11736dx \sin\left(\frac{1}{2}(c + dx)\right) - 16488 \sin\left(\frac{1}{2}(c + dx)\right) - 3912c \sin\left(\frac{3}{2}(c + dx)\right) - 3912dx \sin\left(\frac{3}{2}(c + dx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

Maple [B] time = 0.092, size = 360, normalized size = 2.5

$$\frac{1}{d} \left(a^4 \left(\frac{(\sin(dx + c))^9}{3(\cos(dx + c))^3} - 2 \frac{(\sin(dx + c))^9}{\cos(dx + c)} - 2 \left((\sin(dx + c))^7 + 7/6 (\sin(dx + c))^5 + \frac{35(\sin(dx + c))^3}{24} + \frac{35 \sin(dx + c)}{16} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x)`

[Out] $\frac{1}{d} \left(a^4 \left(\frac{1}{3} \sin(d*x+c)^9 / \cos(d*x+c)^3 - 2 \sin(d*x+c)^9 / \cos(d*x+c) - 2 (\sin(d*x+c)^7 + 7/6 \sin(d*x+c)^5 + 35/24 \sin(d*x+c)^3 + 35/16 \sin(d*x+c)) \cos(d*x+c) + 35/8 d*x + 35/8 c \right) + 4 a^4 \left(\frac{1}{3} \sin(d*x+c)^8 / \cos(d*x+c)^3 - 5/3 \sin(d*x+c)^8 / \cos(d*x+c) - 5/3 (16/5 + \sin(d*x+c)^6 + 6/5 \sin(d*x+c)^4 + 8/5 \sin(d*x+c)^2) \cos(d*x+c) \right) + 6 a^4 \left(\frac{1}{3} \sin(d*x+c)^7 / \cos(d*x+c)^3 - 4/3 \sin(d*x+c)^7 / \cos(d*x+c) - 4/3 (\sin(d*x+c)^5 + 5/4 \sin(d*x+c)^3 + 15/8 \sin(d*x+c)) \cos(d*x+c) + 5/2 d*x + 5/2 c \right) + 4 a^4 \left(\frac{1}{3} \sin(d*x+c)^6 / \cos(d*x+c)^3 - \sin(d*x+c)^6 / \cos(d*x+c) - (8/3 + \sin(d*x+c)^4 + 4/3 \sin(d*x+c)^2) \cos(d*x+c) \right) + a^4 (1/3 \tan(d*x+c)^3 - \tan(d*x+c) + d*x+c) \right)$

Maxima [A] time = 1.67263, size = 321, normalized size = 2.24

$$32 \left(\cos(dx+c)^3 - \frac{9 \cos(dx+c)^2 - 1}{\cos(dx+c)^3} - 9 \cos(dx+c) \right) a^4 + \left(8 \tan(dx+c)^3 + 105 dx + 105 c - \frac{3(13 \tan(dx+c)^3 + 11 \tan(dx+c))}{\tan(dx+c)^4 + 2 \tan(dx+c)^2 + 1} - 7 \right) a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="maxima")`

[Out] $\frac{1}{24} \left(32 (\cos(d*x+c)^3 - (9 \cos(d*x+c)^2 - 1) / \cos(d*x+c)^3 - 9 \cos(d*x+c)) a^4 + (8 \tan(d*x+c)^3 + 105 d*x + 105 c - 3 (13 \tan(d*x+c)^3 + 11 \tan(d*x+c)) / (\tan(d*x+c)^4 + 2 \tan(d*x+c)^2 + 1) - 72 \tan(d*x+c)) a^4 + 24 (2 \tan(d*x+c)^3 + 15 d*x + 15 c - 3 \tan(d*x+c) / (\tan(d*x+c)^2 + 1) - 12 \tan(d*x+c)) a^4 + 8 (\tan(d*x+c)^3 + 3 d*x + 3 c - 3 \tan(d*x+c)) a^4 - 32 a^4 ((6 \cos(d*x+c)^2 - 1) / \cos(d*x+c)^3 + 3 \cos(d*x+c)) \right) / d$

Fricas [A] time = 1.56258, size = 620, normalized size = 4.34

$$6 a^4 \cos(dx+c)^6 - 20 a^4 \cos(dx+c)^5 - 85 a^4 \cos(dx+c)^4 + 214 a^4 \cos(dx+c)^3 + 978 a^4 dx + 32 a^4 - (489 a^4 dx + 721 a^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="fricas")`

[Out] $\frac{-1/24 (6 a^4 \cos(d*x+c)^6 - 20 a^4 \cos(d*x+c)^5 - 85 a^4 \cos(d*x+c)^4 + 214 a^4 \cos(d*x+c)^3 + 978 a^4 d*x + 32 a^4 - (489 a^4 d*x + 721 a^4) \cos(d*x+c)^2 + (489 a^4 d*x - 962 a^4) \cos(d*x+c) - (6 a^4 \cos(d*x+c)^5 + 26 a^4 \cos(d*x+c)^4 - 59 a^4 \cos(d*x+c)^3 + 978 a^4 d*x - 273 a^4 \cos(d*x+c)^2 - 32 a^4 + (489 a^4 d*x - 994 a^4) \cos(d*x+c)) \sin(d*x+c)) / (d \cos(d*x+c)^2 - d \cos(d*x+c) + (d \cos(d*x+c) + 2 d) \sin(d*x+c) - 2 d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.39 $\int (a + a \sin(c + dx))^4 \tan^2(c + dx) dx$

Optimal. Leaf size=113

$$-\frac{4a^4 \cos^3(c + dx)}{3d} + \frac{12a^4 \cos(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} + \frac{31a^4 \sin(c + dx) \cos(c + dx)}{8d} + \frac{8a^4 \cos(c + dx)}{d(1 - \sin(c + dx))}$$

[Out] $(-95*a^4*x)/8 + (12*a^4*\text{Cos}[c + d*x])/d - (4*a^4*\text{Cos}[c + d*x]^3)/(3*d) + (8*a^4*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (31*a^4*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a^4*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.160916, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{4a^4 \cos^3(c + dx)}{3d} + \frac{12a^4 \cos(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} + \frac{31a^4 \sin(c + dx) \cos(c + dx)}{8d} + \frac{8a^4 \cos(c + dx)}{d(1 - \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^4*\text{Tan}[c + d*x]^2, x]$

[Out] $(-95*a^4*x)/8 + (12*a^4*\text{Cos}[c + d*x])/d - (4*a^4*\text{Cos}[c + d*x]^3)/(3*d) + (8*a^4*\text{Cos}[c + d*x])/(d*(1 - \text{Sin}[c + d*x])) + (31*a^4*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a^4*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(4*d)$

Rule 2709

$\text{Int}[(a + b*\text{sin}[e + f*x])^m*\text{tan}[e + f*x]^p, x_Symbol] \rightarrow \text{Dist}[a^p, \text{Int}[\text{ExpandIntegrand}[(\text{Sin}[e + f*x])^{m-p/2}/(a - b*\text{Sin}[e + f*x])^{p/2}], x], x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 2648

$\text{Int}[(a + b*\text{sin}[c + d*x])^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2638

$\text{Int}[\text{sin}[c + d*x], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 2635

$\text{Int}[(b*\text{sin}[c + d*x])^n, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{n-1}/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(c + dx))^4 \tan^2(c + dx) dx &= a^2 \int \left(-8a^2 - \frac{8a^2}{-1 + \sin(c + dx)} - 8a^2 \sin(c + dx) - 7a^2 \sin^2(c + dx) - 4a^2 \sin^3(c + dx) \right) dx \\ &= -8a^4 x - a^4 \int \sin^4(c + dx) dx - (4a^4) \int \sin^3(c + dx) dx - (7a^4) \int \sin^2(c + dx) dx \\ &= -8a^4 x + \frac{8a^4 \cos(c + dx)}{d} + \frac{8a^4 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{7a^4 \cos(c + dx) \sin(c + dx)}{2d} \\ &= -\frac{23a^4 x}{2} + \frac{12a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{8a^4 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{31a^4 \cos(c + dx)}{2d} \\ &= -\frac{95a^4 x}{8} + \frac{12a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{8a^4 \cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{31a^4 \cos(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 1.12581, size = 125, normalized size = 1.11

$$\frac{(a \sin(c + dx) + a)^4 \left(-1140(c + dx) + 192 \sin(2(c + dx)) - 3 \sin(4(c + dx)) + 1056 \cos(c + dx) - 32 \cos(3(c + dx)) + 32 \cos(5(c + dx)) \right)}{96d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^4*(-1140*(c + d*x) + 1056*Cos[c + d*x] - 32*Cos[3*(c + d*x)] + (1536*Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 192*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)])/(96*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)
```

Maple [B] time = 0.06, size = 231, normalized size = 2.

$$\frac{1}{d} \left(a^4 \left(\frac{(\sin(dx + c))^7}{\cos(dx + c)} + \left((\sin(dx + c))^5 + \frac{5(\sin(dx + c))^3}{4} + \frac{15 \sin(dx + c)}{8} \right) \cos(dx + c) - \frac{15 dx}{8} - \frac{15 c}{8} \right) + 4a^4 \left(\frac{(\sin(dx + c))^6}{\cos(dx + c)} + \frac{(\sin(dx + c))^5}{\cos(dx + c)} + \frac{(\sin(dx + c))^4}{\cos(dx + c)} + \frac{(\sin(dx + c))^3}{\cos(dx + c)} + \frac{(\sin(dx + c))^2}{\cos(dx + c)} + \frac{\sin(dx + c)}{\cos(dx + c)} + \frac{1}{\cos(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^4*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+4*a^4*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a^4*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^4*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.58886, size = 244, normalized size = 2.16

$$\frac{32 \left(\cos(dx+c)^3 - \frac{3}{\cos(dx+c)} - 6 \cos(dx+c) \right) a^4 + 3 \left(15 dx + 15 c - \frac{9 \tan(dx+c)^3 + 7 \tan(dx+c)}{\tan(dx+c)^4 + 2 \tan(dx+c)^2 + 1} - 8 \tan(dx+c) \right) a^4 + 72 \left(3 \right)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -1/24*(32*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^4 + 3*(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan(d*x + c))*a^4 + 72*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^4 + 24*(d*x + c - tan(d*x + c))*a^4 - 96*a^4*(1/cos(d*x + c) + cos(d*x + c)))/d

Fricas [A] time = 1.5892, size = 456, normalized size = 4.04

$$\frac{6 a^4 \cos(dx+c)^5 + 32 a^4 \cos(dx+c)^4 - 73 a^4 \cos(dx+c)^3 + 285 a^4 dx - 288 a^4 \cos(dx+c)^2 - 192 a^4 + 3(95 a^4 dx - 127 a^4)}{24(d \cos(dx+c) - d \sin(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="fricas")

[Out] -1/24*(6*a^4*cos(d*x + c)^5 + 32*a^4*cos(d*x + c)^4 - 73*a^4*cos(d*x + c)^3 + 285*a^4*d*x - 288*a^4*cos(d*x + c)^2 - 192*a^4 + 3*(95*a^4*d*x - 127*a^4)*cos(d*x + c) + (6*a^4*cos(d*x + c)^4 - 26*a^4*cos(d*x + c)^3 - 285*a^4*d*x - 99*a^4*cos(d*x + c)^2 + 189*a^4*cos(d*x + c) - 192*a^4)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**2,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="giac")

[Out] Timed out

3.40 $\int (a + a \sin(c + dx))^4 dx$

Optimal. Leaf size=87

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{8a^4 \cos(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{27a^4 \sin(c + dx) \cos(c + dx)}{8d} + \frac{35a^4 x}{8}$$

[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rubi [A] time = 0.0817973, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{8a^4 \cos(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{27a^4 \sin(c + dx) \cos(c + dx)}{8d} + \frac{35a^4 x}{8}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^4,x]

[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rule 2645

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx))^4 dx &= \int (a^4 + 4a^4 \sin(c + dx) + 6a^4 \sin^2(c + dx) + 4a^4 \sin^3(c + dx) + a^4 \sin^4(c + dx)) dx \\
&= a^4 x + a^4 \int \sin^4(c + dx) dx + (4a^4) \int \sin(c + dx) dx + (4a^4) \int \sin^3(c + dx) dx + (6a^4) \int \sin^2(c + dx) dx \\
&= a^4 x - \frac{4a^4 \cos(c + dx)}{d} - \frac{3a^4 \cos(c + dx) \sin(c + dx)}{d} - \frac{a^4 \cos(c + dx) \sin^3(c + dx)}{4d} + \frac{1}{4} (3a^4) \int \sin^2(c + dx) dx \\
&= 4a^4 x - \frac{8a^4 \cos(c + dx)}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{27a^4 \cos(c + dx) \sin(c + dx)}{8d} - \frac{a^4 \cos(c + dx) \sin^3(c + dx)}{4d} \\
&= \frac{35a^4 x}{8} - \frac{8a^4 \cos(c + dx)}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{27a^4 \cos(c + dx) \sin(c + dx)}{8d} - \frac{a^4 \cos(c + dx) \sin^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.425065, size = 57, normalized size = 0.66

$$\frac{a^4(3(-56 \sin(2(c + dx)) + \sin(4(c + dx)) + 140c + 140dx) - 672 \cos(c + dx) + 32 \cos(3(c + dx)))}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^4,x]

[Out] (a^4*(-672*Cos[c + d*x] + 32*Cos[3*(c + d*x)] + 3*(140*c + 140*d*x - 56*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(96*d)

Maple [A] time = 0.029, size = 111, normalized size = 1.3

$$\frac{1}{d} \left(a^4 \left(-\frac{\cos(dx + c)}{4} \left((\sin(dx + c))^3 + \frac{3 \sin(dx + c)}{2} \right) + \frac{3 dx}{8} + \frac{3 c}{8} \right) - \frac{4 a^4 (2 + (\sin(dx + c))^2) \cos(dx + c)}{3} + 6 a^4 (-1/2 \cos(dx + c) \sin(dx + c) + 1/2 dx + 1/2 c) - 4 a^4 \cos(dx + c) + a^4 (dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^4,x)

[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-4/3*a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+6*a^4*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-4*a^4*cos(d*x+c)+a^4*(d*x+c))

Maxima [A] time = 1.11432, size = 146, normalized size = 1.68

$$a^4 x + \frac{4(\cos(dx + c)^3 - 3 \cos(dx + c))a^4}{3d} + \frac{(12 dx + 12 c + \sin(4 dx + 4 c) - 8 \sin(2 dx + 2 c))a^4}{32 d} + \frac{3(2 dx + 2 c - \sin(2 dx + 2 c))a^4}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] a^4*x + 4/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^4/d - 4*a^4*cos(d*x + c)/d

Fricas [A] time = 1.47868, size = 177, normalized size = 2.03

$$\frac{32 a^4 \cos(dx + c)^3 + 105 a^4 dx - 192 a^4 \cos(dx + c) + 3(2 a^4 \cos(dx + c)^3 - 29 a^4 \cos(dx + c)) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/24*(32*a^4*cos(d*x + c)^3 + 105*a^4*d*x - 192*a^4*cos(d*x + c) + 3*(2*a^4*cos(d*x + c)^3 - 29*a^4*cos(d*x + c))*sin(d*x + c))/d

Sympy [A] time = 2.07397, size = 224, normalized size = 2.57

$$\left\{ \begin{array}{l} \frac{3a^4 x \sin^4(c+dx)}{8} + \frac{3a^4 x \sin^2(c+dx) \cos^2(c+dx)}{4} + 3a^4 x \sin^2(c+dx) + \frac{3a^4 x \cos^4(c+dx)}{8} + 3a^4 x \cos^2(c+dx) + a^4 x - \frac{5a^4 \sin^3(c+dx)}{8d} \\ x(a \sin(c) + a)^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**4,x)

[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*cos(c + d*x)**2 + a**4*x - 5*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a**4*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**4*sin(c + d*x)*cos(c + d*x)/d - 8*a**4*cos(c + d*x)**3/(3*d) - 4*a**4*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**4, True))

Giac [A] time = 1.38113, size = 97, normalized size = 1.11

$$\frac{35}{8} a^4 x + \frac{a^4 \cos(3 dx + 3 c)}{3 d} - \frac{7 a^4 \cos(dx + c)}{d} + \frac{a^4 \sin(4 dx + 4 c)}{32 d} - \frac{7 a^4 \sin(2 dx + 2 c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] 35/8*a^4*x + 1/3*a^4*cos(3*d*x + 3*c)/d - 7*a^4*cos(d*x + c)/d + 1/32*a^4*sin(4*d*x + 4*c)/d - 7/4*a^4*sin(2*d*x + 2*c)/d

3.41 $\int \cot^2(c + dx)(a + a \sin(c + dx))^4 dx$

Optimal. Leaf size=116

$$-\frac{4a^4 \cos^3(c + dx)}{3d} + \frac{4a^4 \cos(c + dx)}{d} - \frac{a^4 \cot(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} + \frac{23a^4 \sin(c + dx) \cos(c + dx)}{8d} - \frac{4a^4}{8d}$$

[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rubi [A] time = 0.159022, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 3770, 3767, 8, 2635, 2633}

$$-\frac{4a^4 \cos^3(c + dx)}{3d} + \frac{4a^4 \cos(c + dx)}{d} - \frac{a^4 \cot(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} + \frac{23a^4 \sin(c + dx) \cos(c + dx)}{8d} - \frac{4a^4}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]

[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Exp
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + a \sin(c + dx))^4 dx &= \frac{\int (5a^6 + 4a^6 \csc(c + dx) + a^6 \csc^2(c + dx) - 5a^6 \sin^2(c + dx) - 4a^6 \sin^3(c + dx)) dx}{a^2} \\ &= 5a^4 x + a^4 \int \csc^2(c + dx) dx - a^4 \int \sin^4(c + dx) dx + (4a^4) \int \csc(c + dx) dx \\ &= 5a^4 x - \frac{4a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{5a^4 \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^4 \cos(c + dx)}{d} \\ &= \frac{5a^4 x}{2} - \frac{4a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{a^4}{d} \\ &= \frac{17a^4 x}{8} - \frac{4a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{a^4}{d} \end{aligned}$$

Mathematica [A] time = 1.57393, size = 136, normalized size = 1.17

$$\frac{a^4 \csc\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{1}{2}(c + dx)\right) \left(408c \sin(c + dx) + 408dx \sin(c + dx) + 320 \sin(2(c + dx)) - 32 \sin(4(c + dx)) - 48\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
+ 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)])/(384*d)
```

Maple [A] time = 0.045, size = 127, normalized size = 1.1

$$\frac{a^4 (\cos(dx + c))^3 \sin(dx + c)}{4d} + \frac{25a^4 \cos(dx + c) \sin(dx + c)}{8d} + \frac{17a^4 x}{8} + \frac{17a^4 c}{8d} - \frac{4a^4 (\cos(dx + c))^3}{3d} + 4 \frac{a^4 \ln(\csc(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)+25/8*a^4*cos(d*x+c)*sin(d*x+c)/d+17/8*a^
4*x+17/8/d*a^4*c-4/3*a^4*cos(d*x+c)^3/d+4/d*a^4*ln(csc(d*x+c)-cot(d*x+c))+4
*a^4*cos(d*x+c)/d-a^4*cot(d*x+c)/d
```

Maxima [A] time = 1.57315, size = 158, normalized size = 1.36

$$\frac{128a^4 \cos(dx + c)^3 - 3(4dx + 4c - \sin(4dx + 4c))a^4 - 144(2dx + 2c + \sin(2dx + 2c))a^4 + 96\left(dx + c + \frac{1}{\tan(dx + c)}\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out]
$$\frac{-1/96*(128*a^4*\cos(d*x + c)^3 - 3*(4*d*x + 4*c - \sin(4*d*x + 4*c))*a^4 - 14*4*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/\tan(d*x + c))*a^4 - 192*a^4*(2*\cos(d*x + c) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1))}{d}$$

Fricas [A] time = 1.69257, size = 360, normalized size = 3.1

$$\frac{6 a^4 \cos(dx + c)^5 - 81 a^4 \cos(dx + c)^3 - 48 a^4 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + 48 a^4 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + 51 a^4 \cos(dx + c) - (32 a^4 \cos(dx + c)^3 - 51 a^4 d x - 96 a^4 \cos(dx + c)) \sin(dx + c)}{24 d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\frac{1/24*(6*a^4*\cos(d*x + c)^5 - 81*a^4*\cos(d*x + c)^3 - 48*a^4*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 48*a^4*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 51*a^4*\cos(d*x + c) - (32*a^4*\cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*\cos(d*x + c))*\sin(d*x + c)}{(d*\sin(d*x + c))}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4 \left(\int 4 \sin(c + dx) \cot^2(c + dx) dx + \int 6 \sin^2(c + dx) \cot^2(c + dx) dx + \int 4 \sin^3(c + dx) \cot^2(c + dx) dx + \int \sin^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**4,x)

[Out]
$$a^{**4}*(\text{Integral}(4*\sin(c + d*x)*\cot(c + d*x)**2, x) + \text{Integral}(6*\sin(c + d*x)**2*\cot(c + d*x)**2, x) + \text{Integral}(4*\sin(c + d*x)**3*\cot(c + d*x)**2, x) + \text{Integral}(\sin(c + d*x)**4*\cot(c + d*x)**2, x) + \text{Integral}(\cot(c + d*x)**2, x))$$

Giac [A] time = 1.39615, size = 262, normalized size = 2.26

$$51(dx + c)a^4 + 96a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + 12a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \frac{12\left(8a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a^4\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)} - \frac{2\left(69a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^7 + 93a^4}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}$$

24d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out]
$$\frac{1/24*(51*(d*x + c)*a^4 + 96*a^4*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + 12*a^4*\tan(1/2*d*x + 1/2*c) - 12*(8*a^4*\tan(1/2*d*x + 1/2*c) + a^4)/\tan(1/2*d*x + 1/2*c) - 2*(69*a^4*\tan(1/2*d*x + 1/2*c)^7 + 93*a^4*\tan(1/2*d*x + 1/2*c)^5 - 192*a^4*\tan(1/2*d*x + 1/2*c)^4 - 93*a^4*\tan(1/2*d*x + 1/2*c)^3 - 256*a^4*\tan(1/2*d*x + 1/2*c)^2 - 69*a^4*\tan(1/2*d*x + 1/2*c) - 64*a^4)/(\tan(1/2*d*x + 1/2*c)^2 + 1)^4}{d}$$

3.42 $\int \cot^4(c + dx)(a + a \sin(c + dx))^4 dx$

Optimal. Leaf size=140

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{a^4 \cot^3(c + dx)}{3d} - \frac{5a^4 \cot(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{19a^4 \sin(c + dx) \cos(c + dx)}{8d} + \dots$$

[Out] $(-61a^4x)/8 + (2a^4 \operatorname{ArcTanh}[\cos(c + dx)])/d + (4a^4 \cos(c + dx)^3)/(3d) - (5a^4 \cot(c + dx))/d - (a^4 \cot(c + dx)^3)/(3d) - (2a^4 \cot(c + dx) \operatorname{Csc}(c + dx))/d - (19a^4 \cos(c + dx) \sin(c + dx))/(8d) - (a^4 \cos(c + dx) \sin(c + dx)^3)/(4d)$

Rubi [A] time = 0.224943, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{a^4 \cot^3(c + dx)}{3d} - \frac{5a^4 \cot(c + dx)}{d} - \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d} - \frac{19a^4 \sin(c + dx) \cos(c + dx)}{8d} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\cot(c + dx)^4(a + a \sin(c + dx))^4, x]$

[Out] $(-61a^4x)/8 + (2a^4 \operatorname{ArcTanh}[\cos(c + dx)])/d + (4a^4 \cos(c + dx)^3)/(3d) - (5a^4 \cot(c + dx))/d - (a^4 \cot(c + dx)^3)/(3d) - (2a^4 \cot(c + dx) \operatorname{Csc}(c + dx))/d - (19a^4 \cos(c + dx) \sin(c + dx))/(8d) - (a^4 \cos(c + dx) \sin(c + dx)^3)/(4d)$

Rule 2709

$\operatorname{Int}[(a + b \sin(e + f x))^m \tan(e + f x)^p, x_Symbol] \rightarrow \operatorname{Dist}[a^p, \operatorname{Int}[\operatorname{ExpandIntegrand}[(\sin(e + f x))^p (a + b \sin(e + f x))^{m - p/2}] / (a - b \sin(e + f x))^{p/2}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\}$ && $\operatorname{EqQ}[a^2 - b^2, 0]$ && $\operatorname{IntegersQ}[m, p/2]$ && $(\operatorname{LtQ}[p, 0] \mid \mid \operatorname{GtQ}[m - p/2, 0])$

Rule 3770

$\operatorname{Int}[\operatorname{csc}(c + d x), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos(c + d x)]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 3767

$\operatorname{Int}[\operatorname{csc}(c + d x)^n, x_Symbol] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \cot(c + d x)], x] /;$ $\operatorname{FreeQ}\{c, d, x\}$ && $\operatorname{IGtQ}[n/2, 0]$

Rule 8

$\operatorname{Int}[a x, x_Symbol] \rightarrow \operatorname{Simp}[a x, x] /;$ $\operatorname{FreeQ}[a, x]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}(c + d x) + d x) (b x)^n, x_Symbol] \rightarrow -\operatorname{Simp}[(b \cos(c + d x) + b \operatorname{Csc}(c + d x))^{n - 1} / (d(n - 1)), x] + \operatorname{Dist}[(b^2(n - 2)) / (n - 1), \operatorname{Int}[(b \operatorname{Csc}(c + d x))^{n - 2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, x\}$ && $\operatorname{GtQ}[n, 1]$ &&

IntegerQ[2*n]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_)), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^4(c + dx)(a + a \sin(c + dx))^4 dx &= \frac{\int (-10a^8 - 4a^8 \csc(c + dx) + 4a^8 \csc^2(c + dx) + 4a^8 \csc^3(c + dx) + a^8 \csc^4(c + dx)) dx}{a^8} \\ &= -10a^4x + a^4 \int \csc^4(c + dx) dx + a^4 \int \sin^4(c + dx) dx - (4a^4) \int \csc(c + dx) dx \\ &= -10a^4x + \frac{4a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^4 \cos(c + dx)}{d} - \frac{2a^4 \cot(c + dx) \csc(c + dx)}{d} \\ &= -8a^4x + \frac{2a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{5a^4 \cot(c + dx)}{d} - \frac{a^4 \cot^3(c + dx)}{d} \\ &= -\frac{61a^4x}{8} + \frac{2a^4 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^4 \cos^3(c + dx)}{3d} - \frac{5a^4 \cot(c + dx)}{d} - \frac{a^4 \cot^3(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 5.35963, size = 209, normalized size = 1.49

$$\frac{a^4(\sin(c + dx) + 1)^4 \left(-732(c + dx) - 120 \sin(2(c + dx)) + 3 \sin(4(c + dx)) + 96 \cos(c + dx) + 32 \cos(3(c + dx)) + 224 \tan^2(c + dx) \right)}{96d^2(\cos((c + dx)/2) + \sin((c + dx)/2))^8}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c +
d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c + d*
x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x
]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*(c +
d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^8)
```

Maple [A] time = 0.05, size = 190, normalized size = 1.4

$$-\frac{23a^4(\cos(dx + c))^3 \sin(dx + c)}{4d} - \frac{69a^4 \cos(dx + c) \sin(dx + c)}{8d} - \frac{61a^4x}{8} - \frac{61a^4c}{8d} - \frac{2a^4(\cos(dx + c))^3}{3d} - 2\frac{a^4 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x)`

[Out]
$$-23/4/d*a^4*\cos(d*x+c)^3*\sin(d*x+c)-69/8*a^4*\cos(d*x+c)*\sin(d*x+c)/d-61/8*a^4*x-61/8/d*a^4*c-2/3*a^4*\cos(d*x+c)^3/d-2*a^4*\cos(d*x+c)/d-2/d*a^4*\ln(\csc(d*x+c)-\cot(d*x+c))-6/d*a^4/\sin(d*x+c)*\cos(d*x+c)^5-2/d*a^4/\sin(d*x+c)^2*\cos(d*x+c)^5-1/3*a^4*\cot(d*x+c)^3/d+a^4*\cot(d*x+c)/d$$

Maxima [A] time = 1.52732, size = 294, normalized size = 2.1

$$64 \left(2 \cos(dx + c)^3 + 6 \cos(dx + c) - 3 \log(\cos(dx + c) + 1) + 3 \log(\cos(dx + c) - 1) \right) a^4 + 3 (12 dx + 12 c + \sin(4 dx + 4 c)) a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")`

[Out]
$$\frac{1}{96} * (64 * (2 * \cos(dx + c)^3 + 6 * \cos(dx + c) - 3 * \log(\cos(dx + c) + 1) + 3 * \log(\cos(dx + c) - 1)) * a^4 + 3 * (12 * dx + 12 * c + \sin(4 * dx + 4 * c) + 8 * \sin(2 * dx + 2 * c)) * a^4 - 288 * (3 * dx + 3 * c + (3 * \tan(dx + c)^2 + 2) / (\tan(dx + c)^3 + \tan(dx + c))) * a^4 + 32 * (3 * dx + 3 * c + (3 * \tan(dx + c)^2 - 1) / \tan(dx + c)^3) * a^4 + 96 * a^4 * (2 * \cos(dx + c) / (\cos(dx + c)^2 - 1) - 4 * \cos(dx + c) + 3 * \log(\cos(dx + c) + 1) - 3 * \log(\cos(dx + c) - 1))) / d$$

Fricas [A] time = 1.97689, size = 560, normalized size = 4.

$$6 a^4 \cos(dx + c)^7 - 75 a^4 \cos(dx + c)^5 + 244 a^4 \cos(dx + c)^3 - 183 a^4 \cos(dx + c) - 24 \left(a^4 \cos(dx + c)^2 - a^4 \right) \log\left(\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")`

[Out]
$$-1/24*(6*a^4*\cos(d*x+c)^7-75*a^4*\cos(d*x+c)^5+244*a^4*\cos(d*x+c)^3-183*a^4*\cos(d*x+c)-24*(a^4*\cos(d*x+c)^2-a^4)*\log(1/2*\cos(d*x+c)+1/2)*\sin(d*x+c)+24*(a^4*\cos(d*x+c)^2-a^4)*\log(-1/2*\cos(d*x+c)+1/2)*\sin(d*x+c)-(32*a^4*\cos(d*x+c)^5-183*a^4*d*x*\cos(d*x+c)^2-32*a^4*\cos(d*x+c)^3+183*a^4*d*x+48*a^4*\cos(d*x+c))*\sin(d*x+c)/((d*\cos(d*x+c)^2-d)*\sin(d*x+c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**4,x)`

[Out] Timed out

Giac [B] time = 1.48155, size = 370, normalized size = 2.64

$$a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 12 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 183 (dx + c)a^4 - 48 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + 57 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{24}*(a^4*\tan(1/2*d*x + 1/2*c)^3 + 12*a^4*\tan(1/2*d*x + 1/2*c)^2 - 183*(d*x + c)*a^4 - 48*a^4*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + 57*a^4*\tan(1/2*d*x + 1/2*c) + (88*a^4*\tan(1/2*d*x + 1/2*c)^3 - 57*a^4*\tan(1/2*d*x + 1/2*c)^2 - 12*a^4*\tan(1/2*d*x + 1/2*c) - a^4)/\tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*\tan(1/2*d*x + 1/2*c)^7 + 96*a^4*\tan(1/2*d*x + 1/2*c)^6 + 81*a^4*\tan(1/2*d*x + 1/2*c)^5 + 96*a^4*\tan(1/2*d*x + 1/2*c)^4 - 81*a^4*\tan(1/2*d*x + 1/2*c)^3 + 32*a^4*\tan(1/2*d*x + 1/2*c)^2 - 57*a^4*\tan(1/2*d*x + 1/2*c) + 32*a^4)/(\tan(1/2*d*x + 1/2*c)^2 + 1)^4/d$

3.43 $\int \cot^6(c + dx)(a + a \sin(c + dx))^4 dx$

Optimal. Leaf size=198

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{4a^4 \cos(c + dx)}{d} - \frac{a^4 \cot^5(c + dx)}{5d} - \frac{5a^4 \cot^3(c + dx)}{3d} + \frac{10a^4 \cot(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d}$$

```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
- (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.427749, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$\frac{4a^4 \cos^3(c + dx)}{3d} - \frac{4a^4 \cos(c + dx)}{d} - \frac{a^4 \cot^5(c + dx)}{5d} - \frac{5a^4 \cot^3(c + dx)}{3d} + \frac{10a^4 \cot(c + dx)}{d} + \frac{a^4 \sin^3(c + dx) \cos(c + dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
- (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_
), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
p/2, 0])
```

Rule 3767

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
  [{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
  ]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
  + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
  ]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
  nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
  && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^6(c + dx)(a + a \sin(c + dx))^4 dx &= \frac{\int (14a^{10} - 14a^{10} \csc^2(c + dx) - 8a^{10} \csc^3(c + dx) + 3a^{10} \csc^4(c + dx) + 4a^{10} \csc^5(c + dx)) dx}{d} \\ &= 14a^4x + a^4 \int \csc^6(c + dx) dx - a^4 \int \sin^4(c + dx) dx + (3a^4) \int \csc^4(c + dx) dx \\ &= 14a^4x - \frac{8a^4 \cos(c + dx)}{d} + \frac{4a^4 \cot(c + dx) \csc(c + dx)}{d} - \frac{a^4 \cot(c + dx) \csc^3(c + dx)}{d} \\ &= \frac{25a^4x}{2} + \frac{4a^4 \tanh^{-1}(\cos(c + dx))}{d} - \frac{4a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{10a^4}{3d} \\ &= \frac{97a^4x}{8} + \frac{5a^4 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{4a^4 \cos(c + dx)}{d} - \frac{4a^4 \cos^3(c + dx)}{3d} + \frac{10a^4}{3d} \end{aligned}$$

Mathematica [A] time = 1.5202, size = 283, normalized size = 1.43

$$\frac{a^4(\sin(c + dx) + 1)^4(5820(c + dx) + 480 \sin(2(c + dx)) - 15 \sin(4(c + dx)) - 2400 \cos(c + dx) - 160 \cos(3(c + dx)) - 2752 \cot((c + dx)/2) + 300 \csc((c + dx)/2)^2 - 30 \csc((c + dx)/2)^4 + 1200 \log[\cos((c + dx)/2)] - 1200 \log[\sin((c + dx)/2)] - 300 \sec((c + dx)/2)^2 + 30 \sec((c + dx)/2)^4 + 632 \csc[c + d*x]^3 \sin((c + d*x)/2)^4 + 96 \csc[c + d*x]^5 \sin((c + d*x)/2)^6 - (79 \csc((c + d*x)/2)^4 \sin(c + d*x))/2 - (3 \csc((c + d*x)/2)^6 \sin(c + d*x))/2 + 480 \sin[2*(c + d*x)] - 15 \sin[4*(c + d*x)] - 2752 \tan((c + d*x)/2))}{(480*d*(\cos((c + d*x)/2) + \sin((c + d*x)/2))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(5820*(c + d*x) - 2400*Cos[c + d*x] - 160*Cos[3*(
  c + d*x)] + 2752*Cot[(c + d*x)/2] + 300*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*
  x)/2]^4 + 1200*Log[Cos[(c + d*x)/2]] - 1200*Log[Sin[(c + d*x)/2]] - 300*Sec
  [(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 632*Csc[c + d*x]^3*Sin[(c + d*x)/
  2]^4 + 96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - (79*Csc[(c + d*x)/2]^4*Sin[c
  + d*x])/2 - (3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 480*Sin[2*(c + d*x)] -
  15*Sin[4*(c + d*x)] - 2752*Tan[(c + d*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Si
  n[(c + d*x)/2])^8)
```

Maple [A] time = 0.054, size = 293, normalized size = 1.5

$$\frac{5a^4(\cos(dx+c))^3}{6d} + 7\frac{a^4\sin(dx+c)(\cos(dx+c))^5}{d} - \frac{5a^4\cos(dx+c)}{2d} - \frac{a^4(\cos(dx+c))^5}{2d} - \frac{a^4(\cos(dx+c))^7}{d(\sin(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x)

[Out]
$$-5/6*a^4*\cos(d*x+c)^3/d+7/d*a^4*\sin(d*x+c)*\cos(d*x+c)^5-5/2*a^4*\cos(d*x+c)/d-1/2/d*a^4*\cos(d*x+c)^5-1/d*a^4/\sin(d*x+c)^4*\cos(d*x+c)^7-2/d*a^4/\sin(d*x+c)^3*\cos(d*x+c)^7-1/2/d*a^4/\sin(d*x+c)^2*\cos(d*x+c)^7+97/8*a^4*x+7/d*a^4/\sin(d*x+c)*\cos(d*x+c)^7+35/4/d*a^4*\cos(d*x+c)^3*\sin(d*x+c)+105/8*a^4*\cos(d*x+c)*\sin(d*x+c)/d-1/5*a^4*\cot(d*x+c)^5/d+1/3*a^4*\cot(d*x+c)^3/d-a^4*\cot(d*x+c)/d+97/8/d*a^4*c-5/2/d*a^4*\ln(\csc(d*x+c)-\cot(d*x+c))$$

Maxima [A] time = 1.69153, size = 423, normalized size = 2.14

$$40\left(4\cos(dx+c)^3 - \frac{6\cos(dx+c)}{\cos(dx+c)^2-1} + 24\cos(dx+c) - 15\log(\cos(dx+c)+1) + 15\log(\cos(dx+c)-1)\right)a^4 + 15(1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out]
$$-1/120*(40*(4*\cos(d*x+c)^3 - 6*\cos(d*x+c)/(\cos(d*x+c)^2-1) + 24*\cos(d*x+c) - 15*\log(\cos(d*x+c)+1) + 15*\log(\cos(d*x+c)-1))*a^4 + 15*(15*d*x + 15*c + (15*\tan(d*x+c)^4 + 25*\tan(d*x+c)^2 + 8)/(\tan(d*x+c)^5 + 2*\tan(d*x+c)^3 + \tan(d*x+c)))*a^4 - 120*(15*d*x + 15*c + (15*\tan(d*x+c)^4 + 10*\tan(d*x+c)^2 - 2)/(\tan(d*x+c)^5 + \tan(d*x+c)^3))*a^4 + 8*(15*d*x + 15*c + (15*\tan(d*x+c)^4 - 5*\tan(d*x+c)^2 + 3)/\tan(d*x+c)^5)*a^4 + 30*a^4*(2*(9*\cos(d*x+c)^3 - 7*\cos(d*x+c))/(\cos(d*x+c)^4 - 2*\cos(d*x+c)^2 + 1) - 16*\cos(d*x+c) + 15*\log(\cos(d*x+c)+1) - 15*\log(\cos(d*x+c)-1)))/d$$

Fricas [A] time = 2.08524, size = 770, normalized size = 3.89

$$30a^4\cos(dx+c)^9 - 345a^4\cos(dx+c)^7 + 2231a^4\cos(dx+c)^5 - 3395a^4\cos(dx+c)^3 + 1455a^4\cos(dx+c) + 150$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out]
$$1/120*(30*a^4*\cos(d*x+c)^9 - 345*a^4*\cos(d*x+c)^7 + 2231*a^4*\cos(d*x+c)^5 - 3395*a^4*\cos(d*x+c)^3 + 1455*a^4*\cos(d*x+c) + 150*(a^4*\cos(d*x+c)^4 - 2*a^4*\cos(d*x+c)^2 + a^4)*\log(1/2*\cos(d*x+c) + 1/2)*\sin(d*x+c) - 150*(a^4*\cos(d*x+c)^4 - 2*a^4*\cos(d*x+c)^2 + a^4)*\log(-1/2*\cos(d*x+c) + 1/2)*\sin(d*x+c) - 5*(32*a^4*\cos(d*x+c)^7 - 291*a^4*d*x*\cos(d*x+c)^4 + 32*a^4*\cos(d*x+c)^5 + 582*a^4*d*x*\cos(d*x+c)^2 - 100*a^4*\cos(d*x+c)^3 - 291*a^4*d*x + 60*a^4*\cos(d*x+c))*\sin(d*x+c))/((d*\cos(d*x+c$$

)⁴ - 2*d*cos(d*x + c)² + d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.70284, size = 458, normalized size = 2.31

$$3a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 30a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 85a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 240a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 5820(dx + c)a^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{480} \cdot (3a^4 \tan(1/2dx + 1/2c)^5 + 30a^4 \tan(1/2dx + 1/2c)^4 + 85a^4 \tan(1/2dx + 1/2c)^3 - 240a^4 \tan(1/2dx + 1/2c)^2 + 5820(dx + c)a^4 - 1200a^4 \log(\text{abs}(\tan(1/2dx + 1/2c))) - 2670a^4 \tan(1/2dx + 1/2c) - 40 \cdot (45a^4 \tan(1/2dx + 1/2c)^7 + 192a^4 \tan(1/2dx + 1/2c)^6 + 69a^4 \tan(1/2dx + 1/2c)^5 + 384a^4 \tan(1/2dx + 1/2c)^4 - 69a^4 \tan(1/2dx + 1/2c)^3 + 320a^4 \tan(1/2dx + 1/2c)^2 - 45a^4 \tan(1/2dx + 1/2c) + 128a^4) / (\tan(1/2dx + 1/2c)^2 + 1)^4 + (2740a^4 \tan(1/2dx + 1/2c)^5 + 2670a^4 \tan(1/2dx + 1/2c)^4 + 240a^4 \tan(1/2dx + 1/2c)^3 - 85a^4 \tan(1/2dx + 1/2c)^2 - 30a^4 \tan(1/2dx + 1/2c) - 3a^4) / \tan(1/2dx + 1/2c)^5) / d$

$$3.44 \quad \int \frac{\tan^7(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=130

$$\frac{\tan^8(c+dx)}{8ad} - \frac{35 \tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\tan^7(c+dx) \sec(c+dx)}{8ad} + \frac{7 \tan^5(c+dx) \sec(c+dx)}{48ad} - \frac{35 \tan^3(c+dx) \sec(c+dx)}{192ad}$$

[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c + d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8/(8*a*d)

Rubi [A] time = 0.161036, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^8(c+dx)}{8ad} - \frac{35 \tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\tan^7(c+dx) \sec(c+dx)}{8ad} + \frac{7 \tan^5(c+dx) \sec(c+dx)}{48ad} - \frac{35 \tan^3(c+dx) \sec(c+dx)}{192ad}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]

[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c + d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8/(8*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^7(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^8(c+dx) dx}{a} \\ &= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{7\int \sec(c+dx)\tan^6(c+dx) dx}{8a} + \frac{\text{Subst}\left(\int x^7 dx, x, \tan(c+dx)\right)}{ad} \\ &= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} - \frac{35\int \sec(c+dx)\tan^4 dx}{48a} \\ &= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} \\ &= \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} \\ &= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} \end{aligned}$$

Mathematica [A] time = 1.00892, size = 101, normalized size = 0.78

$$\frac{279\sin^6(c+dx)+87\sin^5(c+dx)-424\sin^4(c+dx)-136\sin^3(c+dx)+249\sin^2(c+dx)+57\sin(c+dx)-48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4} + 105\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Sin[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.059, size = 162, normalized size = 1.3

$$-\frac{1}{96da(\sin(dx+c)-1)^3} - \frac{9}{128da(\sin(dx+c)-1)^2} - \frac{29}{128da(\sin(dx+c)-1)} + \frac{35\ln(\sin(dx+c)-1)}{256da} + \frac{1}{64da(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-9/128/d/a/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c)-1)+35/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-5/48/d/a/(1+sin(d*x+c))^3+19/64/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-35/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.02832, size = 236, normalized size = 1.82

$$\frac{2(279\sin(dx+c)^6+87\sin(dx+c)^5-424\sin(dx+c)^4-136\sin(dx+c)^3+249\sin(dx+c)^2+57\sin(dx+c)-48)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a} + \frac{105\log(\sin(dx+c)+1)}{a} - \frac{105\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out]
$$-1/768*(2*(279*\sin(d*x + c)^6 + 87*\sin(d*x + c)^5 - 424*\sin(d*x + c)^4 - 136*\sin(d*x + c)^3 + 249*\sin(d*x + c)^2 + 57*\sin(d*x + c) - 48)/(a*\sin(d*x + c)^7 + a*\sin(d*x + c)^6 - 3*a*\sin(d*x + c)^5 - 3*a*\sin(d*x + c)^4 + 3*a*\sin(d*x + c)^3 + 3*a*\sin(d*x + c)^2 - a*\sin(d*x + c) - a) + 105*\log(\sin(d*x + c) + 1)/a - 105*\log(\sin(d*x + c) - 1)/a)/d$$

Fricas [A] time = 2.05396, size = 464, normalized size = 3.57

$$\frac{558 \cos(dx + c)^6 - 826 \cos(dx + c)^4 + 476 \cos(dx + c)^2 + 105 (\cos(dx + c)^6 \sin(dx + c) + \cos(dx + c)^6) \log(\sin(dx + c) + 1) - 105 (\cos(dx + c)^6 \sin(dx + c) + \cos(dx + c)^6) \log(-\sin(dx + c) + 1) - 2(87 \cos(dx + c)^4 - 38 \cos(dx + c)^2 + 8) \sin(dx + c) - 112}{768 (ad \cos(dx + c)^6 \sin(dx + c) + a d \cos(dx + c)^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out]
$$-1/768*(558*\cos(d*x + c)^6 - 826*\cos(d*x + c)^4 + 476*\cos(d*x + c)^2 + 105*(\cos(d*x + c)^6*\sin(d*x + c) + \cos(d*x + c)^6)*\log(\sin(d*x + c) + 1) - 105*(\cos(d*x + c)^6*\sin(d*x + c) + \cos(d*x + c)^6)*\log(-\sin(d*x + c) + 1) - 2*(87*\cos(d*x + c)^4 - 38*\cos(d*x + c)^2 + 8)*\sin(d*x + c) - 112)/(a*d*\cos(d*x + c)^6*\sin(d*x + c) + a*d*\cos(d*x + c)^6)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 8.06961, size = 184, normalized size = 1.42

$$\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{2(385 \sin(dx+c)^3 - 807 \sin(dx+c)^2 + 567 \sin(dx+c) - 129)}{a(\sin(dx+c)-1)^3} - \frac{875 \sin(dx+c)^4 + 1964 \sin(dx+c)^3 + 1554 \sin(dx+c)^2 + 396 \sin(dx+c) - 21}{a(\sin(dx+c)+1)^4}}{3072 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out]
$$-1/3072*(420*\log(\text{abs}(\sin(d*x + c) + 1))/a - 420*\log(\text{abs}(\sin(d*x + c) - 1))/a + 2*(385*\sin(d*x + c)^3 - 807*\sin(d*x + c)^2 + 567*\sin(d*x + c) - 129)/(a*(\sin(d*x + c) - 1)^3) - (875*\sin(d*x + c)^4 + 1964*\sin(d*x + c)^3 + 1554*\sin(d*x + c)^2 + 396*\sin(d*x + c) - 21)/(a*(\sin(d*x + c) + 1)^4))/d$$

3.45 $\int \frac{\tan^5(c+dx)}{a+a \sin(c+dx)} dx$

Optimal. Leaf size=106

$$\frac{\tan^6(c+dx)}{6ad} + \frac{5 \tanh^{-1}(\sin(c+dx))}{16ad} - \frac{\tan^5(c+dx) \sec(c+dx)}{6ad} + \frac{5 \tan^3(c+dx) \sec(c+dx)}{24ad} - \frac{5 \tan(c+dx) \sec(c+dx)}{16ad}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
+ (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.13628, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^6(c+dx)}{6ad} + \frac{5 \tanh^{-1}(\sin(c+dx))}{16ad} - \frac{\tan^5(c+dx) \sec(c+dx)}{6ad} + \frac{5 \tan^3(c+dx) \sec(c+dx)}{24ad} - \frac{5 \tan(c+dx) \sec(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
+ (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
- Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]
```

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^5(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^6(c+dx) dx}{a} \\ &= -\frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{5 \int \sec(c+dx)\tan^4(c+dx) dx}{6a} + \frac{\text{Subst}\left(\int x^5 dx, x, \tan(c+dx)\right)}{ad} \\ &= \frac{5 \sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c+dx)}{6ad} - \frac{5 \int \sec(c+dx)\tan^2(c+dx) dx}{8a} \\ &= -\frac{5 \sec(c+dx)\tan(c+dx)}{16ad} + \frac{5 \sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c+dx)}{6ad} \\ &= \frac{5 \tanh^{-1}(\sin(c+dx))}{16ad} - \frac{5 \sec(c+dx)\tan(c+dx)}{16ad} + \frac{5 \sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} \end{aligned}$$

Mathematica [A] time = 0.308239, size = 84, normalized size = 0.79

$$\frac{-\frac{18}{1-\sin(c+dx)} + \frac{48}{\sin(c+dx)+1} + \frac{3}{(1-\sin(c+dx))^2} - \frac{21}{(\sin(c+dx)+1)^2} + \frac{4}{(\sin(c+dx)+1)^3} + 30 \tanh^{-1}(\sin(c+dx))}{96ad}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]

[Out] (30*ArcTanh[Sin[c + d*x]] + 3/(1 - Sin[c + d*x])^2 - 18/(1 - Sin[c + d*x]) + 4/(1 + Sin[c + d*x])^3 - 21/(1 + Sin[c + d*x])^2 + 48/(1 + Sin[c + d*x]))/(96*a*d)

Maple [A] time = 0.056, size = 126, normalized size = 1.2

$$\frac{1}{32 da (\sin(dx+c)-1)^2} + \frac{3}{16 da (\sin(dx+c)-1)} - \frac{5 \ln(\sin(dx+c)-1)}{32 da} + \frac{1}{24 da (1+\sin(dx+c))^3} - \frac{7}{32 da (1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c)),x)

[Out] 1/32/d/a/(sin(d*x+c)-1)^2+3/16/a/d/(sin(d*x+c)-1)-5/32/a/d*ln(sin(d*x+c)-1)+1/24/d/a/(1+sin(d*x+c))^3-7/32/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))+5/32*ln(1+sin(d*x+c))/a/d

Maxima [A] time = 0.999884, size = 176, normalized size = 1.66

$$\frac{2(33 \sin(dx+c)^4 + 9 \sin(dx+c)^3 - 31 \sin(dx+c)^2 - 7 \sin(dx+c) + 8)}{a \sin(dx+c)^5 + a \sin(dx+c)^4 - 2a \sin(dx+c)^3 - 2a \sin(dx+c)^2 + a \sin(dx+c) + a} + \frac{15 \log(\sin(dx+c)+1)}{a} - \frac{15 \log(\sin(dx+c)-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{96} \cdot \frac{2 \cdot (33 \sin(dx+c)^4 + 9 \sin(dx+c)^3 - 31 \sin(dx+c)^2 - 7 \sin(dx+c) + 8)}{a \sin(dx+c)^5 + a \sin(dx+c)^4 - 2a \sin(dx+c)^3 - 2a \sin(dx+c)^2 + a \sin(dx+c) + a} + 15 \cdot \frac{\log(\sin(dx+c) + 1)}{a} - 15 \cdot \frac{\log(\sin(dx+c) - 1)}{a} / d$

Fricas [A] time = 1.8359, size = 398, normalized size = 3.75

$$\frac{66 \cos(dx+c)^4 - 70 \cos(dx+c)^2 + 15 \left(\cos(dx+c)^4 \sin(dx+c) + \cos(dx+c)^4 \right) \log(\sin(dx+c) + 1) - 15 \left(\cos(dx+c)^4 \sin(dx+c) + \cos(dx+c)^4 \right) \log(-\sin(dx+c) + 1) - 2 \cdot (9 \cos(dx+c)^2 - 2) \cdot \sin(dx+c) + 20}{96 \left(ad \cos(dx+c)^4 \sin(dx+c) + ad \cos(dx+c)^4 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{96} \cdot \frac{66 \cos(dx+c)^4 - 70 \cos(dx+c)^2 + 15 \cdot (\cos(dx+c)^4 \sin(dx+c) + \cos(dx+c)^4) \cdot \log(\sin(dx+c) + 1) - 15 \cdot (\cos(dx+c)^4 \sin(dx+c) + \cos(dx+c)^4) \cdot \log(-\sin(dx+c) + 1) - 2 \cdot (9 \cos(dx+c)^2 - 2) \cdot \sin(dx+c) + 20}{a \cdot d \cdot \cos(dx+c)^4 \sin(dx+c) + a \cdot d \cdot \cos(dx+c)^4}$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**5/(sin(c + d*x) + 1), x)/a

Giac [A] time = 3.93359, size = 157, normalized size = 1.48

$$\frac{\frac{30 \log(|\sin(dx+c)+1|)}{a} - \frac{30 \log(|\sin(dx+c)-1|)}{a} + \frac{3(15 \sin(dx+c)^2 - 18 \sin(dx+c) + 5)}{a(\sin(dx+c)-1)^2} - \frac{55 \sin(dx+c)^3 + 69 \sin(dx+c)^2 + 15 \sin(dx+c) - 7}{a(\sin(dx+c)+1)^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{192} \cdot \frac{30 \cdot \log(\text{abs}(\sin(dx+c) + 1))}{a} - \frac{30 \cdot \log(\text{abs}(\sin(dx+c) - 1))}{a} + \frac{3 \cdot (15 \sin(dx+c)^2 - 18 \sin(dx+c) + 5)}{a \cdot (\sin(dx+c) - 1)^2} - \frac{55 \sin(dx+c)^3 + 69 \sin(dx+c)^2 + 15 \sin(dx+c) - 7}{a \cdot (\sin(dx+c) + 1)^3} / d$

$$3.46 \quad \int \frac{\tan^3(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=82

$$\frac{\tan^4(c+dx)}{4ad} - \frac{3 \tanh^{-1}(\sin(c+dx))}{8ad} - \frac{\tan^3(c+dx) \sec(c+dx)}{4ad} + \frac{3 \tan(c+dx) \sec(c+dx)}{8ad}$$

[Out] $(-3 \operatorname{ArcTanh}[\sin[c + d*x]])/(8*a*d) + (3*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*a*d)$
 $- (\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x]^3)/(4*a*d) + \operatorname{Tan}[c + d*x]^4/(4*a*d)$

Rubi [A] time = 0.115661, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^4(c+dx)}{4ad} - \frac{3 \tanh^{-1}(\sin(c+dx))}{8ad} - \frac{\tan^3(c+dx) \sec(c+dx)}{4ad} + \frac{3 \tan(c+dx) \sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tan}[c + d*x]^3/(a + a*\operatorname{Sin}[c + d*x]), x]$

[Out] $(-3*\operatorname{ArcTanh}[\sin[c + d*x]])/(8*a*d) + (3*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*a*d)$
 $- (\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x]^3)/(4*a*d) + \operatorname{Tan}[c + d*x]^4/(4*a*d)$

Rule 2706

$\operatorname{Int}(((g_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(p_.)}/((a_.) + (b_.)*\operatorname{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol) \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[\operatorname{Sec}[e + f*x]^2*(g*\operatorname{Tan}[e + f*x])^p, x], x] - \operatorname{Dist}[1/(b*g), \operatorname{Int}[\operatorname{Sec}[e + f*x]*(g*\operatorname{Tan}[e + f*x])^{(p + 1)}, x], x] /;$ $\operatorname{FreeQ}[\{a, b, e, f, g, p\}, x] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[p, -1]$

Rule 2607

$\operatorname{Int}[\operatorname{sec}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/f, \operatorname{Subst}[\operatorname{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \operatorname{Tan}[e + f*x]], x] /;$ $\operatorname{FreeQ}[\{b, e, f, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m/2] \ \&\& \ !(\operatorname{IntegerQ}[(n - 1)/2] \ \&\& \ \operatorname{LtQ}[0, n, m - 1])$

Rule 30

$\operatorname{Int}[(x_.)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /;$ $\operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 2611

$\operatorname{Int}(((a_.)*\operatorname{sec}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol) \rightarrow \operatorname{Simp}[(b*(a*\operatorname{Sec}[e + f*x])^m*(b*\operatorname{Tan}[e + f*x])^{(n - 1)})/(f*(m + n - 1)), x] - \operatorname{Dist}[(b^2*(n - 1))/(m + n - 1), \operatorname{Int}[(a*\operatorname{Sec}[e + f*x])^m*(b*\operatorname{Tan}[e + f*x])^{(n - 2)}, x], x] /;$ $\operatorname{FreeQ}[\{a, b, e, f, m\}, x] \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{NeQ}[m + n - 1, 0] \ \&\& \ \operatorname{IntegersQ}[2*m, 2*n]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^3(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^4(c+dx) dx}{a} \\
&= -\frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{3\int \sec(c+dx)\tan^2(c+dx) dx}{4a} + \frac{\text{Subst}\left(\int x^3 dx, x, \tan(c+dx)\right)}{ad} \\
&= \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4(c+dx)}{4ad} - \frac{3\int \sec(c+dx) dx}{8a} \\
&= -\frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.164595, size = 54, normalized size = 0.66

$$-\frac{\frac{1}{\sin(c+dx)-1} + \frac{4}{\sin(c+dx)+1} - \frac{1}{(\sin(c+dx)+1)^2} + 3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]

[Out] -(3*ArcTanh[Sin[c + d*x]] + (-1 + Sin[c + d*x])^(-1) - (1 + Sin[c + d*x])^(-2) + 4/(1 + Sin[c + d*x]))/(8*a*d)

Maple [A] time = 0.053, size = 90, normalized size = 1.1

$$-\frac{1}{8da(\sin(dx+c)-1)} + \frac{3\ln(\sin(dx+c)-1)}{16da} + \frac{1}{8da(1+\sin(dx+c))^2} - \frac{1}{2da(1+\sin(dx+c))} - \frac{3\ln(1+\sin(dx+c))}{16da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c)),x)

[Out] -1/8/a/d/(sin(d*x+c)-1)+3/16/a/d*ln(sin(d*x+c)-1)+1/8/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-3/16*ln(1+sin(d*x+c))/a/d

Maxima [A] time = 0.999972, size = 120, normalized size = 1.46

$$-\frac{\frac{2(5\sin(dx+c)^2+\sin(dx+c)-2)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}}{16d} + \frac{3\log(\sin(dx+c)+1)}{a} - \frac{3\log(\sin(dx+c)-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/16*(2*(5*sin(d*x + c)^2 + sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 3*log(sin(d*x + c) + 1)/a - 3*log(sin(d*x + c) - 1)/a)/d

Fricas [A] time = 1.34329, size = 338, normalized size = 4.12

$$\frac{10 \cos(dx + c)^2 + 3(\cos(dx + c)^2 \sin(dx + c) + \cos(dx + c)^2) \log(\sin(dx + c) + 1) - 3(\cos(dx + c)^2 \sin(dx + c) + \cos(dx + c)^2)}{16(ad \cos(dx + c)^2 \sin(dx + c) + ad \cos(dx + c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/16*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*sin(d*x + c) - 6)/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^3(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**3/(sin(c + d*x) + 1), x)/a

Giac [A] time = 2.11468, size = 130, normalized size = 1.59

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a} - \frac{6 \log(|\sin(dx+c)-1|)}{a} + \frac{2(3 \sin(dx+c)-1)}{a(\sin(dx+c)-1)} - \frac{9 \sin(dx+c)^2 + 2 \sin(dx+c) - 3}{a(\sin(dx+c)+1)^2}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(3*sin(d*x + c) - 1)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 2*sin(d*x + c) - 3)/(a*(sin(d*x + c) + 1)^2))/d

$$3.47 \quad \int \frac{\tan(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=37

$$\frac{1}{2d(a \sin(c+dx) + a)} + \frac{\tanh^{-1}(\sin(c+dx))}{2ad}$$

[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + 1/(2*d*(a + a*Sin[c + d*x]))

Rubi [A] time = 0.067309, antiderivative size = 58, normalized size of antiderivative = 1.57, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{\sec^2(c+dx)}{2ad} + \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{\tan(c+dx) \sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]

[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + Sec[c + d*x]^2/(2*a*d) - (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^2(c+dx) dx}{a} \\ &= -\frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \sec(c+dx) dx}{2a} + \frac{\text{Subst}(\int x dx, x, \sec(c+dx))}{ad} \\ &= \frac{\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{\sec^2(c+dx)}{2ad} - \frac{\sec(c+dx)\tan(c+dx)}{2ad} \end{aligned}$$

Mathematica [A] time = 0.038978, size = 28, normalized size = 0.76

$$\frac{\frac{1}{\sin(c+dx)+1} + \tanh^{-1}(\sin(c+dx))}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x]), x]

[Out] (ArcTanh[Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(2*a*d)

Maple [A] time = 0.058, size = 54, normalized size = 1.5

$$-\frac{\ln(\sin(dx+c)-1)}{4da} + \frac{1}{2da(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+a*sin(d*x+c)), x)

[Out] -1/4/a/d*ln(sin(d*x+c)-1)+1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d

Maxima [A] time = 1.08759, size = 63, normalized size = 1.7

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} + \frac{2}{a\sin(dx+c)+a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)), x, algorithm="maxima")

[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a + 2/(a*sin(d*x + c) + a))/d

Fricas [A] time = 1.63615, size = 163, normalized size = 4.41

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1) - (\sin(dx+c)+1)\log(-\sin(dx+c)+1) + 2}{4(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} * ((\sin(d*x + c) + 1) * \log(\sin(d*x + c) + 1) - (\sin(d*x + c) + 1) * \log(-\sin(d*x + c) + 1) + 2) / (a*d*\sin(d*x + c) + a*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)/(sin(c + d*x) + 1), x)/a

Giac [A] time = 1.34095, size = 78, normalized size = 2.11

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)-1|)}{a} - \frac{\sin(dx+c)-1}{a(\sin(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{4} * (\log(\text{abs}(\sin(d*x + c) + 1)) / a - \log(\text{abs}(\sin(d*x + c) - 1)) / a - (\sin(d*x + c) - 1) / (a * (\sin(d*x + c) + 1))) / d$

$$3.48 \quad \int \frac{\cot(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)

Rubi [A] time = 0.0390786, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]

[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(p_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{a+a \sin(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, a \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, a \sin(c+dx)\right)}{ad} \\ &= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1 + \sin(c+dx))}{ad} \end{aligned}$$

Mathematica [A] time = 0.0193949, size = 32, normalized size = 1.

$$\frac{\log(\sin(c + dx))}{ad} - \frac{\log(\sin(c + dx) + 1)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]

[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)

Maple [A] time = 0.023, size = 33, normalized size = 1.

$$\frac{\ln(\sin(dx + c))}{da} - \frac{\ln(1 + \sin(dx + c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d

Maxima [A] time = 1.04724, size = 42, normalized size = 1.31

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d

Fricas [A] time = 1.40674, size = 74, normalized size = 2.31

$$\frac{\log\left(\frac{1}{2} \sin(dx + c)\right) - \log(\sin(dx + c) + 1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)/(sin(c + d*x) + 1), x)/a

Giac [A] time = 1.2827, size = 45, normalized size = 1.41

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d

$$3.49 \quad \int \frac{\cot^3(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)

Rubi [A] time = 0.0677649, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2706, 2606, 30, 8}

$$\frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]

[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_.)]^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.)), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{a+a \sin(c+dx)} dx &= -\frac{\int \cot(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot(c+dx) \csc^2(c+dx) dx}{a} \\ &= \frac{\text{Subst}(\int 1 dx, x, \csc(c+dx))}{ad} - \frac{\text{Subst}(\int x dx, x, \csc(c+dx))}{ad} \\ &= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} \end{aligned}$$

Mathematica [A] time = 0.032614, size = 24, normalized size = 0.75

$$-\frac{(\csc(c + dx) - 2) \csc(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]

[Out] -((-2 + Csc[c + d*x])*Csc[c + d*x])/(2*a*d)

Maple [A] time = 0.032, size = 30, normalized size = 0.9

$$-\frac{1}{da} \left(-(\sin(dx + c))^{-1} + \frac{1}{2(\sin(dx + c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c)),x)

[Out] -1/d/a*(-1/sin(d*x+c)+1/2/sin(d*x+c)^2)

Maxima [A] time = 1.11797, size = 35, normalized size = 1.09

$$\frac{2 \sin(dx + c) - 1}{2ad \sin(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)

Fricas [A] time = 1.37048, size = 73, normalized size = 2.28

$$-\frac{2 \sin(dx + c) - 1}{2(ad \cos(dx + c)^2 - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^3(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**3/(sin(c + d*x) + 1), x)/a

Giac [A] time = 1.32292, size = 35, normalized size = 1.09

$$\frac{2 \sin(dx + c) - 1}{2ad \sin(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)

$$3.50 \quad \int \frac{\cot^5(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad} + \frac{\csc^3(c+dx)}{3ad} - \frac{\csc(c+dx)}{ad}$$

[Out] $-\text{Cot}[c + d*x]^4/(4*a*d) - \text{Csc}[c + d*x]/(a*d) + \text{Csc}[c + d*x]^3/(3*a*d)$

Rubi [A] time = 0.0889617, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^4(c+dx)}{4ad} + \frac{\csc^3(c+dx)}{3ad} - \frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^5/(a + a*\text{Sin}[c + d*x]), x]$

[Out] $-\text{Cot}[c + d*x]^4/(4*a*d) - \text{Csc}[c + d*x]/(a*d) + \text{Csc}[c + d*x]^3/(3*a*d)$

Rule 2706

$\text{Int}[(g_*)\tan[(e_*) + (f_*)(x_)]^{(p_*)}/((a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]), x_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[\text{Sec}[e + f*x]^2*(g*\text{Tan}[e + f*x])^p, x], x] - \text{Dist}[1/(b*g), \text{Int}[\text{Sec}[e + f*x]*(g*\text{Tan}[e + f*x])^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[p, -1]$

Rule 2607

$\text{Int}[\sec[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\tan[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, e, f, n\}, x] \&\& \text{IntegerQ}[m/2] \&\& !(\text{IntegerQ}[(n - 1)/2] \&\& \text{LtQ}[0, n, m - 1])$

Rule 30

$\text{Int}[(x_*)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}/(m + 1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 2606

$\text{Int}[(a_*)\sec[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\tan[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m - 1)}*(-1 + x^2)^{(n - 1)/2}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n - 1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rubi steps

$$\begin{aligned} \int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^3(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^3(c+dx) \csc^2(c+dx) dx}{a} \\ &= -\frac{\text{Subst}\left(\int x^3 dx, x, -\cot(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int (-1+x^2) dx, x, \csc(c+dx)\right)}{ad} \\ &= -\frac{\cot^4(c+dx)}{4ad} - \frac{\csc(c+dx)}{ad} + \frac{\csc^3(c+dx)}{3ad} \end{aligned}$$

Mathematica [A] time = 0.0486413, size = 30, normalized size = 0.59

$$-\frac{(\csc(c+dx)-1)^3(3\csc(c+dx)+5)}{12ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]), x]

[Out] -((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(12*a*d)

Maple [A] time = 0.085, size = 49, normalized size = 1.

$$\frac{1}{da} \left(-(\sin(dx+c))^{-1} - \frac{1}{4(\sin(dx+c))^4} + \frac{1}{3(\sin(dx+c))^3} + \frac{1}{2(\sin(dx+c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c)), x)

[Out] 1/d/a*(-1/sin(d*x+c)-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)

Maxima [A] time = 1.10562, size = 62, normalized size = 1.22

$$\frac{12 \sin(dx+c)^3 - 6 \sin(dx+c)^2 - 4 \sin(dx+c) + 3}{12 ad \sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)), x, algorithm="maxima")

[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(d*x + c)^4)

Fricas [A] time = 1.40033, size = 162, normalized size = 3.18

$$\frac{6 \cos(dx+c)^2 - 4(3 \cos(dx+c)^2 - 2) \sin(dx+c) - 3}{12(ad \cos(dx+c)^4 - 2ad \cos(dx+c)^2 + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $-1/12*(6*\cos(d*x + c)^2 - 4*(3*\cos(d*x + c)^2 - 2)*\sin(d*x + c) - 3)/(a*d*\cos(d*x + c)^4 - 2*a*d*\cos(d*x + c)^2 + a*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{\sin(c+dx)+1} dx$$

a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**5/(sin(c + d*x) + 1), x)/a

Giac [A] time = 1.32141, size = 62, normalized size = 1.22

$$\frac{12 \sin(dx + c)^3 - 6 \sin(dx + c)^2 - 4 \sin(dx + c) + 3}{12 ad \sin(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] $-1/12*(12*\sin(d*x + c)^3 - 6*\sin(d*x + c)^2 - 4*\sin(d*x + c) + 3)/(a*d*\sin(d*x + c)^4)$

3.51 $\int \frac{\cot^7(c+dx)}{a+a \sin(c+dx)} dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad} + \frac{\csc^5(c+dx)}{5ad} - \frac{2 \csc^3(c+dx)}{3ad} + \frac{\csc(c+dx)}{ad}$$

[Out] $-\text{Cot}[c + d*x]^6/(6*a*d) + \text{Csc}[c + d*x]/(a*d) - (2*\text{Csc}[c + d*x]^3)/(3*a*d) + \text{Csc}[c + d*x]^5/(5*a*d)$

Rubi [A] time = 0.0933341, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^6(c+dx)}{6ad} + \frac{\csc^5(c+dx)}{5ad} - \frac{2 \csc^3(c+dx)}{3ad} + \frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^7/(a + a*\text{Sin}[c + d*x]), x]$

[Out] $-\text{Cot}[c + d*x]^6/(6*a*d) + \text{Csc}[c + d*x]/(a*d) - (2*\text{Csc}[c + d*x]^3)/(3*a*d) + \text{Csc}[c + d*x]^5/(5*a*d)$

Rule 2706

$\text{Int}[(g_*)*\tan[(e_*) + (f_*)*(x_)]^{(p_*)}/((a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[\text{Sec}[e + f*x]^2*(g*\text{Tan}[e + f*x])^p, x], x] - \text{Dist}[1/(b*g), \text{Int}[\text{Sec}[e + f*x]*(g*\text{Tan}[e + f*x])^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[p, -1]$

Rule 2607

$\text{Int}[\sec[(e_*) + (f_*)*(x_)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)*(x_)]^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, e, f, n\}, x] \&\& \text{IntegerQ}[m/2] \&\& !(\text{IntegerQ}[(n - 1)/2] \&\& \text{LtQ}[0, n, m - 1])$

Rule 30

$\text{Int}[(x_*)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 2606

$\text{Int}[(a_*)*\sec[(e_*) + (f_*)*(x_)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)*(x_)]^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)}*(-1 + x^2)^{((n-1)/2)}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 194

$\text{Int}[(a_*) + (b_*)*(x_*)^{(n_*)}]^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^5(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^5(c+dx) \csc^2(c+dx) dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^5 dx, x, -\cot(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int (-1+x^2)^2 dx, x, \csc(c+dx)\right)}{ad} \\
&= -\frac{\cot^6(c+dx)}{6ad} + \frac{\text{Subst}\left(\int (1-2x^2+x^4) dx, x, \csc(c+dx)\right)}{ad} \\
&= -\frac{\cot^6(c+dx)}{6ad} + \frac{\csc(c+dx)}{ad} - \frac{2 \csc^3(c+dx)}{3ad} + \frac{\csc^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A] time = 0.149153, size = 61, normalized size = 0.9

$$\frac{\csc^6(c+dx)(78 \sin(c+dx) - 5(7 \sin(3(c+dx))) - 3 \sin(5(c+dx)) + 5) - 15 \cos(4(c+dx))}{240ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]

[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)

Maple [A] time = 0.095, size = 67, normalized size = 1.

$$\frac{1}{da} \left((\sin(dx+c))^{-1} + \frac{1}{5(\sin(dx+c))^5} + \frac{1}{2(\sin(dx+c))^4} - \frac{1}{6(\sin(dx+c))^6} - \frac{2}{3(\sin(dx+c))^3} - \frac{1}{2(\sin(dx+c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(1/sin(d*x+c)+1/5/sin(d*x+c)^5+1/2/sin(d*x+c)^4-1/6/sin(d*x+c)^6-2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)

Maxima [A] time = 1.12848, size = 89, normalized size = 1.31

$$\frac{30 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 15 \sin(dx+c)^2 + 6 \sin(dx+c) - 5}{30 ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)

Fricas [A] time = 1.41345, size = 248, normalized size = 3.65

$$\frac{15 \cos(dx+c)^4 - 15 \cos(dx+c)^2 - 2(15 \cos(dx+c)^4 - 20 \cos(dx+c)^2 + 8) \sin(dx+c) + 5}{30(ad \cos(dx+c)^6 - 3ad \cos(dx+c)^4 + 3ad \cos(dx+c)^2 - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos
(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)
^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4623, size = 89, normalized size = 1.31

$$\frac{30 \sin(dx + c)^5 - 15 \sin(dx + c)^4 - 20 \sin(dx + c)^3 + 15 \sin(dx + c)^2 + 6 \sin(dx + c) - 5}{30 ad \sin(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

$$3.52 \quad \int \frac{\cot^9(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=84

$$-\frac{\cot^8(c+dx)}{8ad} + \frac{\csc^7(c+dx)}{7ad} - \frac{3 \csc^5(c+dx)}{5ad} + \frac{\csc^3(c+dx)}{ad} - \frac{\csc(c+dx)}{ad}$$

[Out] $-\text{Cot}[c + d*x]^8/(8*a*d) - \text{Csc}[c + d*x]/(a*d) + \text{Csc}[c + d*x]^3/(a*d) - (3*\text{Csc}[c + d*x]^5)/(5*a*d) + \text{Csc}[c + d*x]^7/(7*a*d)$

Rubi [A] time = 0.0997991, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^8(c+dx)}{8ad} + \frac{\csc^7(c+dx)}{7ad} - \frac{3 \csc^5(c+dx)}{5ad} + \frac{\csc^3(c+dx)}{ad} - \frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^9/(a + a*\text{Sin}[c + d*x]), x]$

[Out] $-\text{Cot}[c + d*x]^8/(8*a*d) - \text{Csc}[c + d*x]/(a*d) + \text{Csc}[c + d*x]^3/(a*d) - (3*\text{Csc}[c + d*x]^5)/(5*a*d) + \text{Csc}[c + d*x]^7/(7*a*d)$

Rule 2706

$\text{Int}[(g_*)\tan[(e_*) + (f_*)(x_)]^{(p_*)}/((a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]), x_Symbol] := \text{Dist}[1/a, \text{Int}[\text{Sec}[e + f*x]^2*(g*\text{Tan}[e + f*x])^p, x], x] - \text{Dist}[1/(b*g), \text{Int}[\text{Sec}[e + f*x]*(g*\text{Tan}[e + f*x])^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[p, -1]$

Rule 2607

$\text{Int}[\sec[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\tan[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, e, f, n\}, x] \&\& \text{IntegerQ}[m/2] \&\& !(\text{IntegerQ}[(n - 1)/2] \&\& \text{LtQ}[0, n, m - 1])$

Rule 30

$\text{Int}[(x_*)^{(m_*)}, x_Symbol] := \text{Simp}[x^{(m + 1)}/(m + 1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 2606

$\text{Int}[(a_*)\sec[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\tan[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] := \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m - 1)}*(-1 + x^2)^{((n - 1)/2)}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n - 1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 194

$\text{Int}[(a_*) + (b_*)(x_*)^{(n_*)}]^{(p_*)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^7(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^7(c+dx) \csc^2(c+dx) dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^7 dx, x, -\cot(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int (-1+x^2)^3 dx, x, \csc(c+dx)\right)}{ad} \\
&= -\frac{\cot^8(c+dx)}{8ad} + \frac{\text{Subst}\left(\int (-1+3x^2-3x^4+x^6) dx, x, \csc(c+dx)\right)}{ad} \\
&= -\frac{\cot^8(c+dx)}{8ad} - \frac{\csc(c+dx)}{ad} + \frac{\csc^3(c+dx)}{ad} - \frac{3\csc^5(c+dx)}{5ad} + \frac{\csc^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] time = 0.208641, size = 77, normalized size = 0.92

$$\frac{\csc^8(c+dx)(-513\sin(c+dx) + 371\sin(3(c+dx)) - 105\sin(5(c+dx)) + 35\sin(7(c+dx)) - 245\cos(2(c+dx)) - 35\cos(4(c+dx)))}{2240ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]

[Out] (Csc[c + d*x]^8*(-245*Cos[2*(c + d*x)] - 35*Cos[6*(c + d*x)] - 513*Sin[c + d*x] + 371*Sin[3*(c + d*x)] - 105*Sin[5*(c + d*x)] + 35*Sin[7*(c + d*x)]))/ (2240*a*d)

Maple [A] time = 0.115, size = 87, normalized size = 1.

$$\frac{1}{da} \left(\frac{1}{7 (\sin(dx+c))^7} - (\sin(dx+c))^{-1} - \frac{1}{8 (\sin(dx+c))^8} - \frac{3}{5 (\sin(dx+c))^5} - \frac{3}{4 (\sin(dx+c))^4} + \frac{1}{2 (\sin(dx+c))^6} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(1/7/sin(d*x+c)^7-1/sin(d*x+c)-1/8/sin(d*x+c)^8-3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4+1/2/sin(d*x+c)^6+1/sin(d*x+c)^3+1/2/sin(d*x+c)^2)

Maxima [A] time = 1.10832, size = 116, normalized size = 1.38

$$\frac{280 \sin(dx+c)^7 - 140 \sin(dx+c)^6 - 280 \sin(dx+c)^5 + 210 \sin(dx+c)^4 + 168 \sin(dx+c)^3 - 140 \sin(dx+c)^2 - 40 \sin(dx+c) + 35}{280 ad \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c) + 35)/(a*d*sin(d*x + c)^8)

Fricas [A] time = 1.51708, size = 343, normalized size = 4.08

$$\frac{140 \cos(dx + c)^6 - 210 \cos(dx + c)^4 + 140 \cos(dx + c)^2 - 8(35 \cos(dx + c)^6 - 70 \cos(dx + c)^4 + 56 \cos(dx + c)^2 - 16) \sin(dx + c) - 35}{280(ad \cos(dx + c)^8 - 4ad \cos(dx + c)^6 + 6ad \cos(dx + c)^4 - 4ad \cos(dx + c)^2 + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/280*(140*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 140*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^6 - 70*cos(d*x + c)^4 + 56*cos(d*x + c)^2 - 16)*sin(d*x + c) - 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.43815, size = 116, normalized size = 1.38

$$\frac{280 \sin(dx + c)^7 - 140 \sin(dx + c)^6 - 280 \sin(dx + c)^5 + 210 \sin(dx + c)^4 + 168 \sin(dx + c)^3 - 140 \sin(dx + c)^2 + 35}{280 ad \sin(dx + c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c) + 35)/(a*d*sin(d*x + c)^8)

3.53 $\int \frac{\tan^6(c+dx)}{a+a \sin(c+dx)} dx$

Optimal. Leaf size=84

$$\frac{\tan^7(c+dx)}{7ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^3(c+dx)}{ad} + \frac{\sec(c+dx)}{ad}$$

[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Sec[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)

Rubi [A] time = 0.0973616, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^7(c+dx)}{7ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^3(c+dx)}{ad} + \frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]

[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Sec[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 194

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^6(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^7(c+dx) dx}{a} \\
&= \frac{\text{Subst}\left(\int x^6 dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int (-1+x^2)^3 dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{\tan^7(c+dx)}{7ad} - \frac{\text{Subst}\left(\int (-1+3x^2-3x^4+x^6) dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{\tan^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] time = 0.319019, size = 146, normalized size = 1.74

$$\frac{\sec^5(c+dx)(2432\sin(c+dx) - 1905\sin(2(c+dx)) + 320\sin(3(c+dx)) - 1524\sin(4(c+dx)) + 960\sin(5(c+dx)) - 381\sin(6(c+dx)))}{7920a*d*(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^6/(a + a*Sin[c + d*x]), x]

[Out] (Sec[c + d*x]^5*(2912 - 7620*Cos[c + d*x] + 3760*Cos[2*(c + d*x)] - 3810*Cos[3*(c + d*x)] + 1440*Cos[4*(c + d*x)] - 762*Cos[5*(c + d*x)] + 80*Cos[6*(c + d*x)] + 2432*Sin[c + d*x] - 1905*Sin[2*(c + d*x)] + 320*Sin[3*(c + d*x)] - 1524*Sin[4*(c + d*x)] + 960*Sin[5*(c + d*x)] - 381*Sin[6*(c + d*x)])/(7920*a*d*(1 + Sin[c + d*x]))

Maple [B] time = 0.057, size = 175, normalized size = 2.1

$$128 \frac{1}{da} \left(-\frac{1}{1280 (\tan(1/2 dx + c/2) - 1)^5} - \frac{1}{512 (\tan(1/2 dx + c/2) - 1)^4} + \frac{1}{512 (\tan(1/2 dx + c/2) - 1)^2} - \frac{1}{2048 \tan(1/2 dx + c/2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^6/(a+a*sin(d*x+c)), x)

[Out] 128/d/a*(-1/1280/(tan(1/2*d*x+1/2*c)-1)^5-1/512/(tan(1/2*d*x+1/2*c)-1)^4+1/512/(tan(1/2*d*x+1/2*c)-1)^2-5/2048/(tan(1/2*d*x+1/2*c)-1)-1/448/(tan(1/2*d*x+1/2*c)+1)^7+1/128/(tan(1/2*d*x+1/2*c)+1)^6-9/1280/(tan(1/2*d*x+1/2*c)+1)^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4+1/512/(tan(1/2*d*x+1/2*c)+1)^3+3/1024/(tan(1/2*d*x+1/2*c)+1)^2+5/2048/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] time = 1.09477, size = 456, normalized size = 5.43

$$\begin{aligned}
&32 \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - \frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right. \\
&35 \left(a + \frac{2a \sin(dx+c)}{\cos(dx+c)+1} - \frac{4a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10a \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20a \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{20a \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{5a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{10a \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)), x, algorithm="maxima")

```
[Out] 32/35*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 10*a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 2*a*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.67288, size = 248, normalized size = 2.95

$$\frac{5 \cos(dx + c)^6 + 15 \cos(dx + c)^4 - 5 \cos(dx + c)^2 + 2(15 \cos(dx + c)^4 - 10 \cos(dx + c)^2 + 3) \sin(dx + c) + 1}{35(ad \cos(dx + c)^5 \sin(dx + c) + ad \cos(dx + c)^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(d*x + c)^6 + 15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^5*sin(d*x + c) + a*d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^6(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 5.31986, size = 232, normalized size = 2.76

$$\frac{7\left(25 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 120 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 210 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 140 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 33\right)}{a\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)^5} - \frac{175 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 1260 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 3815 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 6020 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 4641 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1792 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 281}{a\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^7} / d$$

560 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/560*(7*(25*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 210*tan(1/2*d*x + 1/2*c)^2 - 140*tan(1/2*d*x + 1/2*c) + 33)/(a*(tan(1/2*d*x + 1/2*c) - 1)^5) - (175*tan(1/2*d*x + 1/2*c)^6 + 1260*tan(1/2*d*x + 1/2*c)^5 + 3815*tan(1/2*d*x + 1/2*c)^4 + 6020*tan(1/2*d*x + 1/2*c)^3 + 4641*tan(1/2*d*x + 1/2*c)^2 + 1792*tan(1/2*d*x + 1/2*c) + 281)/(a*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.54 $\int \frac{\tan^4(c+dx)}{a+a \sin(c+dx)} dx$

Optimal. Leaf size=69

$$\frac{\tan^5(c+dx)}{5ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{2 \sec^3(c+dx)}{3ad} - \frac{\sec(c+dx)}{ad}$$

[Out] $-(\text{Sec}[c + d*x]/(a*d)) + (2*\text{Sec}[c + d*x]^3)/(3*a*d) - \text{Sec}[c + d*x]^5/(5*a*d) + \text{Tan}[c + d*x]^5/(5*a*d)$

Rubi [A] time = 0.0918369, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^5(c+dx)}{5ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{2 \sec^3(c+dx)}{3ad} - \frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[c + d*x]^4/(a + a*\text{Sin}[c + d*x]), x]$

[Out] $-(\text{Sec}[c + d*x]/(a*d)) + (2*\text{Sec}[c + d*x]^3)/(3*a*d) - \text{Sec}[c + d*x]^5/(5*a*d) + \text{Tan}[c + d*x]^5/(5*a*d)$

Rule 2706

$\text{Int}[(g_*)\text{tan}[(e_*) + (f_*)(x_)]^{(p_*)}/((a_*) + (b_*)\text{sin}[(e_*) + (f_*)(x_)]), x_Symbol] := \text{Dist}[1/a, \text{Int}[\text{Sec}[e + f*x]^2*(g*\text{Tan}[e + f*x])^p, x], x] - \text{Dist}[1/(b*g), \text{Int}[\text{Sec}[e + f*x]*(g*\text{Tan}[e + f*x])^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[p, -1]$

Rule 2607

$\text{Int}[\text{sec}[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\text{tan}[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, e, f, n\}, x] \&\& \text{IntegerQ}[m/2] \&\& !(\text{IntegerQ}[(n - 1)/2] \&\& \text{LtQ}[0, n, m - 1])$

Rule 30

$\text{Int}[(x_*)^{(m_*)}, x_Symbol] := \text{Simp}[x^{(m + 1)}/(m + 1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 2606

$\text{Int}[(a_*)\text{sec}[(e_*) + (f_*)(x_)]^{(m_*)}*((b_*)\text{tan}[(e_*) + (f_*)(x_)]^{(n_*)}), x_Symbol] := \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m - 1)}*(-1 + x^2)^{((n - 1)/2)}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n - 1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 194

$\text{Int}[(a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^5(c+dx) dx}{a} \\ &= \frac{\text{Subst}\left(\int x^4 dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int (-1+x^2)^2 dx, x, \sec(c+dx)\right)}{ad} \\ &= \frac{\tan^5(c+dx)}{5ad} - \frac{\text{Subst}\left(\int (1-2x^2+x^4) dx, x, \sec(c+dx)\right)}{ad} \\ &= -\frac{\sec(c+dx)}{ad} + \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^5(c+dx)}{5ad} \end{aligned}$$

Mathematica [A] time = 0.319816, size = 106, normalized size = 1.54

$$\frac{\sec^3(c+dx)(-64\sin(c+dx) - 178\sin(2(c+dx)) + 192\sin(3(c+dx)) - 89\sin(4(c+dx)) - 534\cos(c+dx) + 288\cos(2(c+dx)))}{960ad(\sin(c+dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]

[Out] -(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c + d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)])/(960*a*d*(1 + Sin[c + d*x]))

Maple [B] time = 0.055, size = 130, normalized size = 1.9

$$32 \frac{1}{da} \left(-\frac{1}{192 (\tan(1/2 dx + c/2) - 1)^3} - \frac{1}{128 (\tan(1/2 dx + c/2) - 1)^2} + \frac{3}{256 \tan(1/2 dx + c/2) - 256} - \frac{1}{80 (\tan(1/2 dx + c/2) + 1)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^4/(a+a*sin(d*x+c)),x)

[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/256/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] time = 1.08124, size = 289, normalized size = 4.19

$$\frac{16 \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} - \frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + 1 \right)}{15 \left(a + \frac{2 a \sin(dx+c)}{\cos(dx+c)+1} - \frac{2 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6 a \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6 a \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{2 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{2 a \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x

$$+ c)^3/(\cos(dx + c) + 1)^3 + 6*a*\sin(dx + c)^5/(\cos(dx + c) + 1)^5 + 2*a*\sin(dx + c)^6/(\cos(dx + c) + 1)^6 - 2*a*\sin(dx + c)^7/(\cos(dx + c) + 1)^7 - a*\sin(dx + c)^8/(\cos(dx + c) + 1)^8)*d$$

Fricas [A] time = 1.51105, size = 194, normalized size = 2.81

$$\frac{3 \cos(dx + c)^4 + 6 \cos(dx + c)^2 + 4(3 \cos(dx + c)^2 - 1) \sin(dx + c) - 1}{15(ad \cos(dx + c)^3 \sin(dx + c) + ad \cos(dx + c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^4/(a+a*sin(dx+c)),x, algorithm="fricas")

[Out] -1/15*(3*cos(dx + c)^4 + 6*cos(dx + c)^2 + 4*(3*cos(dx + c)^2 - 1)*sin(dx + c) - 1)/(a*d*cos(dx + c)^3*sin(dx + c) + a*d*cos(dx + c)^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**4/(a+a*sin(dx+c)),x)

[Out] Integral(tan(c + dx)**4/(sin(c + dx) + 1), x)/a

Giac [A] time = 2.56709, size = 162, normalized size = 2.35

$$\frac{5 \left(9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 24 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 11 \right)}{a \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right)^3} - \frac{45 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 240 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 490 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 320 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 73}{a \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right)^5}$$

$120 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^4/(a+a*sin(dx+c)),x, algorithm="giac")

[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

$$3.55 \quad \int \frac{\tan^2(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=50

$$\frac{\tan^3(c+dx)}{3ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)}{ad}$$

[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)

Rubi [A] time = 0.0876481, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2706, 2607, 30, 2606}

$$\frac{\tan^3(c+dx)}{3ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx) dx}{a} - \frac{\int \sec(c+dx)\tan^3(c+dx) dx}{a} \\ &= \frac{\text{Subst}\left(\int x^2 dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int (-1+x^2) dx, x, \sec(c+dx)\right)}{ad} \\ &= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad} \end{aligned}$$

Mathematica [B] time = 0.164187, size = 106, normalized size = 2.12

$$\frac{8\sin(c+dx) - 5\sin(2(c+dx)) - 10\cos(c+dx) + 2\cos(2(c+dx)) + 6}{12ad(\sin(c+dx) + 1)\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))

Maple [A] time = 0.047, size = 70, normalized size = 1.4

$$8 \frac{1}{da} \left(-\frac{1}{16} (\tan(1/2 dx + c/2) - 1)^{-1} - \frac{1}{12} (\tan(1/2 dx + c/2) + 1)^{-3} + \frac{1}{8} (\tan(1/2 dx + c/2) + 1)^{-2} + \frac{1}{16} (\tan(1/2 dx + c/2) + 1)^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c)),x)

[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))

Maxima [A] time = 1.09485, size = 122, normalized size = 2.44

$$\frac{4 \left(\frac{2 \sin(dx+c)}{\cos(dx+c)+1} + 1 \right)}{3 \left(a + \frac{2 a \sin(dx+c)}{\cos(dx+c)+1} - \frac{2 a \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)

Fricas [A] time = 1.37095, size = 127, normalized size = 2.54

$$\frac{\cos(dx+c)^2 + 2 \sin(dx+c) + 1}{3(ad \cos(dx+c) \sin(dx+c) + ad \cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^2(c+dx)}{\sin(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**2/(sin(c + d*x) + 1), x)/a

Giac [A] time = 1.65929, size = 92, normalized size = 1.84

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} - \frac{3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 12 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 5}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d

$$3.56 \quad \int \frac{1}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=23

$$-\frac{\cos(c+dx)}{d(a \sin(c+dx)+a)}$$

[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))

Rubi [A] time = 0.0116354, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2648}

$$-\frac{\cos(c+dx)}{d(a \sin(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^(-1), x]

[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))

Rule 2648

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{a+a \sin(c+dx)} dx = -\frac{\cos(c+dx)}{d(a+a \sin(c+dx))}$$

Mathematica [B] time = 0.0414675, size = 48, normalized size = 2.09

$$\frac{2 \sin\left(\frac{1}{2}(c+dx)\right) \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{d(a \sin(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^(-1), x]

[Out] (2*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*(a + a*Sin[c + d*x]))

Maple [A] time = 0.022, size = 22, normalized size = 1.

$$-2 \frac{1}{da (\tan(1/2 dx + c/2) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+a*sin(d*x+c)),x)`

[Out] `-2/a/d/(tan(1/2*d*x+1/2*c)+1)`

Maxima [A] time = 1.07772, size = 36, normalized size = 1.57

$$-\frac{2}{\left(a + \frac{a \sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sin(d*x+c)),x, algorithm="maxima")`

[Out] `-2/((a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)`

Fricas [A] time = 1.46038, size = 108, normalized size = 4.7

$$-\frac{\cos(dx+c) - \sin(dx+c) + 1}{ad \cos(dx+c) + ad \sin(dx+c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sin(d*x+c)),x, algorithm="fricas")`

[Out] `-(cos(d*x + c) - sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)`

Sympy [A] time = 1.72599, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{ad \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} & \text{for } d \neq 0 \\ \frac{x}{a \sin(c) + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sin(d*x+c)),x)`

[Out] `Piecewise((-2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x/(a*sin(c) + a), True))`

Giac [A] time = 1.27038, size = 28, normalized size = 1.22

$$-\frac{2}{ad \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sin(d*x+c)),x, algorithm="giac")`

[Out] `-2/(a*d*(tan(1/2*d*x + 1/2*c) + 1))`

$$3.57 \quad \int \frac{\cot^2(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad}$$

[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)

Rubi [A] time = 0.0512723, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)

Rule 2706

Int[((g_)*tan[(e_) + (f_)*(x_)]^(p_))/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(c+dx)}{a+a \sin(c+dx)} dx &= -\frac{\int \csc(c+dx) dx}{a} + \frac{\int \csc^2(c+dx) dx}{a} \\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\text{Subst}(\int 1 dx, x, \cot(c+dx))}{ad} \\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad} \end{aligned}$$

Mathematica [B] time = 0.218073, size = 69, normalized size = 2.38

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\sin(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(2*a*d)

Maple [A] time = 0.068, size = 56, normalized size = 1.9

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c)),x)

[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 1.08722, size = 95, normalized size = 3.28

$$\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Fricas [B] time = 1.53889, size = 173, normalized size = 5.97

$$\frac{\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\cos(dx+c)}{2ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**2/(sin(c + d*x) + 1), x)/a

Giac [B] time = 1.2855, size = 88, normalized size = 3.03

$$\frac{\frac{2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} - \frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a} - \frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d

$$3.58 \quad \int \frac{\cot^4(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=58

$$-\frac{\cot^3(c+dx)}{3ad} - \frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx) \csc(c+dx)}{2ad}$$

[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)

Rubi [A] time = 0.088263, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^3(c+dx)}{3ad} - \frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx) \csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]

[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^2(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^2(c+dx) \csc^2(c+dx) dx}{a} \\ &= \frac{\cot(c+dx) \csc(c+dx)}{2ad} + \frac{\int \csc(c+dx) dx}{2a} + \frac{\text{Subst}\left(\int x^2 dx, x, -\cot(c+dx)\right)}{ad} \\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx) \csc(c+dx)}{2ad} \end{aligned}$$

Mathematica [B] time = 0.491465, size = 124, normalized size = 2.14

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right) \sec\left(\frac{1}{2}(c+dx)\right) \left(\csc\left(\frac{1}{2}(c+dx)\right) + \sec\left(\frac{1}{2}(c+dx)\right)\right)^2 \left(\cos(3(c+dx)) + (3-6\sin(c+dx))\cos(c+dx)\right)}{96ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]

[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(96*a*d*(1 + Sin[c + d*x]))

Maple [B] time = 0.075, size = 132, normalized size = 2.3

$$\frac{1}{24da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 - \frac{1}{8da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 - \frac{1}{8da} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{8da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} + \frac{1}{2da} \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c)),x)

[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/8/d/a*tan(1/2*d*x+1/2*c)+1/8/d/a/tan(1/2*d*x+1/2*c)+1/2/d/a*ln(tan(1/2*d*x+1/2*c))-1/24/d/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2

Maxima [B] time = 1.02688, size = 209, normalized size = 3.6

$$\frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)^3}{a \sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c

$\cos(dx + c) + 1)^2 - 1) * (\cos(dx + c) + 1)^3 / (a * \sin(dx + c)^3) / d$

Fricas [B] time = 1.41589, size = 311, normalized size = 5.36

$$\frac{4 \cos(dx + c)^3 - 3(\cos(dx + c)^2 - 1) \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + 3(\cos(dx + c)^2 - 1) \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 6 \cos(dx + c) \sin(dx + c)}{12(ad \cos(dx + c)^2 - ad) \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**4/(sin(c + d*x) + 1), x)/a

Giac [B] time = 1.41595, size = 171, normalized size = 2.95

$$\frac{12 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} + \frac{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 3a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3} - \frac{22 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}$$

24 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2*c)^3))/d

$$3.59 \quad \int \frac{\cot^6(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=82

$$-\frac{\cot^5(c+dx)}{5ad} + \frac{3 \tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\cot^3(c+dx) \csc(c+dx)}{4ad} - \frac{3 \cot(c+dx) \csc(c+dx)}{8ad}$$

[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)

Rubi [A] time = 0.10552, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^5(c+dx)}{5ad} + \frac{3 \tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\cot^3(c+dx) \csc(c+dx)}{4ad} - \frac{3 \cot(c+dx) \csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]

[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^4(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^4(c+dx) \csc^2(c+dx) dx}{a} \\
&= \frac{\cot^3(c+dx) \csc(c+dx)}{4ad} + \frac{3 \int \cot^2(c+dx) \csc(c+dx) dx}{4a} + \frac{\text{Subst}\left(\int x^4 dx, x, -\cot(c+dx)\right)}{ad} \\
&= -\frac{\cot^5(c+dx)}{5ad} - \frac{3 \cot(c+dx) \csc(c+dx)}{8ad} + \frac{\cot^3(c+dx) \csc(c+dx)}{4ad} - \frac{3 \int \csc(c+dx) dx}{8a} \\
&= \frac{3 \tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot^5(c+dx)}{5ad} - \frac{3 \cot(c+dx) \csc(c+dx)}{8ad} + \frac{\cot^3(c+dx) \csc(c+dx)}{4ad}
\end{aligned}$$

Mathematica [B] time = 0.743001, size = 189, normalized size = 2.3

$$\csc^5(c+dx) \left(20 \sin(2(c+dx)) - 50 \sin(4(c+dx)) + 80 \cos(c+dx) + 40 \cos(3(c+dx)) + 8 \cos(5(c+dx)) + 150 \sin(c+dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]

[Out] -(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c + d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(640*a*d)

Maple [B] time = 0.088, size = 208, normalized size = 2.5

$$\frac{1}{160da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^5 - \frac{1}{64da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 - \frac{1}{32da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 + \frac{1}{8da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{1}{16da} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c)),x)

[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/32/d/a*tan(1/2*d*x+1/2*c)^3+1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/16/d/a*tan(1/2*d*x+1/2*c)-1/16/d/a/tan(1/2*d*x+1/2*c)-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d*x+1/2*c)^4-3/8/d/a*ln(tan(1/2*d*x+1/2*c))+1/32/d/a/tan(1/2*d*x+1/2*c)^3-1/8/d/a/tan(1/2*d*x+1/2*c)^2

Maxima [B] time = 1.01987, size = 316, normalized size = 3.85

$$\frac{20 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{20 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a \sin(dx+c)^5}$$

320 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{320} \left(\frac{20 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin^2(dx+c)}{(\cos(dx+c)+1)^2} - \frac{10 \sin^3(dx+c)}{(\cos(dx+c)+1)^3} - \frac{5 \sin^4(dx+c)}{(\cos(dx+c)+1)^4} + \frac{2 \sin^5(dx+c)}{(\cos(dx+c)+1)^5} \right) / a - 120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) / a + \frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin^2(dx+c)}{(\cos(dx+c)+1)^2} - \frac{40 \sin^3(dx+c)}{(\cos(dx+c)+1)^3} - \frac{20 \sin^4(dx+c)}{(\cos(dx+c)+1)^4} - \frac{2 (\cos(dx+c)+1)^5}{a \sin(dx+c)^5} \right) / d$

Fricas [B] time = 1.39609, size = 431, normalized size = 5.26

$$\frac{16 \cos(dx+c)^5 - 15 (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 15 (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 15 (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c)}{80 (ad \cos(dx+c)^4 - 2 ad \cos(dx+c)^2 + a^2 d^2 \sin(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] $-\frac{1}{80} \left(\frac{16 \cos(dx+c)^5 - 15 (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 15 (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 10 (5 \cos(dx+c)^3 - 3 \cos(dx+c)) \sin(dx+c)}{(a d \cos(dx+c)^4 - 2 a d \cos(dx+c)^2 + a^2 d^2 \sin(dx+c)^2)} \right)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^6(c+dx)}{\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**6/(sin(c + d*x) + 1), x)/a

Giac [B] time = 1.57756, size = 252, normalized size = 3.07

$$\frac{120 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} - \frac{2 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 5 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 10 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 40 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 20 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 274 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^5}$$

320 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] $-\frac{1}{320} \left(\frac{120 \log(\tan(1/2 dx + 1/2 c))}{a} - \frac{(2 a^4 \tan(1/2 dx + 1/2 c)^5 - 5 a^4 \tan(1/2 dx + 1/2 c)^4 - 10 a^4 \tan(1/2 dx + 1/2 c)^3 + 40 a^4 \tan(1/2 dx + 1/2 c)^2 + 20 a^4 \tan(1/2 dx + 1/2 c) - 274 a^4 \tan(1/2 dx + 1/2 c))}{a^5} \right)$

$$\frac{+ 1/2*c)^5 - 20*\tan(1/2*d*x + 1/2*c)^4 - 40*\tan(1/2*d*x + 1/2*c)^3 + 10*\tan(1/2*d*x + 1/2*c)^2 + 5*\tan(1/2*d*x + 1/2*c) - 2}{(a*\tan(1/2*d*x + 1/2*c)^5)}/d$$

$$3.60 \quad \int \frac{\cot^8(c+dx)}{a+a \sin(c+dx)} dx$$

Optimal. Leaf size=106

$$\frac{\cot^7(c+dx)}{7ad} - \frac{5 \tanh^{-1}(\cos(c+dx))}{16ad} + \frac{\cot^5(c+dx) \csc(c+dx)}{6ad} - \frac{5 \cot^3(c+dx) \csc(c+dx)}{24ad} + \frac{5 \cot(c+dx) \csc(c+dx)}{16ad}$$

[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Cot[c + d*x]^5*Csc[c + d*x])/(6*a*d)

Rubi [A] time = 0.127459, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\cot^7(c+dx)}{7ad} - \frac{5 \tanh^{-1}(\cos(c+dx))}{16ad} + \frac{\cot^5(c+dx) \csc(c+dx)}{6ad} - \frac{5 \cot^3(c+dx) \csc(c+dx)}{24ad} + \frac{5 \cot(c+dx) \csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]

[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Cot[c + d*x]^5*Csc[c + d*x])/(6*a*d)

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)} dx &= -\frac{\int \cot^6(c+dx) \csc(c+dx) dx}{a} + \frac{\int \cot^6(c+dx) \csc^2(c+dx) dx}{a} \\ &= \frac{\cot^5(c+dx) \csc(c+dx)}{6ad} + \frac{5 \int \cot^4(c+dx) \csc(c+dx) dx}{6a} + \frac{\text{Subst}\left(\int x^6 dx, x, -\cot(c+dx)\right)}{ad} \\ &= -\frac{\cot^7(c+dx)}{7ad} - \frac{5 \cot^3(c+dx) \csc(c+dx)}{24ad} + \frac{\cot^5(c+dx) \csc(c+dx)}{6ad} - \frac{5 \int \cot^2(c+dx) \csc(c+dx) dx}{8a} \\ &= -\frac{\cot^7(c+dx)}{7ad} + \frac{5 \cot(c+dx) \csc(c+dx)}{16ad} - \frac{5 \cot^3(c+dx) \csc(c+dx)}{24ad} + \frac{\cot^5(c+dx) \csc(c+dx)}{6ad} \\ &= -\frac{5 \tanh^{-1}(\cos(c+dx))}{16ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{5 \cot(c+dx) \csc(c+dx)}{16ad} - \frac{5 \cot^3(c+dx) \csc(c+dx)}{24ad} \end{aligned}$$

Mathematica [B] time = 0.914319, size = 284, normalized size = 2.68

$$\frac{\csc^5(c+dx) \left(\csc\left(\frac{1}{2}(c+dx)\right) + \sec\left(\frac{1}{2}(c+dx)\right) \right)^2 \left(-1190 \sin(2(c+dx)) + 392 \sin(4(c+dx)) - 462 \sin(6(c+dx)) + 168 \sin(8(c+dx)) \right)}{16ad^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]), x]

[Out] -(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)] + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)])/(86016*a*d*(1 + Sin[c + d*x]))

Maple [B] time = 0.102, size = 284, normalized size = 2.7

$$\frac{1}{896da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^7 - \frac{1}{384da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^6 - \frac{1}{128da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^5 + \frac{3}{128da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 + \frac{3}{128da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 - \frac{15}{128da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{5}{128da} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{896da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{5}{128da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{1}{128da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{5}{16da} \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{1}{384da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} - \frac{3}{128da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{15}{128da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{1}{896da} \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^8/(a+a*sin(d*x+c)), x)

[Out] 1/896/d/a*tan(1/2*d*x+1/2*c)^7-1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/128/d/a*tan(1/2*d*x+1/2*c)^5+3/128/d/a*tan(1/2*d*x+1/2*c)^4+3/128/d/a*tan(1/2*d*x+1/2*c)^3-15/128/d/a*tan(1/2*d*x+1/2*c)^2-5/128/d/a*tan(1/2*d*x+1/2*c)-1/896/d/a/tan(1/2*d*x+1/2*c)+5/128/d/a/tan(1/2*d*x+1/2*c)+1/128/d/a/tan(1/2*d*x+1/2*c)^5-3/128/d/a/tan(1/2*d*x+1/2*c)^4+5/16/d/a*ln(tan(1/2*d*x+1/2*c))+1/384/d/a/tan(1/2*d*x+1/2*c)^6-3/128/d/a/tan(1/2*d*x+1/2*c)^3+15/128/d/a/tan(1/2*d*x+1/2*c)^2

Maxima [B] time = 1.12476, size = 425, normalized size = 4.01

$$\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{315 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{63 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{63 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{7 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a} - \frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1} + \frac{21 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{63 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{63 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{7 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a}$$

2688 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d

Fricas [B] time = 1.57927, size = 545, normalized size = 5.14

$$\frac{96 \cos(dx+c)^7 - 105 (\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 105 (\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 14 (33 \cos(dx+c)^5 - 40 \cos(dx+c)^3 + 15 \cos(dx+c)) \sin(dx+c)}{672 (ad \cos(dx+c)^7 - 3a^2d \cos(dx+c)^5 + 3a^4d \cos(dx+c)^3 - a^6d \cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c)/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**8/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [B] time = 1.97865, size = 329, normalized size = 3.1

$$\frac{840 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} + \frac{3 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 7 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 21 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 63 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 63 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 315 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 315 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 105 a^6}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*tan(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```


$$3.61 \quad \int \frac{\tan^7(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=189

$$\frac{a^3}{80d(a \sin(c+dx)+a)^5} - \frac{5a^2}{64d(a \sin(c+dx)+a)^4} + \frac{21}{256d(a^2 - a^2 \sin(c+dx))} + \frac{35}{256d(a^2 \sin(c+dx)+a^2)} - \frac{7 \tan(c+dx)}{256d(a^2 \sin(c+dx)+a^2)}$$

[Out] $(-7*\text{ArcTanh}[\text{Sin}[c+d*x]])/(128*a^2*d) + a/(192*d*(a - a*\text{Sin}[c+d*x])^3) - 1/(32*d*(a - a*\text{Sin}[c+d*x])^2) + a^3/(80*d*(a + a*\text{Sin}[c+d*x])^5) - (5*a^2)/(64*d*(a + a*\text{Sin}[c+d*x])^4) + (19*a)/(96*d*(a + a*\text{Sin}[c+d*x])^3) - 1/(4*d*(a + a*\text{Sin}[c+d*x])^2) + 21/(256*d*(a^2 - a^2*\text{Sin}[c+d*x])) + 35/(256*d*(a^2 + a^2*\text{Sin}[c+d*x]))$

Rubi [A] time = 0.148605, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{a^3}{80d(a \sin(c+dx)+a)^5} - \frac{5a^2}{64d(a \sin(c+dx)+a)^4} + \frac{21}{256d(a^2 - a^2 \sin(c+dx))} + \frac{35}{256d(a^2 \sin(c+dx)+a^2)} - \frac{7 \tan(c+dx)}{256d(a^2 \sin(c+dx)+a^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[c+d*x]^7/(a+a*\text{Sin}[c+d*x])^2, x]$

[Out] $(-7*\text{ArcTanh}[\text{Sin}[c+d*x]])/(128*a^2*d) + a/(192*d*(a - a*\text{Sin}[c+d*x])^3) - 1/(32*d*(a - a*\text{Sin}[c+d*x])^2) + a^3/(80*d*(a + a*\text{Sin}[c+d*x])^5) - (5*a^2)/(64*d*(a + a*\text{Sin}[c+d*x])^4) + (19*a)/(96*d*(a + a*\text{Sin}[c+d*x])^3) - 1/(4*d*(a + a*\text{Sin}[c+d*x])^2) + 21/(256*d*(a^2 - a^2*\text{Sin}[c+d*x])) + 35/(256*d*(a^2 + a^2*\text{Sin}[c+d*x]))$

Rule 2707

$\text{Int}[(a + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)} \tan[(e_*) + (f_*)(x_*)]^{(p_*)}, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a+x)^{(m-(p+1)/2)})/(a-x)^{((p+1)/2)}, x], x, b*\text{Sin}[e+f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[(p+1)/2]$

Rule 88

$\text{Int}[(a + (b_*)(x_*)^{(m_*)} * ((c_*) + (d_*)(x_*)^{(n_*)} * ((e_*) + (f_*)(x_*)^{(p_*)}))^{(q_*)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 206

$\text{Int}[(a + (b_*)(x_*)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)^6} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{64(a-x)^4} - \frac{1}{16(a-x)^3} + \frac{21}{256a(a-x)^2} - \frac{a^3}{16(a+x)^6} + \frac{5a^2}{16(a+x)^5} - \frac{19a}{32(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{35}{256a(a+x)^2}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a^3}{80d(a+a\sin(c+dx))^5} - \frac{5a}{64d(a+a\sin(c+dx))^4} \\
&= -\frac{7 \tanh^{-1}(\sin(c+dx))}{128a^2d} + \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a}{80d(a+a\sin(c+dx))^5}
\end{aligned}$$

Mathematica [A] time = 1.69255, size = 112, normalized size = 0.59

$$\frac{210 \tanh^{-1}(\sin(c+dx)) - \frac{2(105 \sin^7(c+dx) - 750 \sin^6(c+dx) - 815 \sin^5(c+dx) + 560 \sin^4(c+dx) + 1039 \sin^3(c+dx) + 78 \sin^2(c+dx) - 393 \sin(c+dx) - 144)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^5}}{3840a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]

[Out] -(210*ArcTanh[Sin[c + d*x]] - (2*(-144 - 393*Sin[c + d*x] + 78*Sin[c + d*x]^2 + 1039*Sin[c + d*x]^3 + 560*Sin[c + d*x]^4 - 815*Sin[c + d*x]^5 - 750*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^5))/(3840*a^2*d)

Maple [A] time = 0.096, size = 180, normalized size = 1.

$$-\frac{1}{192da^2(\sin(dx+c)-1)^3} - \frac{1}{32da^2(\sin(dx+c)-1)^2} - \frac{21}{256da^2(\sin(dx+c)-1)} + \frac{7 \ln(\sin(dx+c)-1)}{256da^2} + \frac{1}{80da^2(1+\sin(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x)

[Out] -1/192/d/a^2/(sin(d*x+c)-1)^3-1/32/d/a^2/(sin(d*x+c)-1)^2-21/256/d/a^2/(sin(d*x+c)-1)+7/256/d/a^2*ln(sin(d*x+c)-1)+1/80/d/a^2/(1+sin(d*x+c))^5-5/64/d/a^2/(1+sin(d*x+c))^4+19/96/d/a^2/(1+sin(d*x+c))^3-1/4/d/a^2/(1+sin(d*x+c))^2+35/256/d/a^2/(1+sin(d*x+c))-7/256*ln(1+sin(d*x+c))/a^2/d

Maxima [A] time = 1.10553, size = 273, normalized size = 1.44

$$\frac{2(105 \sin(dx+c)^7 - 750 \sin(dx+c)^6 - 815 \sin(dx+c)^5 + 560 \sin(dx+c)^4 + 1039 \sin(dx+c)^3 + 78 \sin(dx+c)^2 - 393 \sin(dx+c) - 144)}{a^2 \sin(dx+c)^8 + 2a^2 \sin(dx+c)^7 - 2a^2 \sin(dx+c)^6 - 6a^2 \sin(dx+c)^5 + 6a^2 \sin(dx+c)^3 + 2a^2 \sin(dx+c)^2 - 2a^2 \sin(dx+c) - a^2} - \frac{105 \log(\sin(dx+c)+1)}{a^2} + \frac{105}{3840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

```
[Out] 1/3840*(2*(105*sin(d*x + c)^7 - 750*sin(d*x + c)^6 - 815*sin(d*x + c)^5 + 560*sin(d*x + c)^4 + 1039*sin(d*x + c)^3 + 78*sin(d*x + c)^2 - 393*sin(d*x + c) - 144)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x + c)^6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2 - 2*a^2*sin(d*x + c) - a^2) - 105*log(sin(d*x + c) + 1)/a^2 + 105*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.72815, size = 599, normalized size = 3.17

$$1500 \cos(dx + c)^6 - 3380 \cos(dx + c)^4 + 2104 \cos(dx + c)^2 - 105 (\cos(dx + c)^8 - 2 \cos(dx + c)^6 \sin(dx + c) - 2 \cos(dx + c)^6) \log(\sin(dx + c) + 1) + 105 (\cos(dx + c)^8 - 2 \cos(dx + c)^6 \sin(dx + c) - 2 \cos(dx + c)^6) \log(-\sin(dx + c) + 1) - 2(105 \cos(dx + c)^6 + 500 \cos(dx + c)^4 - 276 \cos(dx + c)^2 + 64) \sin(dx + c) - 512 / (a^2 d \cos(dx + c)^8 - 2 a^2 d \cos(dx + c)^6 \sin(dx + c) - 2 a^2 d \cos(dx + c)^6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(1500*cos(d*x + c)^6 - 3380*cos(d*x + c)^4 + 2104*cos(d*x + c)^2 - 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 500*cos(d*x + c)^4 - 276*cos(d*x + c)^2 + 64)*sin(d*x + c) - 512)/(a^2*d*cos(d*x + c)^8 - 2*a^2*d*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 9.84355, size = 197, normalized size = 1.04

$$\frac{420 \log(|\sin(dx+c)+1|)}{a^2} - \frac{420 \log(|\sin(dx+c)-1|)}{a^2} + \frac{10(77 \sin(dx+c)^3 - 105 \sin(dx+c)^2 + 27 \sin(dx+c) + 9)}{a^2(\sin(dx+c)-1)^3} - \frac{959 \sin(dx+c)^5 + 6895 \sin(dx+c)^4 + 14150 \sin(dx+c)^3 + 13710 \sin(dx+c)^2 + 6555 \sin(dx+c) + 1251}{a^2(\sin(dx+c)+1)^5} / 15360 d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/15360*(420*log(abs(sin(d*x + c) + 1))/a^2 - 420*log(abs(sin(d*x + c) - 1))/a^2 + 10*(77*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 27*sin(d*x + c) + 9)/(a^2*(sin(d*x + c) - 1)^3) - (959*sin(d*x + c)^5 + 6895*sin(d*x + c)^4 + 14150*sin(d*x + c)^3 + 13710*sin(d*x + c)^2 + 6555*sin(d*x + c) + 1251)/(a^2*(sin(d*x + c) + 1)^5))/d
```

3.62 $\int \frac{\tan^5(c+dx)}{(a+a \sin(c+dx))^2} dx$

Optimal. Leaf size=146

$$\frac{a^2}{32d(a \sin(c+dx) + a)^4} - \frac{5}{64d(a^2 - a^2 \sin(c+dx))} - \frac{5}{32d(a^2 \sin(c+dx) + a^2)} + \frac{5 \tanh^{-1}(\sin(c+dx))}{64a^2d} - \frac{7a}{48d(a \sin(c+dx) + a)^3}$$

[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^2 + a^2*Sin[c + d*x]))

Rubi [A] time = 0.107958, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{a^2}{32d(a \sin(c+dx) + a)^4} - \frac{5}{64d(a^2 - a^2 \sin(c+dx))} - \frac{5}{32d(a^2 \sin(c+dx) + a^2)} + \frac{5 \tanh^{-1}(\sin(c+dx))}{64a^2d} - \frac{7a}{48d(a \sin(c+dx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]

[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^2 + a^2*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)^(p_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^5} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{32(a-x)^3} - \frac{5}{64a(a-x)^2} - \frac{a^2}{8(a+x)^5} + \frac{7a}{16(a+x)^4} - \frac{1}{2(a+x)^3} + \frac{5}{32a(a+x)^2} + \frac{5}{64a(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(c+dx))^3} + \frac{5}{4d(a+a\sin(c+dx))^2} \\
&= \frac{5 \tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7}{48d(a+a\sin(c+dx))^3}
\end{aligned}$$

Mathematica [A] time = 0.449695, size = 91, normalized size = 0.62

$$\frac{-15 \sin^5(c+dx) + 66 \sin^4(c+dx) + 74 \sin^3(c+dx) - 14 \sin^2(c+dx) - 47 \sin(c+dx) - 16}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^4} + 15 \tanh^{-1}(\sin(c+dx))}{192a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]

[Out] (15*ArcTanh[Sin[c + d*x]] + (-16 - 47*Sin[c + d*x] - 14*Sin[c + d*x]^2 + 74*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 - 15*Sin[c + d*x]^5)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^4))/(192*a^2*d)

Maple [A] time = 0.084, size = 144, normalized size = 1.

$$\frac{1}{64da^2(\sin(dx+c)-1)^2} + \frac{5}{64da^2(\sin(dx+c)-1)} - \frac{5 \ln(\sin(dx+c)-1)}{128da^2} + \frac{1}{32da^2(1+\sin(dx+c))^4} - \frac{1}{48da^2(1+\sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x)

[Out] 1/64/d/a^2/(sin(d*x+c)-1)^2+5/64/d/a^2/(sin(d*x+c)-1)-5/128/d/a^2*ln(sin(d*x+c)-1)+1/32/d/a^2/(1+sin(d*x+c))^4-7/48/d/a^2/(1+sin(d*x+c))^3+1/4/d/a^2/(1+sin(d*x+c))^2-5/32/d/a^2/(1+sin(d*x+c))+5/128*ln(1+sin(d*x+c))/a^2/d

Maxima [A] time = 1.05414, size = 225, normalized size = 1.54

$$\frac{2(15 \sin(dx+c)^5 - 66 \sin(dx+c)^4 - 74 \sin(dx+c)^3 + 14 \sin(dx+c)^2 + 47 \sin(dx+c) + 16)}{a^2 \sin(dx+c)^6 + 2a^2 \sin(dx+c)^5 - a^2 \sin(dx+c)^4 - 4a^2 \sin(dx+c)^3 - a^2 \sin(dx+c)^2 + 2a^2 \sin(dx+c) + a^2} - \frac{15 \log(\sin(dx+c)+1)}{a^2} + \frac{15 \log(\sin(dx+c)-1)}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/384*(2*(15*sin(d*x + c)^5 - 66*sin(d*x + c)^4 - 74*sin(d*x + c)^3 + 14*sin(d*x + c)^2 + 47*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x + c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - 15*log(sin(dx+c)+1)/a^2 + 15*log(sin(dx+c)-1)/a^2)

$c)^5 - a^2 \sin(dx + c)^4 - 4a^2 \sin(dx + c)^3 - a^2 \sin(dx + c)^2 + 2a^2 \sin(dx + c) + a^2) - 15 \log(\sin(dx + c) + 1)/a^2 + 15 \log(\sin(dx + c) - 1)/a^2)/d$

Fricas [A] time = 1.52801, size = 532, normalized size = 3.64

$$\frac{132 \cos(dx + c)^4 - 236 \cos(dx + c)^2 - 15(\cos(dx + c)^6 - 2 \cos(dx + c)^4 \sin(dx + c) - 2 \cos(dx + c)^4) \log(\sin(dx + c) + 1) + 15(\cos(dx + c)^6 - 2 \cos(dx + c)^4 \sin(dx + c) - 2 \cos(dx + c)^4) \log(\sin(dx + c) - 1)}{384(a^2 d \cos(dx + c) + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/384*(132*\cos(dx + c)^4 - 236*\cos(dx + c)^2 - 15*(\cos(dx + c)^6 - 2*\cos(dx + c)^4*\sin(dx + c) - 2*\cos(dx + c)^4)*\log(\sin(dx + c) + 1) + 15*(\cos(dx + c)^6 - 2*\cos(dx + c)^4*\sin(dx + c) - 2*\cos(dx + c)^4)*\log(-\sin(dx + c) + 1) - 2*(15*\cos(dx + c)^4 + 44*\cos(dx + c)^2 - 12)*\sin(dx + c) + 72)/(a^2*d*\cos(dx + c)^6 - 2*a^2*d*\cos(dx + c)^4*\sin(dx + c) - 2*a^2*d*\cos(dx + c)^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2

Giac [A] time = 5.28893, size = 170, normalized size = 1.16

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^2} - \frac{60 \log(|\sin(dx+c)-1|)}{a^2} + \frac{6(15 \sin(dx+c)^2 - 10 \sin(dx+c) - 1)}{a^2(\sin(dx+c)-1)^2} - \frac{125 \sin(dx+c)^4 + 740 \sin(dx+c)^3 + 1086 \sin(dx+c)^2 + 676 \sin(dx+c) + 157}{a^2(\sin(dx+c)+1)^4}}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] $1/1536*(60*\log(\text{abs}(\sin(dx + c) + 1))/a^2 - 60*\log(\text{abs}(\sin(dx + c) - 1))/a^2 + 6*(15*\sin(dx + c)^2 - 10*\sin(dx + c) - 1)/(a^2*(\sin(dx + c) - 1)^2) - (125*\sin(dx + c)^4 + 740*\sin(dx + c)^3 + 1086*\sin(dx + c)^2 + 676*\sin(dx + c) + 157)/(a^2*(\sin(dx + c) + 1)^4))/d$

3.63 $\int \frac{\tan^3(c+dx)}{(a+a \sin(c+dx))^2} dx$

Optimal. Leaf size=104

$$\frac{1}{16d(a^2 - a^2 \sin(c + dx))} + \frac{3}{16d(a^2 \sin(c + dx) + a^2)} - \frac{\tanh^{-1}(\sin(c + dx))}{8a^2d} + \frac{a}{12d(a \sin(c + dx) + a)^3} - \frac{1}{4d(a \sin(c + dx) + a)}$$

```
[Out] -ArcTanh[Sin[c + d*x]]/(8*a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4*d
*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.085784, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{1}{16d(a^2 - a^2 \sin(c + dx))} + \frac{3}{16d(a^2 \sin(c + dx) + a^2)} - \frac{\tanh^{-1}(\sin(c + dx))}{8a^2d} + \frac{a}{12d(a \sin(c + dx) + a)^3} - \frac{1}{4d(a \sin(c + dx) + a)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(8*a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4*d
*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p
_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x
_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^4} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{16a(a-x)^2} - \frac{a}{4(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{3}{16a(a+x)^2} - \frac{1}{8a(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d(a^2-a^2\sin(c+dx))} + \frac{3}{16d(a^2+a^2\sin(c+dx))} \\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d} + \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d(a^2-a^2\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.310755, size = 70, normalized size = 0.67

$$-\frac{\frac{3}{1-\sin(c+dx)} - \frac{9}{\sin(c+dx)+1} + \frac{12}{(\sin(c+dx)+1)^2} - \frac{4}{(\sin(c+dx)+1)^3} + 6 \tanh^{-1}(\sin(c+dx))}{48a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]

[Out] -(6*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 4/(1 + Sin[c + d*x])^3 + 12/(1 + Sin[c + d*x])^2 - 9/(1 + Sin[c + d*x]))/(48*a^2*d)

Maple [A] time = 0.083, size = 108, normalized size = 1.

$$-\frac{1}{16da^2(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{16da^2} + \frac{1}{12da^2(1+\sin(dx+c))^3} - \frac{1}{4da^2(1+\sin(dx+c))^2} + \frac{3}{16da^2(1+\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x)

[Out] -1/16/d/a^2/(sin(d*x+c)-1)+1/16/d/a^2*ln(sin(d*x+c)-1)+1/12/d/a^2/(1+sin(d*x+c))^3-1/4/d/a^2/(1+sin(d*x+c))^2+3/16/d/a^2/(1+sin(d*x+c))-1/16*ln(1+sin(d*x+c))/a^2/d

Maxima [A] time = 1.05494, size = 149, normalized size = 1.43

$$\frac{2(3\sin(dx+c)^3-6\sin(dx+c)^2-7\sin(dx+c)-2)}{a^2\sin(dx+c)^4+2a^2\sin(dx+c)^3-2a^2\sin(dx+c)-a^2} - \frac{3\log(\sin(dx+c)+1)}{a^2} + \frac{3\log(\sin(dx+c)-1)}{a^2}$$

48d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/48*(2*(3*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 7*sin(d*x + c) - 2)/(a^2*sin(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(d*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d

Fricas [A] time = 1.53143, size = 467, normalized size = 4.49

$$\frac{12 \cos(dx+c)^2 - 3(\cos(dx+c)^4 - 2 \cos(dx+c)^2 \sin(dx+c) - 2 \cos(dx+c)^2) \log(\sin(dx+c)+1) + 3(\cos(dx+c)^4 - 2 \cos(dx+c)^2 \sin(dx+c) - 2 \cos(dx+c)^2) \log(-\sin(dx+c)+1) - 2(3 \cos(dx+c)^2 + 4) \sin(dx+c) - 16}{48(a^2 d \cos(dx+c)^4 - 2 a^2 d \cos(dx+c)^2 \sin(dx+c) - 2 a^2 d \cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/48*(12*cos(d*x + c)^2 - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*cos(d*x + c)^2 + 4)*sin(d*x + c) - 16)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

a^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2

Giac [A] time = 2.99776, size = 138, normalized size = 1.33

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a^2} - \frac{6 \log(|\sin(dx+c)-1|)}{a^2} + \frac{6 \sin(dx+c)}{a^2(\sin(dx+c)-1)} - \frac{11 \sin(dx+c)^3 + 51 \sin(dx+c)^2 + 45 \sin(dx+c) + 13}{a^2(\sin(dx+c)+1)^3}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/96*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2 + 6*sin(d*x + c)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 51*sin(d*x + c)^2 + 45*sin(d*x + c) + 13)/(a^2*(sin(d*x + c) + 1)^3))/d

$$3.64 \quad \int \frac{\tan(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=60

$$-\frac{1}{4d(a^2 \sin(c+dx) + a^2)} + \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d} + \frac{1}{4d(a \sin(c+dx) + a)^2}$$

[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(a^2 + a^2*Sin[c + d*x]))

Rubi [A] time = 0.0483769, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2707, 77, 206}

$$-\frac{1}{4d(a^2 \sin(c+dx) + a^2)} + \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d} + \frac{1}{4d(a \sin(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]

[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(a^2 + a^2*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^3} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^3} + \frac{1}{4a(a+x)^2} + \frac{1}{4a(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{1}{4d(a+a\sin(c+dx))^2} - \frac{1}{4d(a^2+a^2\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2} dx, x, a\sin(c+dx)\right)}{4ad} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d} + \frac{1}{4d(a+a\sin(c+dx))^2} - \frac{1}{4d(a^2+a^2\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.0807824, size = 36, normalized size = 0.6

$$\frac{\tanh^{-1}(\sin(c+dx)) - \frac{\sin(c+dx)}{(\sin(c+dx)+1)^2}}{4a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]

[Out] (ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(1 + Sin[c + d*x])^2)/(4*a^2*d)

Maple [A] time = 0.082, size = 72, normalized size = 1.2

$$-\frac{\ln(\sin(dx+c)-1)}{8da^2} + \frac{1}{4da^2(1+\sin(dx+c))^2} - \frac{1}{4da^2(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{8da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^2,x)

[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)+1/4/d/a^2/(1+sin(d*x+c))^2-1/4/d/a^2/(1+sin(d*x+c))+1/8*ln(1+sin(d*x+c))/a^2/d

Maxima [A] time = 1.02483, size = 95, normalized size = 1.58

$$-\frac{\frac{2\sin(dx+c)}{a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2} - \frac{\log(\sin(dx+c)+1)}{a^2} + \frac{\log(\sin(dx+c)-1)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/8*(2*sin(d*x + c)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d

Fricas [A] time = 1.45633, size = 274, normalized size = 4.57

$$\frac{(\cos(dx+c)^2 - 2 \sin(dx+c) - 2) \log(\sin(dx+c) + 1) - (\cos(dx+c)^2 - 2 \sin(dx+c) - 2) \log(-\sin(dx+c) + 1) + 2 \sin(dx+c)}{8(a^2 d \cos(dx+c)^2 - 2 a^2 d \sin(dx+c) - 2 a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

a^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x)

[Out] Integral(tan(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2

Giac [A] time = 1.84808, size = 122, normalized size = 2.03

$$\frac{\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) + 2\right|\right)}{a^2} - \frac{\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) - 2\right|\right)}{a^2} - \frac{\frac{1}{\sin(dx+c)} + \sin(dx+c) + 6}{a^2\left(\frac{1}{\sin(dx+c)} + \sin(dx+c) + 2\right)}$$

$16d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/16*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2))/a^2 - log(abs(1/sin(d*x + c) + sin(d*x + c) - 2))/a^2 - (1/sin(d*x + c) + sin(d*x + c) + 6)/(a^2*(1/sin(d*x + c) + sin(d*x + c) + 2)))/d

$$3.65 \quad \int \frac{\cot(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=52

$$\frac{1}{d(a^2 \sin(c+dx) + a^2)} + \frac{\log(\sin(c+dx))}{a^2 d} - \frac{\log(\sin(c+dx) + 1)}{a^2 d}$$

[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2 *Sin[c + d*x]))

Rubi [A] time = 0.0504909, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 44}

$$\frac{1}{d(a^2 \sin(c+dx) + a^2)} + \frac{\log(\sin(c+dx))}{a^2 d} - \frac{\log(\sin(c+dx) + 1)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]

[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2 *Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2 x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\log(\sin(c+dx))}{a^2 d} - \frac{\log(1 + \sin(c+dx))}{a^2 d} + \frac{1}{d(a^2 + a^2 \sin(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.0548377, size = 36, normalized size = 0.69

$$\frac{1}{\sin(c+dx)+1} + \frac{\log(\sin(c+dx)) - \log(\sin(c+dx) + 1)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]

[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*d)

Maple [A] time = 0.034, size = 50, normalized size = 1.

$$\frac{1}{da^2(1 + \sin(dx + c))} - \frac{\ln(1 + \sin(dx + c))}{da^2} + \frac{\ln(\sin(dx + c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^2/d+ln(sin(d*x+c))/a^2/d

Maxima [A] time = 1.11919, size = 62, normalized size = 1.19

$$\frac{\frac{1}{a^2 \sin(dx+c)+a^2} - \frac{\log(\sin(dx+c)+1)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c)) /a^2)/d

Fricas [A] time = 1.50947, size = 162, normalized size = 3.12

$$\frac{(\sin(dx + c) + 1) \log\left(\frac{1}{2} \sin(dx + c)\right) - (\sin(dx + c) + 1) \log(\sin(dx + c) + 1) + 1}{a^2 d \sin(dx + c) + a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

$$a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2

Giac [A] time = 1.7951, size = 61, normalized size = 1.17

$$\frac{a \left(\frac{\log\left(\left| -\frac{a}{a \sin(dx+c)+a} + 1 \right| \right)}{a^3} + \frac{1}{(a \sin(dx+c)+a)a^2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2))/d

$$3.66 \quad \int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=65

$$-\frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \csc(c+dx)}{a^2d} + \frac{2 \log(\sin(c+dx))}{a^2d} - \frac{2 \log(\sin(c+dx)+1)}{a^2d}$$

[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)

Rubi [A] time = 0.0593806, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 77}

$$-\frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \csc(c+dx)}{a^2d} + \frac{2 \log(\sin(c+dx))}{a^2d} - \frac{2 \log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]

[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3} - \frac{2}{ax^2} + \frac{2}{a^2x} - \frac{2}{a^2(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{2 \csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \log(\sin(c+dx))}{a^2d} - \frac{2 \log(1 + \sin(c+dx))}{a^2d} \end{aligned}$$

Mathematica [A] time = 0.0701507, size = 49, normalized size = 0.75

$$\frac{-\csc^2(c+dx) + 4 \csc(c+dx) + 4 \log(\sin(c+dx)) - 4 \log(\sin(c+dx)+1)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]

[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(2*a^2*d)

Maple [A] time = 0.106, size = 66, normalized size = 1.

$$-2 \frac{\ln(1 + \sin(dx + c))}{a^2 d} - \frac{1}{2 a^2 d (\sin(dx + c))^2} + 2 \frac{1}{a^2 d \sin(dx + c)} + 2 \frac{\ln(\sin(dx + c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x)

[Out] -2*ln(1+sin(d*x+c))/a^2/d-1/2/d/a^2/sin(d*x+c)^2+2/d/a^2/sin(d*x+c)+2*ln(sin(d*x+c))/a^2/d

Maxima [A] time = 1.38376, size = 74, normalized size = 1.14

$$\frac{\frac{4 \log(\sin(dx+c)+1)}{a^2} - \frac{4 \log(\sin(dx+c))}{a^2} - \frac{4 \sin(dx+c)-1}{a^2 \sin(dx+c)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*(4*log(sin(d*x + c) + 1)/a^2 - 4*log(sin(d*x + c))/a^2 - (4*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^2))/d

Fricas [A] time = 1.54804, size = 204, normalized size = 3.14

$$\frac{4 \left(\cos(dx + c)^2 - 1 \right) \log\left(\frac{1}{2} \sin(dx + c)\right) - 4 \left(\cos(dx + c)^2 - 1 \right) \log(\sin(dx + c) + 1) - 4 \sin(dx + c) + 1}{2 \left(a^2 d \cos(dx + c)^2 - a^2 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*(4*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*(cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) - 4*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

$$a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2

Giac [A] time = 2.10221, size = 155, normalized size = 2.38

$$\frac{\frac{32 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^2} - \frac{16 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^2} + \frac{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^4} + \frac{24 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out]
$$-1/8*(32*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^2 - 16*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))))/a^2 + (a^2*\tan(1/2*d*x + 1/2*c)^2 - 8*a^2*\tan(1/2*d*x + 1/2*c))/a^4 + (24*\tan(1/2*d*x + 1/2*c)^2 - 8*\tan(1/2*d*x + 1/2*c) + 1)/(a^2*\tan(1/2*d*x + 1/2*c)^2)/d$$

$$3.67 \quad \int \frac{\cot^5(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^2(c+dx)}{2a^2d}$$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) - \text{Csc}[c + d*x]^4/(4*a^2*d)$

Rubi [A] time = 0.0511339, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 43}

$$-\frac{\csc^4(c+dx)}{4a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^5/(a + a*\text{Sin}[c + d*x])^2, x]$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) - \text{Csc}[c + d*x]^4/(4*a^2*d)$

Rule 2707

$\text{Int}[(a + (b \sin(e + f x))^m) \tan(e + f x)^p, x] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p (a + x)^{m - (p + 1)/2}) / (a - x)^{(p + 1)/2}, x], x, b \sin[e + f x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \text{ \&\& EqQ}[a^2 - b^2, 0] \text{ \&\& IntegerQ}[(p + 1)/2]$

Rule 43

$\text{Int}[(a + (b x)^m) (c + (d x)^n), x] := \text{Int}[\text{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \text{ \&\& NeQ}[b c - a d, 0] \text{ \&\& IGtQ}[m, 0] \text{ \&\& (!IntegerQ}[n] || (\text{EqQ}[c, 0] \text{ \&\& LeQ}[7 m + 4 n + 4, 0]) || \text{LtQ}[9 m + 5 (n + 1), 0] || \text{GtQ}[m + n + 2, 0])]$

Rubi steps

$$\begin{aligned} \int \frac{\cot^5(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{4a^2d} \end{aligned}$$

Mathematica [A] time = 0.074456, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(8 \sin(c+dx) + 3 \cos(2(c+dx)) - 6)}{12a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]

[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)

Maple [A] time = 0.092, size = 39, normalized size = 0.7

$$\frac{1}{da^2} \left(-\frac{1}{4 (\sin(dx + c))^4} + \frac{2}{3 (\sin(dx + c))^3} - \frac{1}{2 (\sin(dx + c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(-1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)

Maxima [A] time = 0.955802, size = 49, normalized size = 0.89

$$\frac{6 \sin(dx + c)^2 - 8 \sin(dx + c) + 3}{12 a^2 d \sin(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)

Fricas [A] time = 1.4656, size = 138, normalized size = 2.51

$$\frac{6 \cos(dx + c)^2 + 8 \sin(dx + c) - 9}{12 (a^2 d \cos(dx + c)^4 - 2 a^2 d \cos(dx + c)^2 + a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1} dx$$

$$a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**2,x)

[Out] `Integral(cot(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2`

Giac [A] time = 1.9624, size = 49, normalized size = 0.89

$$\frac{6 \sin(dx + c)^2 - 8 \sin(dx + c) + 3}{12 a^2 d \sin(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")`

[Out] `-1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)`

$$3.68 \quad \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^2d} + \frac{2 \csc^5(c+dx)}{5a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{2a^2d}$$

[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)

Rubi [A] time = 0.0571898, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 75}

$$-\frac{\csc^6(c+dx)}{6a^2d} + \frac{2 \csc^5(c+dx)}{5a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]

[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 75

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILTQ[n + p + 2, 0] && GtQ[n + 2*p, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)}{x^7} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^7} - \frac{2a^3}{x^6} + \frac{2a}{x^4} - \frac{1}{x^3}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{2 \csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d} \end{aligned}$$

Mathematica [A] time = 0.07344, size = 73, normalized size = 1.

$$-\frac{\csc^6(c+dx)}{6a^2d} + \frac{2 \csc^5(c+dx)}{5a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]

[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)

Maple [A] time = 0.108, size = 49, normalized size = 0.7

$$\frac{1}{da^2} \left(\frac{2}{5 (\sin(dx + c))^5} - \frac{1}{6 (\sin(dx + c))^6} - \frac{2}{3 (\sin(dx + c))^3} + \frac{1}{2 (\sin(dx + c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(2/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)

Maxima [A] time = 1.07194, size = 62, normalized size = 0.85

$$\frac{15 \sin(dx + c)^4 - 20 \sin(dx + c)^3 + 12 \sin(dx + c) - 5}{30 a^2 d \sin(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*sin(d*x + c)^6)

Fricas [A] time = 1.40071, size = 234, normalized size = 3.21

$$\frac{15 \cos(dx + c)^4 - 30 \cos(dx + c)^2 + 4(5 \cos(dx + c)^2 - 2) \sin(dx + c) + 10}{30 (a^2 d \cos(dx + c)^6 - 3 a^2 d \cos(dx + c)^4 + 3 a^2 d \cos(dx + c)^2 - a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 + 4*(5*cos(d*x + c)^2 - 2)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 2.36406, size = 62, normalized size = 0.85

$$\frac{15 \sin(dx + c)^4 - 20 \sin(dx + c)^3 + 12 \sin(dx + c) - 5}{30 a^2 d \sin(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*sin(d*x + c)^6)

$$3.69 \quad \int \frac{\cot^9(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=127

$$-\frac{\csc^8(c+dx)}{8a^2d} + \frac{2 \csc^7(c+dx)}{7a^2d} + \frac{\csc^6(c+dx)}{6a^2d} - \frac{4 \csc^5(c+dx)}{5a^2d} + \frac{\csc^4(c+dx)}{4a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^2(c+dx)}{2a^2d}$$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) + \text{Csc}[c + d*x]^4/(4*a^2*d) - (4*\text{Csc}[c + d*x]^5)/(5*a^2*d) + \text{Csc}[c + d*x]^6/(6*a^2*d) + (2*\text{Csc}[c + d*x]^7)/(7*a^2*d) - \text{Csc}[c + d*x]^8/(8*a^2*d)$

Rubi [A] time = 0.0738176, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^8(c+dx)}{8a^2d} + \frac{2 \csc^7(c+dx)}{7a^2d} + \frac{\csc^6(c+dx)}{6a^2d} - \frac{4 \csc^5(c+dx)}{5a^2d} + \frac{\csc^4(c+dx)}{4a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^9/(a + a*\text{Sin}[c + d*x])^2, x]$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) + \text{Csc}[c + d*x]^4/(4*a^2*d) - (4*\text{Csc}[c + d*x]^5)/(5*a^2*d) + \text{Csc}[c + d*x]^6/(6*a^2*d) + (2*\text{Csc}[c + d*x]^7)/(7*a^2*d) - \text{Csc}[c + d*x]^8/(8*a^2*d)$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m * \tan(e + f*x)^p, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^9(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)^2}{x^9} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^9} - \frac{2a^5}{x^8} - \frac{a^4}{x^7} + \frac{4a^3}{x^6} - \frac{a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{4a^2d} - \frac{4 \csc^5(c+dx)}{5a^2d} + \frac{\csc^6(c+dx)}{6a^2d} + \frac{2 \csc^7(c+dx)}{7a^2d} \end{aligned}$$

Mathematica [A] time = 0.142755, size = 78, normalized size = 0.61

$$\frac{\csc^2(c+dx) \left(-105 \csc^6(c+dx) + 240 \csc^5(c+dx) + 140 \csc^4(c+dx) - 672 \csc^3(c+dx) + 210 \csc^2(c+dx) + 560 \csc(c+dx)\right)}{840a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]

[Out] (Csc[c + d*x]^2*(-420 + 560*Csc[c + d*x] + 210*Csc[c + d*x]^2 - 672*Csc[c + d*x]^3 + 140*Csc[c + d*x]^4 + 240*Csc[c + d*x]^5 - 105*Csc[c + d*x]^6))/(840*a^2*d)

Maple [A] time = 0.125, size = 79, normalized size = 0.6

$$\frac{1}{4a^2} \left(\frac{2}{7 (\sin(dx+c))^7} - \frac{1}{8 (\sin(dx+c))^8} - \frac{4}{5 (\sin(dx+c))^5} + \frac{1}{4 (\sin(dx+c))^4} + \frac{1}{6 (\sin(dx+c))^6} + \frac{2}{3 (\sin(dx+c))^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(2/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8-4/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4+1/6/sin(d*x+c)^6+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)

Maxima [A] time = 1.76225, size = 103, normalized size = 0.81

$$\frac{420 \sin(dx+c)^6 - 560 \sin(dx+c)^5 - 210 \sin(dx+c)^4 + 672 \sin(dx+c)^3 - 140 \sin(dx+c)^2 - 240 \sin(dx+c) + 105}{840 a^2 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*x + c)^8)

Fricas [A] time = 1.44614, size = 331, normalized size = 2.61

$$\frac{420 \cos(dx+c)^6 - 1050 \cos(dx+c)^4 + 700 \cos(dx+c)^2 + 16(35 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + 8) \sin(dx+c) - 175}{840 (a^2 d \cos(dx+c)^8 - 4 a^2 d \cos(dx+c)^6 + 6 a^2 d \cos(dx+c)^4 - 4 a^2 d \cos(dx+c)^2 + a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/840*(420*cos(d*x + c)^6 - 1050*cos(d*x + c)^4 + 700*cos(d*x + c)^2 + 16*(35*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 8)*sin(d*x + c) - 175)/(a^2*d*cos(d*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^2 + a^2*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 2.22548, size = 103, normalized size = 0.81

$$\frac{420 \sin(dx + c)^6 - 560 \sin(dx + c)^5 - 210 \sin(dx + c)^4 + 672 \sin(dx + c)^3 - 140 \sin(dx + c)^2 - 240 \sin(dx + c) + 105}{840 a^2 d \sin(dx + c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*x + c)^8)

3.70 $\int \frac{\cot^{11}(c+dx)}{(a+a \sin(c+dx))^2} dx$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^2d} + \frac{2 \csc^9(c+dx)}{9a^2d} + \frac{\csc^8(c+dx)}{4a^2d} - \frac{6 \csc^7(c+dx)}{7a^2d} + \frac{6 \csc^5(c+dx)}{5a^2d} - \frac{\csc^4(c+dx)}{2a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{a^2d}$$

[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*a^2*d)

Rubi [A] time = 0.0811955, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^2d} + \frac{2 \csc^9(c+dx)}{9a^2d} + \frac{\csc^8(c+dx)}{4a^2d} - \frac{6 \csc^7(c+dx)}{7a^2d} + \frac{6 \csc^5(c+dx)}{5a^2d} - \frac{\csc^4(c+dx)}{2a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]

[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*a^2*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^{11}(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^3}{x^{11}} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^{11}} - \frac{2a^7}{x^{10}} - \frac{2a^6}{x^9} + \frac{6a^5}{x^8} - \frac{6a^3}{x^6} + \frac{2a^2}{x^5} + \frac{2a}{x^4} - \frac{1}{x^3}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2 \csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{2a^2d} + \frac{6 \csc^5(c+dx)}{5a^2d} - \frac{6 \csc^7(c+dx)}{7a^2d} + \frac{\csc^8(c+dx)}{4a^2d} \end{aligned}$$

Mathematica [A] time = 0.205023, size = 88, normalized size = 0.61

$$\frac{\csc^2(c + dx) \left(-126 \csc^8(c + dx) + 280 \csc^7(c + dx) + 315 \csc^6(c + dx) - 1080 \csc^5(c + dx) + 1512 \csc^3(c + dx) - 630 \right)}{1260 a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]

[Out] (Csc[c + d*x]^2*(630 - 840*Csc[c + d*x] - 630*Csc[c + d*x]^2 + 1512*Csc[c + d*x]^3 - 1080*Csc[c + d*x]^5 + 315*Csc[c + d*x]^6 + 280*Csc[c + d*x]^7 - 126*Csc[c + d*x]^8))/(1260*a^2*d)

Maple [A] time = 0.139, size = 89, normalized size = 0.6

$$\frac{1}{d a^2} \left(-\frac{1}{10 (\sin(dx + c))^{10}} - \frac{6}{7 (\sin(dx + c))^7} + \frac{1}{4 (\sin(dx + c))^8} + \frac{6}{5 (\sin(dx + c))^5} - \frac{1}{2 (\sin(dx + c))^4} + \frac{2}{9 (\sin(dx + c))^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(-1/10/sin(d*x+c)^10-6/7/sin(d*x+c)^7+1/4/sin(d*x+c)^8+6/5/sin(d*x+c)^5-1/2/sin(d*x+c)^4+2/9/sin(d*x+c)^9-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)

Maxima [A] time = 1.83669, size = 116, normalized size = 0.8

$$\frac{630 \sin(dx + c)^8 - 840 \sin(dx + c)^7 - 630 \sin(dx + c)^6 + 1512 \sin(dx + c)^5 - 1080 \sin(dx + c)^3 + 315 \sin(dx + c)}{1260 a^2 d \sin(dx + c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x + c) - 126)/(a^2*d*sin(d*x + c)^10)

Fricas [A] time = 1.67582, size = 431, normalized size = 2.97

$$\frac{630 \cos(dx + c)^8 - 1890 \cos(dx + c)^6 + 1890 \cos(dx + c)^4 - 945 \cos(dx + c)^2 + 8 (105 \cos(dx + c)^6 - 126 \cos(dx + c)^4 - 945 \cos(dx + c)^2 + 189)}{1260 (a^2 d \cos(dx + c)^{10} - 5 a^2 d \cos(dx + c)^8 + 10 a^2 d \cos(dx + c)^6 - 10 a^2 d \cos(dx + c)^4 + 5 a^2 d \cos(dx + c)^2 - 189)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/1260*(630*cos(d*x + c)^8 - 1890*cos(d*x + c)^6 + 1890*cos(d*x + c)^4 - 945*cos(d*x + c)^2 + 8*(105*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x + c) + 189)/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - 189)

$x + c)^2 - a^2d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.93118, size = 116, normalized size = 0.8

$$\frac{630 \sin(dx + c)^8 - 840 \sin(dx + c)^7 - 630 \sin(dx + c)^6 + 1512 \sin(dx + c)^5 - 1080 \sin(dx + c)^3 + 315 \sin(dx + c)^2 + 126}{1260 a^2 d \sin(dx + c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x + c) - 126)/(a^2*d*sin(d*x + c)^10)

$$3.71 \quad \int \frac{\cot^{13}(c+dx)}{(a+a \sin(c+dx))^2} dx$$

Optimal. Leaf size=199

$$-\frac{\csc^{12}(c+dx)}{12a^2d} + \frac{2 \csc^{11}(c+dx)}{11a^2d} + \frac{3 \csc^{10}(c+dx)}{10a^2d} - \frac{8 \csc^9(c+dx)}{9a^2d} - \frac{\csc^8(c+dx)}{4a^2d} + \frac{12 \csc^7(c+dx)}{7a^2d} - \frac{\csc^6(c+dx)}{3a^2d}$$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^2*d) - (8*\text{Csc}[c + d*x]^5)/(5*a^2*d) - \text{Csc}[c + d*x]^6/(3*a^2*d) + (12*\text{Csc}[c + d*x]^7)/(7*a^2*d) - \text{Csc}[c + d*x]^8/(4*a^2*d) - (8*\text{Csc}[c + d*x]^9)/(9*a^2*d) + (3*\text{Csc}[c + d*x]^10)/(10*a^2*d) + (2*\text{Csc}[c + d*x]^11)/(11*a^2*d) - \text{Csc}[c + d*x]^12/(12*a^2*d)$

Rubi [A] time = 0.102394, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^2d} + \frac{2 \csc^{11}(c+dx)}{11a^2d} + \frac{3 \csc^{10}(c+dx)}{10a^2d} - \frac{8 \csc^9(c+dx)}{9a^2d} - \frac{\csc^8(c+dx)}{4a^2d} + \frac{12 \csc^7(c+dx)}{7a^2d} - \frac{\csc^6(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^13/(a + a*\text{Sin}[c + d*x])^2, x]$

[Out] $-\text{Csc}[c + d*x]^2/(2*a^2*d) + (2*\text{Csc}[c + d*x]^3)/(3*a^2*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^2*d) - (8*\text{Csc}[c + d*x]^5)/(5*a^2*d) - \text{Csc}[c + d*x]^6/(3*a^2*d) + (12*\text{Csc}[c + d*x]^7)/(7*a^2*d) - \text{Csc}[c + d*x]^8/(4*a^2*d) - (8*\text{Csc}[c + d*x]^9)/(9*a^2*d) + (3*\text{Csc}[c + d*x]^10)/(10*a^2*d) + (2*\text{Csc}[c + d*x]^11)/(11*a^2*d) - \text{Csc}[c + d*x]^12/(12*a^2*d)$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m*\tan(e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ $\text{FreeQ}\{a, b, e, f, m\}, x$ && $\text{Eq}[a^2 - b^2, 0]$ && $\text{IntegerQ}[(p + 1)/2]$

Rule 88

$\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, p\}, x$ && $\text{IntegersQ}[m, n]$ && $(\text{IntegerQ}[p] \mid \mid (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rubi steps

$$\begin{aligned} \int \frac{\cot^{13}(c+dx)}{(a+a \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^4}{x^{13}} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{13}} - \frac{2a^9}{x^{12}} - \frac{3a^8}{x^{11}} + \frac{8a^7}{x^{10}} + \frac{2a^6}{x^9} - \frac{12a^5}{x^8} + \frac{2a^4}{x^7} + \frac{8a^3}{x^6} - \frac{3a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2 \csc^3(c+dx)}{3a^2d} + \frac{3 \csc^4(c+dx)}{4a^2d} - \frac{8 \csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{3a^2d} + \frac{12 \csc^7(c+dx)}{7a^2d} - \frac{\csc^8(c+dx)}{4a^2d} + \frac{8 \csc^9(c+dx)}{9a^2d} - \frac{3 \csc^{10}(c+dx)}{10a^2d} + \frac{2 \csc^{11}(c+dx)}{11a^2d} - \frac{\csc^{12}(c+dx)}{12a^2d} \end{aligned}$$

Mathematica [A] time = 0.329218, size = 118, normalized size = 0.59

$$\frac{\csc^2(c + dx) (1155 \csc^{10}(c + dx) - 2520 \csc^9(c + dx) - 4158 \csc^8(c + dx) + 12320 \csc^7(c + dx) + 3465 \csc^6(c + dx) - 23760 \csc^5(c + dx) + 1155 \csc^4(c + dx))}{13860 a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]

[Out] -(Csc[c + d*x]^2*(6930 - 9240*Csc[c + d*x] - 10395*Csc[c + d*x]^2 + 22176*Csc[c + d*x]^3 + 4620*Csc[c + d*x]^4 - 23760*Csc[c + d*x]^5 + 3465*Csc[c + d*x]^6 + 12320*Csc[c + d*x]^7 - 4158*Csc[c + d*x]^8 - 2520*Csc[c + d*x]^9 + 1155*Csc[c + d*x]^10))/(13860*a^2*d)

Maple [A] time = 0.165, size = 119, normalized size = 0.6

$$\frac{1}{da^2} \left(\frac{3}{10 (\sin(dx + c))^{10}} + \frac{12}{7 (\sin(dx + c))^7} + \frac{2}{11 (\sin(dx + c))^{11}} - \frac{1}{4 (\sin(dx + c))^8} - \frac{8}{5 (\sin(dx + c))^5} + \frac{3}{4 (\sin(dx + c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(3/10/sin(d*x+c)^10+12/7/sin(d*x+c)^7+2/11/sin(d*x+c)^11-1/4/sin(d*x+c)^8-8/5/sin(d*x+c)^5+3/4/sin(d*x+c)^4-8/9/sin(d*x+c)^9-1/12/sin(d*x+c)^12-1/3/sin(d*x+c)^6+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)

Maxima [A] time = 1.83541, size = 157, normalized size = 0.79

$$\frac{6930 \sin(dx + c)^{10} - 9240 \sin(dx + c)^9 - 10395 \sin(dx + c)^8 + 22176 \sin(dx + c)^7 + 4620 \sin(dx + c)^6 - 23760 \sin(dx + c)^5 + 1155 \sin(dx + c)^4}{13860 a^2 d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x + c) + 1155)/(a^2*d*sin(d*x + c)^12)

Fricas [A] time = 1.65369, size = 541, normalized size = 2.72

$$\frac{6930 \cos(dx + c)^{10} - 24255 \cos(dx + c)^8 + 32340 \cos(dx + c)^6 - 24255 \cos(dx + c)^4 + 9702 \cos(dx + c)^2 + 8(1155 \cos(dx + c) - 1)}{13860 (a^2 d \cos(dx + c)^{12} - 6 a^2 d \cos(dx + c)^{10} + 15 a^2 d \cos(dx + c)^8 - 20 a^2 d \cos(dx + c)^6 + 15 a^2 d \cos(dx + c)^4 - 6 a^2 d \cos(dx + c)^2 + a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="fricas")


```
[Out] 1/13860*(6930*cos(d*x + c)^10 - 24255*cos(d*x + c)^8 + 32340*cos(d*x + c)^6
- 24255*cos(d*x + c)^4 + 9702*cos(d*x + c)^2 + 8*(1155*cos(d*x + c)^8 - 18
48*cos(d*x + c)^6 + 1584*cos(d*x + c)^4 - 704*cos(d*x + c)^2 + 128)*sin(d*x
+ c) - 1617)/(a^2*d*cos(d*x + c)^12 - 6*a^2*d*cos(d*x + c)^10 + 15*a^2*d*c
os(d*x + c)^8 - 20*a^2*d*cos(d*x + c)^6 + 15*a^2*d*cos(d*x + c)^4 - 6*a^2*d
*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1608, size = 157, normalized size = 0.79

$$\frac{6930 \sin(dx + c)^{10} - 9240 \sin(dx + c)^9 - 10395 \sin(dx + c)^8 + 22176 \sin(dx + c)^7 + 4620 \sin(dx + c)^6 - 23760 \sin(dx + c)^5 + 3465 \sin(dx + c)^4 + 12320 \sin(dx + c)^3 - 4158 \sin(dx + c)^2 - 2520 \sin(dx + c) + 1155}{13860 a^2 d \sin(dx + c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
+ 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
+ c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

$$3.72 \quad \int \frac{\tan^5(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=171

$$\frac{a^2}{40d(a \sin(c+dx)+a)^5} - \frac{1}{32d(a^3 - a^3 \sin(c+dx))} - \frac{5}{128d(a^3 \sin(c+dx)+a^3)} + \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} - \frac{7a}{64d(a \sin(c+dx)+a)^4}$$

[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))

Rubi [A] time = 0.123351, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{a^2}{40d(a \sin(c+dx)+a)^5} - \frac{1}{32d(a^3 - a^3 \sin(c+dx))} - \frac{5}{128d(a^3 \sin(c+dx)+a^3)} + \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} - \frac{7a}{64d(a \sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]

[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)^(p_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3} dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^6} dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a(a-x)^3} - \frac{1}{32a^2(a-x)^2} - \frac{a^2}{8(a+x)^6} + \frac{7a}{16(a+x)^5} - \frac{1}{2(a+x)^4} + \frac{5}{32a(a+x)^3} + \frac{5}{128a^2(a+x)^2} + \frac{1}{128a^3}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))^4} + \frac{1}{6d(a+a\sin(c+dx))^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))^4} + \frac{1}{6d(a+a\sin(c+dx))^3}$$

Mathematica [A] time = 0.925661, size = 102, normalized size = 0.6

$$\frac{15 \tanh^{-1}(\sin(c+dx)) - \frac{15 \sin^6(c+dx) + 45 \sin^5(c+dx) - 620 \sin^4(c+dx) - 540 \sin^3(c+dx) + 157 \sin^2(c+dx) + 351 \sin(c+dx) + 112}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^5}}{1920a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]

[Out] (15*ArcTanh[Sin[c + d*x]] - (112 + 351*Sin[c + d*x] + 157*Sin[c + d*x]^2 - 540*Sin[c + d*x]^3 - 620*Sin[c + d*x]^4 + 45*Sin[c + d*x]^5 + 15*Sin[c + d*x]^6)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^5))/(1920*a^3*d)

Maple [A] time = 0.106, size = 162, normalized size = 1.

$$\frac{1}{128da^3(\sin(dx+c)-1)^2} + \frac{1}{32da^3(\sin(dx+c)-1)} - \frac{\ln(\sin(dx+c)-1)}{256da^3} + \frac{1}{40da^3(1+\sin(dx+c))^5} - \frac{1}{64da^3(1+\sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x)

[Out] 1/128/d/a^3/(sin(d*x+c)-1)^2+1/32/d/a^3/(sin(d*x+c)-1)-1/256/d/a^3*ln(sin(d*x+c)-1)+1/40/d/a^3/(1+sin(d*x+c))^5-7/64/d/a^3/(1+sin(d*x+c))^4+1/6/d/a^3/(1+sin(d*x+c))^3-5/64/d/a^3/(1+sin(d*x+c))^2-5/128/d/a^3/(1+sin(d*x+c))+1/256*ln(1+sin(d*x+c))/a^3/d

Maxima [A] time = 1.96124, size = 254, normalized size = 1.49

$$\frac{2(15 \sin(dx+c)^6 + 45 \sin(dx+c)^5 - 620 \sin(dx+c)^4 - 540 \sin(dx+c)^3 + 157 \sin(dx+c)^2 + 351 \sin(dx+c) + 112)}{a^3 \sin(dx+c)^7 + 3a^3 \sin(dx+c)^6 + a^3 \sin(dx+c)^5 - 5a^3 \sin(dx+c)^4 - 5a^3 \sin(dx+c)^3 + a^3 \sin(dx+c)^2 + 3a^3 \sin(dx+c) + a^3} - \frac{15 \log(\sin(dx+c)+1)}{a^3} + \frac{15 \log(\sin(dx+c)-1)}{a^3}$$

$$3840d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/3840*(2*(15*sin(d*x + c)^6 + 45*sin(d*x + c)^5 - 620*sin(d*x + c)^4 - 540*sin(d*x + c)^3 + 157*sin(d*x + c)^2 + 351*sin(d*x + c) + 112)/(a^3*sin(d*x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 15*log(sin(d*x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)

$$x + c)^7 + 3a^3 \sin(dx + c)^6 + a^3 \sin(dx + c)^5 - 5a^3 \sin(dx + c)^4 - 5a^3 \sin(dx + c)^3 + a^3 \sin(dx + c)^2 + 3a^3 \sin(dx + c) + a^3) - 15 \log(\sin(dx + c) + 1)/a^3 + 15 \log(\sin(dx + c) - 1)/a^3)/d$$

Fricas [A] time = 1.67077, size = 657, normalized size = 3.84

$$30 \cos(dx + c)^6 + 1150 \cos(dx + c)^4 - 2076 \cos(dx + c)^2 - 15(3 \cos(dx + c)^6 - 4 \cos(dx + c)^4 + (\cos(dx + c)^6 - 4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^5/(a+a*sin(dx+c))^3,x, algorithm="fricas")

[Out]
$$-1/3840*(30*\cos(dx + c)^6 + 1150*\cos(dx + c)^4 - 2076*\cos(dx + c)^2 - 15*(3*\cos(dx + c)^6 - 4*\cos(dx + c)^4 + (\cos(dx + c)^6 - 4*\cos(dx + c)^4)*\sin(dx + c))*\log(\sin(dx + c) + 1) + 15*(3*\cos(dx + c)^6 - 4*\cos(dx + c)^4 + (\cos(dx + c)^6 - 4*\cos(dx + c)^4)*\sin(dx + c))*\log(-\sin(dx + c) + 1) - 18*(5*\cos(dx + c)^4 + 50*\cos(dx + c)^2 - 16)*\sin(dx + c) + 672)/(3*a^3*d*\cos(dx + c)^6 - 4*a^3*d*\cos(dx + c)^4 + (a^3*d*\cos(dx + c)^6 - 4*a^3*d*\cos(dx + c)^4)*\sin(dx + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**5/(a+a*sin(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 4.82376, size = 184, normalized size = 1.08

$$\frac{60 \log(|\sin(dx+c)+1|)}{a^3} - \frac{60 \log(|\sin(dx+c)-1|)}{a^3} + \frac{30(3 \sin(dx+c)^2 + 10 \sin(dx+c) - 9)}{a^3(\sin(dx+c)-1)^2} - \frac{137 \sin(dx+c)^5 + 1285 \sin(dx+c)^4 + 4970 \sin(dx+c)^3 + 6010 \sin(dx+c)^2 + 3245 \sin(dx+c) + 673}{a^3(\sin(dx+c)+1)^5} / 15360 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^5/(a+a*sin(dx+c))^3,x, algorithm="giac")

[Out]
$$1/15360*(60*\log(\text{abs}(\sin(dx + c) + 1))/a^3 - 60*\log(\text{abs}(\sin(dx + c) - 1)))/a^3 + 30*(3*\sin(dx + c)^2 + 10*\sin(dx + c) - 9)/(a^3*(\sin(dx + c) - 1)^2) - (137*\sin(dx + c)^5 + 1285*\sin(dx + c)^4 + 4970*\sin(dx + c)^3 + 6010*\sin(dx + c)^2 + 3245*\sin(dx + c) + 673)/(a^3*(\sin(dx + c) + 1)^5))/d$$

$$3.73 \quad \int \frac{\tan^3(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=126

$$\frac{1}{32d(a^3 - a^3 \sin(c + dx))} + \frac{1}{16d(a^3 \sin(c + dx) + a^3)} - \frac{\tanh^{-1}(\sin(c + dx))}{32a^3d} + \frac{a}{16d(a \sin(c + dx) + a)^4} - \frac{1}{6d(a \sin(c + dx) + a)}$$

[Out] -ArcTanh[Sin[c + d*x]]/(32*a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(6*d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(a^3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))

Rubi [A] time = 0.090512, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{1}{32d(a^3 - a^3 \sin(c + dx))} + \frac{1}{16d(a^3 \sin(c + dx) + a^3)} - \frac{\tanh^{-1}(\sin(c + dx))}{32a^3d} + \frac{a}{16d(a \sin(c + dx) + a)^4} - \frac{1}{6d(a \sin(c + dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]

[Out] -ArcTanh[Sin[c + d*x]]/(32*a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(6*d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(a^3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^5} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{32a^2(a-x)^2} - \frac{a}{4(a+x)^5} + \frac{1}{2(a+x)^4} - \frac{3}{16a(a+x)^3} - \frac{1}{16a^2(a+x)^2} - \frac{1}{32a^2(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{1}{32d(a^3-a^3)} \\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d} + \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 0.363215, size = 82, normalized size = 0.65

$$-\frac{3}{1-\sin(c+dx)} - \frac{6}{\sin(c+dx)+1} - \frac{9}{(\sin(c+dx)+1)^2} + \frac{16}{(\sin(c+dx)+1)^3} - \frac{6}{(\sin(c+dx)+1)^4} + 3 \tanh^{-1}(\sin(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]

[Out] -(3*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 6/(1 + Sin[c + d*x])^4 + 16/(1 + Sin[c + d*x])^3 - 9/(1 + Sin[c + d*x])^2 - 6/(1 + Sin[c + d*x]))/(96*a^3*d)

Maple [A] time = 0.092, size = 126, normalized size = 1.

$$-\frac{1}{32da^3(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{64da^3} + \frac{1}{16da^3(1+\sin(dx+c))^4} - \frac{1}{6da^3(1+\sin(dx+c))^3} + \frac{3}{32da^3(1+\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x)

[Out] -1/32/d/a^3/(sin(d*x+c)-1)+1/64/d/a^3*ln(sin(d*x+c)-1)+1/16/d/a^3/(1+sin(d*x+c))^4-1/6/d/a^3/(1+sin(d*x+c))^3+3/32/d/a^3/(1+sin(d*x+c))^2+1/16/d/a^3/(1+sin(d*x+c))-1/64*ln(1+sin(d*x+c))/a^3/d

Maxima [A] time = 1.93841, size = 197, normalized size = 1.56

$$\frac{2(3\sin(dx+c)^4+9\sin(dx+c)^3-25\sin(dx+c)^2-27\sin(dx+c)-8)}{a^3\sin(dx+c)^5+3a^3\sin(dx+c)^4+2a^3\sin(dx+c)^3-2a^3\sin(dx+c)^2-3a^3\sin(dx+c)-a^3} - \frac{3\log(\sin(dx+c)+1)}{a^3} + \frac{3\log(\sin(dx+c)-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/192*(2*(3*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 27*sin(d*x + c) - 8)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*x + c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 3*log(sin(dx+c)+1)/a^3 + 3*log(sin(dx+c)-1)/a^3

$c)^3 - 2a^3 \sin(dx + c)^2 - 3a^3 \sin(dx + c) - a^3) - 3 \log(\sin(dx + c) + 1) / a^3 + 3 \log(\sin(dx + c) - 1) / a^3) / d$

Fricas [A] time = 1.58082, size = 586, normalized size = 4.65

$$\frac{6 \cos(dx + c)^4 + 38 \cos(dx + c)^2 - 3(3 \cos(dx + c)^4 - 4 \cos(dx + c)^2 + (\cos(dx + c)^4 - 4 \cos(dx + c)^2) \sin(dx + c))}{192(3 a^3 d \cos(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^3/(a+a*sin(dx+c))^3,x, algorithm="fricas")

[Out] 1/192*(6*cos(dx + c)^4 + 38*cos(dx + c)^2 - 3*(3*cos(dx + c)^4 - 4*cos(dx + c)^2 + (cos(dx + c)^4 - 4*cos(dx + c)^2)*sin(dx + c))*log(sin(dx + c) + 1) + 3*(3*cos(dx + c)^4 - 4*cos(dx + c)^2 + (cos(dx + c)^4 - 4*cos(dx + c)^2)*sin(dx + c))*log(-sin(dx + c) + 1) - 18*(cos(dx + c)^2 + 2)*sin(dx + c) - 60)/(3*a^3*d*cos(dx + c)^4 - 4*a^3*d*cos(dx + c)^2 + (a^3*d*cos(dx + c)^4 - 4*a^3*d*cos(dx + c)^2)*sin(dx + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{\frac{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**3/(a+a*sin(dx+c))**3,x)

[Out] Integral(tan(c + dx)**3/(sin(c + dx)**3 + 3*sin(c + dx)**2 + 3*sin(c + dx) + 1), x)/a**3

Giac [A] time = 2.55233, size = 154, normalized size = 1.22

$$\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^3} - \frac{12 \log(|\sin(dx+c)-1|)}{a^3} + \frac{12(\sin(dx+c)+1)}{a^3(\sin(dx+c)-1)} - \frac{25 \sin(dx+c)^4 + 148 \sin(dx+c)^3 + 366 \sin(dx+c)^2 + 260 \sin(dx+c) + 65}{a^3(\sin(dx+c)+1)^4}}{768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^3/(a+a*sin(dx+c))^3,x, algorithm="giac")

[Out] -1/768*(12*log(abs(sin(dx + c) + 1))/a^3 - 12*log(abs(sin(dx + c) - 1))/a^3 + 12*(sin(dx + c) + 1)/(a^3*(sin(dx + c) - 1)) - (25*sin(dx + c)^4 + 148*sin(dx + c)^3 + 366*sin(dx + c)^2 + 260*sin(dx + c) + 65)/(a^3*(sin(dx + c) + 1)^4))/d

$$3.74 \quad \int \frac{\tan(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=82

$$-\frac{1}{8d(a^3 \sin(c+dx) + a^3)} + \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{1}{8ad(a \sin(c+dx) + a)^2} + \frac{1}{6d(a \sin(c+dx) + a)^3}$$

[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))

Rubi [A] time = 0.0571133, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2707, 77, 206}

$$-\frac{1}{8d(a^3 \sin(c+dx) + a^3)} + \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{1}{8ad(a \sin(c+dx) + a)^2} + \frac{1}{6d(a \sin(c+dx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]

[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^4} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^4} + \frac{1}{4a(a+x)^3} + \frac{1}{8a^2(a+x)^2} + \frac{1}{8a^2(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{1}{6d(a+a\sin(c+dx))^3} - \frac{1}{8ad(a+a\sin(c+dx))^2} - \frac{1}{8d(a^3+a^3\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2} dx, x, a\sin(c+dx)\right)}{8d} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d} + \frac{1}{6d(a+a\sin(c+dx))^3} - \frac{1}{8ad(a+a\sin(c+dx))^2} - \frac{1}{8d(a^3+a^3\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.145046, size = 52, normalized size = 0.63

$$\frac{3 \tanh^{-1}(\sin(c+dx)) - \frac{3 \sin^2(c+dx) + 9 \sin(c+dx) + 2}{(\sin(c+dx)+1)^3}}{24a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^3, x]

[Out] (3*ArcTanh[Sin[c + d*x]] - (2 + 9*Sin[c + d*x] + 3*Sin[c + d*x]^2)/(1 + Sin[c + d*x])^3)/(24*a^3*d)

Maple [A] time = 0.095, size = 90, normalized size = 1.1

$$-\frac{\ln(\sin(dx+c)-1)}{16da^3} + \frac{1}{6da^3(1+\sin(dx+c))^3} - \frac{1}{8da^3(1+\sin(dx+c))^2} - \frac{1}{8da^3(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{16da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^3, x)

[Out] -1/16/d/a^3*ln(sin(d*x+c)-1)+1/6/d/a^3/(1+sin(d*x+c))^3-1/8/d/a^3/(1+sin(d*x+c))^2-1/8/d/a^3/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c))/a^3/d

Maxima [A] time = 2.14301, size = 132, normalized size = 1.61

$$\frac{\frac{2(3\sin(dx+c)^2+9\sin(dx+c)+2)}{a^3\sin(dx+c)^3+3a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3} - \frac{3\log(\sin(dx+c)+1)}{a^3} + \frac{3\log(\sin(dx+c)-1)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3, x, algorithm="maxima")

[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 3*a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3 + 3*log(sin(d*x + c) - 1)/a^3)/d

Fricas [B] time = 1.55306, size = 409, normalized size = 4.99

$$\frac{6 \cos(dx+c)^2 - 3(3 \cos(dx+c)^2 + (\cos(dx+c)^2 - 4) \sin(dx+c) - 4) \log(\sin(dx+c) + 1) + 3(3 \cos(dx+c)^2 + (\cos(dx+c)^2 - 4) \sin(dx+c) - 4) \log(-\sin(dx+c) + 1) - 18 \sin(dx+c) - 10}{48(3a^3d \cos(dx+c)^2 - 4a^3d + (a^3d \cos(dx+c)^2 - 4a^3d) \sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 10)/(3*a^3*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{\frac{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**3,x)

[Out] Integral(tan(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

Giac [A] time = 2.18984, size = 109, normalized size = 1.33

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a^3} - \frac{6 \log(|\sin(dx+c)-1|)}{a^3} - \frac{11 \sin(dx+c)^3 + 45 \sin(dx+c)^2 + 69 \sin(dx+c) + 19}{a^3(\sin(dx+c)+1)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 - (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 19)/(a^3*(sin(d*x + c) + 1)^3))/d

$$3.75 \quad \int \frac{\cot(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=74

$$\frac{1}{d(a^3 \sin(c+dx) + a^3)} + \frac{\log(\sin(c+dx))}{a^3 d} - \frac{\log(\sin(c+dx) + 1)}{a^3 d} + \frac{1}{2ad(a \sin(c+dx) + a)^2}$$

[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))

Rubi [A] time = 0.0582604, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 44}

$$\frac{1}{d(a^3 \sin(c+dx) + a^3)} + \frac{\log(\sin(c+dx))}{a^3 d} - \frac{\log(\sin(c+dx) + 1)}{a^3 d} + \frac{1}{2ad(a \sin(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]

[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3 x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\log(\sin(c+dx))}{a^3 d} - \frac{\log(1 + \sin(c+dx))}{a^3 d} + \frac{1}{2ad(a + a \sin(c+dx))^2} + \frac{1}{d(a^3 + a^3 \sin(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.175268, size = 52, normalized size = 0.7

$$\frac{2 \sin(c+dx)+3}{(\sin(c+dx)+1)^2} + 2 \log(\sin(c+dx)) - 2 \log(\sin(c+dx) + 1)$$

$$2a^3 d$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]

[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(2*a^3*d)

Maple [A] time = 0.037, size = 68, normalized size = 0.9

$$\frac{1}{2da^3(1+\sin(dx+c))^2} + \frac{1}{da^3(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{da^3} + \frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^3,x)

[Out] 1/2/d/a^3/(1+sin(d*x+c))^2+1/d/a^3/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^3/d+ln(sin(d*x+c))/a^3/d

Maxima [A] time = 1.76862, size = 97, normalized size = 1.31

$$\frac{\frac{2 \sin(dx+c)+3}{a^3 \sin(dx+c)^2+2a^3 \sin(dx+c)+a^3} - \frac{2 \log(\sin(dx+c)+1)}{a^3} + \frac{2 \log(\sin(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) - 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d

Fricas [A] time = 1.6075, size = 284, normalized size = 3.84

$$\frac{2(\cos(dx+c)^2 - 2\sin(dx+c) - 2)\log\left(\frac{1}{2}\sin(dx+c)\right) - 2(\cos(dx+c)^2 - 2\sin(dx+c) - 2)\log(\sin(dx+c)+1) - 2}{2(a^3d\cos(dx+c)^2 - 2a^3d\sin(dx+c) - 2a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) - 3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{\frac{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

Giac [A] time = 1.9801, size = 80, normalized size = 1.08

$$-\frac{\frac{2 \log(|\sin(dx+c)+1|)}{a^3} - \frac{2 \log(|\sin(dx+c)|)}{a^3} - \frac{2 \sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d

3.76 $\int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^3} dx$

Optimal. Leaf size=86

$$\frac{2}{d(a^3 \sin(c+dx) + a^3)} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{3 \csc(c+dx)}{a^3d} + \frac{5 \log(\sin(c+dx))}{a^3d} - \frac{5 \log(\sin(c+dx)+1)}{a^3d}$$

[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]))

Rubi [A] time = 0.0697399, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 77}

$$\frac{2}{d(a^3 \sin(c+dx) + a^3)} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{3 \csc(c+dx)}{a^3d} + \frac{5 \log(\sin(c+dx))}{a^3d} - \frac{5 \log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]

[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^2} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{3}{a^2x^2} + \frac{5}{a^3x} - \frac{2}{a^2(a+x)^2} - \frac{5}{a^3(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{3 \csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{5 \log(\sin(c+dx))}{a^3d} - \frac{5 \log(1 + \sin(c+dx))}{a^3d} + \frac{2}{d(a^3 + a^3 \sin(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.19094, size = 61, normalized size = 0.71

$$\frac{\frac{4}{\sin(c+dx)+1} - \csc^2(c+dx) + 6 \csc(c+dx) + 10 \log(\sin(c+dx)) - 10 \log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]

[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 10*Log[Sin[c + d*x]] - 10*Log[1 + Sin[c + d*x]] + 4/(1 + Sin[c + d*x]))/(2*a^3*d)

Maple [A] time = 0.129, size = 84, normalized size = 1.

$$2 \frac{1}{da^3(1 + \sin(dx + c))} - 5 \frac{\ln(1 + \sin(dx + c))}{da^3} - \frac{1}{2da^3(\sin(dx + c))^2} + 3 \frac{1}{da^3 \sin(dx + c)} + 5 \frac{\ln(\sin(dx + c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x)

[Out] 2/d/a^3/(1+sin(d*x+c))-5*ln(1+sin(d*x+c))/a^3/d-1/2/d/a^3/sin(d*x+c)^2+3/d/a^3/sin(d*x+c)+5*ln(sin(d*x+c))/a^3/d

Maxima [A] time = 1.95102, size = 108, normalized size = 1.26

$$\frac{\frac{10 \sin(dx+c)^2+5 \sin(dx+c)-1}{a^3 \sin(dx+c)^3+a^3 \sin(dx+c)^2} - \frac{10 \log(\sin(dx+c)+1)}{a^3} + \frac{10 \log(\sin(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((10*sin(d*x + c)^2 + 5*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2) - 10*log(sin(d*x + c) + 1)/a^3 + 10*log(sin(d*x + c))/a^3)/d

Fricas [A] time = 1.54676, size = 393, normalized size = 4.57

$$\frac{10 \cos(dx + c)^2 + 10 (\cos(dx + c)^2 + (\cos(dx + c)^2 - 1) \sin(dx + c) - 1) \log\left(\frac{1}{2} \sin(dx + c)\right) - 10 (\cos(dx + c)^2 + (\cos(dx + c)^2 - 1) \sin(dx + c) - 1) \log(\sin(dx + c) + 1) - 5 \sin(dx + c) - 9}{2(a^3d \cos(dx + c)^2 - a^3d + (a^3d \cos(dx + c)^2 - a^3d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(10*cos(d*x + c)^2 + 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - 1)*log(1/2*sin(d*x + c)) - 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 5*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^2 - a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\frac{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

Giac [A] time = 1.73369, size = 208, normalized size = 2.42

$$\frac{80 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^3} - \frac{40 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} - \frac{30 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 40 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 53 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 10 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 a^3} + \frac{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/8*(80*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 40*log(abs(tan(1/2*d*x + 1/2*c))))/a^3 - (30*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 53*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6/d

$$3.77 \quad \int \frac{\cot^5(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=96

$$-\frac{\csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{2 \csc^2(c+dx)}{a^3d} + \frac{4 \csc(c+dx)}{a^3d} + \frac{4 \log(\sin(c+dx))}{a^3d} - \frac{4 \log(\sin(c+dx)+1)}{a^3d}$$

[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)

Rubi [A] time = 0.0712619, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{2 \csc^2(c+dx)}{a^3d} + \frac{4 \csc(c+dx)}{a^3d} + \frac{4 \log(\sin(c+dx))}{a^3d} - \frac{4 \log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]

[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^5(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5} - \frac{3}{x^4} + \frac{4}{ax^3} - \frac{4}{a^2x^2} + \frac{4}{a^3x} - \frac{4}{a^3(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{4 \csc(c+dx)}{a^3d} - \frac{2 \csc^2(c+dx)}{a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{4 \log(\sin(c+dx))}{a^3d} - \frac{4 \log(\sin(c+dx)+1)}{a^3d} \end{aligned}$$

Mathematica [A] time = 0.327739, size = 69, normalized size = 0.72

$$-\frac{\csc^4(c+dx) + 4 \csc^3(c+dx) - 8 \csc^2(c+dx) + 16 \csc(c+dx) + 16 \log(\sin(c+dx)) - 16 \log(\sin(c+dx)+1)}{4a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]

[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 16*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)

Maple [A] time = 0.126, size = 97, normalized size = 1.

$$-4 \frac{\ln(1 + \sin(dx + c))}{a^3 d} - \frac{1}{4 a^3 d (\sin(dx + c))^4} + \frac{1}{a^3 d (\sin(dx + c))^3} - 2 \frac{1}{a^3 d (\sin(dx + c))^2} + 4 \frac{1}{a^3 d \sin(dx + c)} + 4 \frac{\ln(\sin(dx + c))}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x)

[Out] -4*ln(1+sin(d*x+c))/a^3/d-1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d*x+c)^3-2/d/a^3/sin(d*x+c)^2+4/d/a^3/sin(d*x+c)+4*ln(sin(d*x+c))/a^3/d

Maxima [A] time = 2.05557, size = 101, normalized size = 1.05

$$\frac{\frac{16 \log(\sin(dx+c)+1)}{a^3} - \frac{16 \log(\sin(dx+c))}{a^3} - \frac{16 \sin(dx+c)^3 - 8 \sin(dx+c)^2 + 4 \sin(dx+c) - 1}{a^3 \sin(dx+c)^4}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d

Fricas [A] time = 1.65155, size = 348, normalized size = 3.62

$$\frac{8 \cos(dx + c)^2 + 16 (\cos(dx + c)^4 - 2 \cos(dx + c)^2 + 1) \log\left(\frac{1}{2} \sin(dx + c)\right) - 16 (\cos(dx + c)^4 - 2 \cos(dx + c)^2 + 1) \log(\sin(dx + c))}{4 (a^3 d \cos(dx + c)^4 - 2 a^3 d \cos(dx + c)^2 + a^3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c) + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

Giac [A] time = 2.57501, size = 235, normalized size = 2.45

$$\frac{1536 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^3} - \frac{768 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} + \frac{1600 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 456 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 108 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 24 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 3}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4}$$

192 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d*x + 1/2*c))))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d

$$3.78 \quad \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^3d} + \frac{3 \csc^5(c+dx)}{5a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{3a^3d}$$

[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)

Rubi [A] time = 0.0574368, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 43}

$$-\frac{\csc^6(c+dx)}{6a^3d} + \frac{3 \csc^5(c+dx)}{5a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]

[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^7} - \frac{3a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{3 \csc^5(c+dx)}{5a^3d} - \frac{\csc^6(c+dx)}{6a^3d} \end{aligned}$$

Mathematica [A] time = 0.098613, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx) \left(-10 \csc^3(c+dx) + 36 \csc^2(c+dx) - 45 \csc(c+dx) + 20\right)}{60a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]

[Out] (Csc[c + d*x]^3*(20 - 45*Csc[c + d*x] + 36*Csc[c + d*x]^2 - 10*Csc[c + d*x]^3))/(60*a^3*d)

Maple [A] time = 0.119, size = 49, normalized size = 0.7

$$\frac{1}{da^3} \left(\frac{3}{5 (\sin(dx + c))^5} - \frac{3}{4 (\sin(dx + c))^4} - \frac{1}{6 (\sin(dx + c))^6} + \frac{1}{3 (\sin(dx + c))^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x)

[Out] 1/d/a^3*(3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6+1/3/sin(d*x+c)^3)

Maxima [A] time = 1.53887, size = 62, normalized size = 0.85

$$\frac{20 \sin(dx + c)^3 - 45 \sin(dx + c)^2 + 36 \sin(dx + c) - 10}{60 a^3 d \sin(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*sin(d*x + c)^6)

Fricas [A] time = 1.46143, size = 208, normalized size = 2.85

$$\frac{45 \cos(dx + c)^2 - 4(5 \cos(dx + c)^2 - 14) \sin(dx + c) - 55}{60 (a^3 d \cos(dx + c)^6 - 3 a^3 d \cos(dx + c)^4 + 3 a^3 d \cos(dx + c)^2 - a^3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/60*(45*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 14)*sin(d*x + c) - 55)/(a^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64026, size = 62, normalized size = 0.85

$$\frac{20 \sin(dx + c)^3 - 45 \sin(dx + c)^2 + 36 \sin(dx + c) - 10}{60 a^3 d \sin(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

$$3.79 \quad \int \frac{\cot^9(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=109

$$-\frac{\csc^8(c+dx)}{8a^3d} + \frac{3 \csc^7(c+dx)}{7a^3d} - \frac{\csc^6(c+dx)}{3a^3d} - \frac{2 \csc^5(c+dx)}{5a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d} - \frac{\csc^3(c+dx)}{3a^3d}$$

[Out] $-\text{Csc}[c + d*x]^3/(3*a^3*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^3*d) - (2*\text{Csc}[c + d*x]^5)/(5*a^3*d) - \text{Csc}[c + d*x]^6/(3*a^3*d) + (3*\text{Csc}[c + d*x]^7)/(7*a^3*d) - \text{Csc}[c + d*x]^8/(8*a^3*d)$

Rubi [A] time = 0.0689768, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 75}

$$-\frac{\csc^8(c+dx)}{8a^3d} + \frac{3 \csc^7(c+dx)}{7a^3d} - \frac{\csc^6(c+dx)}{3a^3d} - \frac{2 \csc^5(c+dx)}{5a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d} - \frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^9/(a + a*\text{Sin}[c + d*x])^3, x]$

[Out] $-\text{Csc}[c + d*x]^3/(3*a^3*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^3*d) - (2*\text{Csc}[c + d*x]^5)/(5*a^3*d) - \text{Csc}[c + d*x]^6/(3*a^3*d) + (3*\text{Csc}[c + d*x]^7)/(7*a^3*d) - \text{Csc}[c + d*x]^8/(8*a^3*d)$

Rule 2707

$\text{Int}[(a + b*\sin(e + f*x))^m * \tan(e + f*x)^p, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{m - (p + 1)/2})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[(p + 1)/2]$

Rule 75

$\text{Int}[(d*x)^n * (a + b*x) * (e + f*x)^p, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[b*e + a*f, 0] \ \&\& \ !(\text{ILtQ}[n + p + 2, 0] \ \&\& \ \text{GtQ}[n + 2*p, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\cot^9(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)}{x^9} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^9} - \frac{3a^4}{x^8} + \frac{2a^3}{x^7} + \frac{2a^2}{x^6} - \frac{3a}{x^5} + \frac{1}{x^4}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d} - \frac{2 \csc^5(c+dx)}{5a^3d} - \frac{\csc^6(c+dx)}{3a^3d} + \frac{3 \csc^7(c+dx)}{7a^3d} - \frac{\csc^8(c+dx)}{8a^3d} \end{aligned}$$

Mathematica [A] time = 0.0721869, size = 68, normalized size = 0.62

$$\frac{\csc^3(c+dx)(105 \csc^5(c+dx) - 360 \csc^4(c+dx) + 280 \csc^3(c+dx) + 336 \csc^2(c+dx) - 630 \csc(c+dx) + 280)}{840a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]

[Out] $-(\text{Csc}[c + d*x]^3(280 - 630*\text{Csc}[c + d*x] + 336*\text{Csc}[c + d*x]^2 + 280*\text{Csc}[c + d*x]^3 - 360*\text{Csc}[c + d*x]^4 + 105*\text{Csc}[c + d*x]^5))/(840*a^3*d)$

Maple [A] time = 0.136, size = 69, normalized size = 0.6

$$\frac{1}{da^3} \left(\frac{3}{7 (\sin(dx+c))^7} - \frac{1}{8 (\sin(dx+c))^8} - \frac{2}{5 (\sin(dx+c))^5} + \frac{3}{4 (\sin(dx+c))^4} - \frac{1}{3 (\sin(dx+c))^6} - \frac{1}{3 (\sin(dx+c))^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x)

[Out] $1/d/a^3*(3/7/\sin(d*x+c)^7-1/8/\sin(d*x+c)^8-2/5/\sin(d*x+c)^5+3/4/\sin(d*x+c)^4-1/3/\sin(d*x+c)^6-1/3/\sin(d*x+c)^3)$

Maxima [A] time = 1.21198, size = 89, normalized size = 0.82

$$\frac{280 \sin(dx+c)^5 - 630 \sin(dx+c)^4 + 336 \sin(dx+c)^3 + 280 \sin(dx+c)^2 - 360 \sin(dx+c) + 105}{840 a^3 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/840*(280*\sin(d*x + c)^5 - 630*\sin(d*x + c)^4 + 336*\sin(d*x + c)^3 + 280*\sin(d*x + c)^2 - 360*\sin(d*x + c) + 105)/(a^3*d*\sin(d*x + c)^8)$

Fricas [A] time = 1.47629, size = 302, normalized size = 2.77

$$\frac{630 \cos(dx+c)^4 - 980 \cos(dx+c)^2 - 8(35 \cos(dx+c)^4 - 112 \cos(dx+c)^2 + 32) \sin(dx+c) + 245}{840(a^3 d \cos(dx+c)^8 - 4 a^3 d \cos(dx+c)^6 + 6 a^3 d \cos(dx+c)^4 - 4 a^3 d \cos(dx+c)^2 + a^3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $1/840*(630*\cos(d*x + c)^4 - 980*\cos(d*x + c)^2 - 8*(35*\cos(d*x + c)^4 - 112*\cos(d*x + c)^2 + 32)*\sin(d*x + c) + 245)/(a^3*d*\cos(d*x + c)^8 - 4*a^3*d*\cos(d*x + c)^6 + 6*a^3*d*\cos(d*x + c)^4 - 4*a^3*d*\cos(d*x + c)^2 + a^3*d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61935, size = 89, normalized size = 0.82

$$\frac{280 \sin(dx + c)^5 - 630 \sin(dx + c)^4 + 336 \sin(dx + c)^3 + 280 \sin(dx + c)^2 - 360 \sin(dx + c) + 105}{840 a^3 d \sin(dx + c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

$$3.80 \quad \int \frac{\cot^{11}(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^3d} + \frac{\csc^9(c+dx)}{3a^3d} - \frac{\csc^8(c+dx)}{8a^3d} - \frac{5 \csc^7(c+dx)}{7a^3d} + \frac{5 \csc^6(c+dx)}{6a^3d} + \frac{\csc^5(c+dx)}{5a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{3a^3d}$$

[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*d)

Rubi [A] time = 0.0805763, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^3d} + \frac{\csc^9(c+dx)}{3a^3d} - \frac{\csc^8(c+dx)}{8a^3d} - \frac{5 \csc^7(c+dx)}{7a^3d} + \frac{5 \csc^6(c+dx)}{6a^3d} + \frac{\csc^5(c+dx)}{5a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]

[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^{11}(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^2}{x^{11}} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^7}{x^{11}} - \frac{3a^6}{x^{10}} + \frac{a^5}{x^9} + \frac{5a^4}{x^8} - \frac{5a^3}{x^7} - \frac{a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3 \csc^4(c+dx)}{4a^3d} + \frac{\csc^5(c+dx)}{5a^3d} + \frac{5 \csc^6(c+dx)}{6a^3d} - \frac{5 \csc^7(c+dx)}{7a^3d} - \frac{\csc^8(c+dx)}{8a^3d} \end{aligned}$$

Mathematica [A] time = 0.11307, size = 88, normalized size = 0.61

$$\frac{\csc^3(c + dx) (-84 \csc^7(c + dx) + 280 \csc^6(c + dx) - 105 \csc^5(c + dx) - 600 \csc^4(c + dx) + 700 \csc^3(c + dx) + 168 \csc^2(c + dx) - 84 \csc(c + dx))}{840 a^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]

[Out] (Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 168*Csc[c + d*x]^2 + 700*Csc[c + d*x]^3 - 600*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5 + 280*Csc[c + d*x]^6 - 84*Csc[c + d*x]^7))/(840*a^3*d)

Maple [A] time = 0.153, size = 89, normalized size = 0.6

$$\frac{1}{da^3} \left(-\frac{1}{10 (\sin(dx + c))^{10}} - \frac{5}{7 (\sin(dx + c))^7} - \frac{1}{8 (\sin(dx + c))^8} + \frac{1}{5 (\sin(dx + c))^5} - \frac{3}{4 (\sin(dx + c))^4} + \frac{1}{3 (\sin(dx + c))^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x)

[Out] 1/d/a^3*(-1/10/sin(d*x+c)^10-5/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8+1/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3)

Maxima [A] time = 1.96466, size = 116, normalized size = 0.8

$$\frac{280 \sin(dx + c)^7 - 630 \sin(dx + c)^6 + 168 \sin(dx + c)^5 + 700 \sin(dx + c)^4 - 600 \sin(dx + c)^3 - 105 \sin(dx + c)^2 - 84 \sin(dx + c)}{840 a^3 d \sin(dx + c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*sin(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c) - 84)/(a^3*d*sin(d*x + c)^10)

Fricas [A] time = 1.59062, size = 398, normalized size = 2.74

$$\frac{630 \cos(dx + c)^6 - 1190 \cos(dx + c)^4 + 595 \cos(dx + c)^2 - 8(35 \cos(dx + c)^6 - 126 \cos(dx + c)^4 + 72 \cos(dx + c)^2 - 16) \sin(dx + c) - 119}{840 (a^3 d \cos(dx + c)^{10} - 5 a^3 d \cos(dx + c)^8 + 10 a^3 d \cos(dx + c)^6 - 10 a^3 d \cos(dx + c)^4 + 5 a^3 d \cos(dx + c)^2 - 16) \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/840*(630*cos(d*x + c)^6 - 1190*cos(d*x + c)^4 + 595*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x + c) - 119)/(a^3*d*cos(d*x + c)^10 - 5*a^3*d*cos(d*x + c)^8 + 10*a^3*d*cos(d*x + c)^6 - 10*a^3*d*cos(d*x + c)^4 + 5*a^3*d*cos(d*x + c)^2 - 16)

$$x + c)^6 - 10a^3d\cos(dx + c)^4 + 5a^3d\cos(dx + c)^2 - a^3d$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**11/(a+a*sin(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 2.01292, size = 116, normalized size = 0.8

$$\frac{280 \sin(dx + c)^7 - 630 \sin(dx + c)^6 + 168 \sin(dx + c)^5 + 700 \sin(dx + c)^4 - 600 \sin(dx + c)^3 - 105 \sin(dx + c)^2 + 280 \sin(dx + c) - 84}{840 a^3 d \sin(dx + c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^11/(a+a*sin(dx+c))^3,x, algorithm="giac")

[Out] 1/840*(280*sin(dx + c)^7 - 630*sin(dx + c)^6 + 168*sin(dx + c)^5 + 700*sin(dx + c)^4 - 600*sin(dx + c)^3 - 105*sin(dx + c)^2 + 280*sin(dx + c) - 84)/(a^3*d*sin(dx + c)^10)

$$3.81 \quad \int \frac{\cot^{13}(c+dx)}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=145

$$-\frac{\csc^{12}(c+dx)}{12a^3d} + \frac{3 \csc^{11}(c+dx)}{11a^3d} - \frac{8 \csc^9(c+dx)}{9a^3d} + \frac{3 \csc^8(c+dx)}{4a^3d} + \frac{6 \csc^7(c+dx)}{7a^3d} - \frac{4 \csc^6(c+dx)}{3a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d}$$

[Out] $-\text{Csc}[c + d*x]^3/(3*a^3*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^3*d) - (4*\text{Csc}[c + d*x]^6)/(3*a^3*d) + (6*\text{Csc}[c + d*x]^7)/(7*a^3*d) + (3*\text{Csc}[c + d*x]^8)/(4*a^3*d) - (8*\text{Csc}[c + d*x]^9)/(9*a^3*d) + (3*\text{Csc}[c + d*x]^11)/(11*a^3*d) - \text{Csc}[c + d*x]^12/(12*a^3*d)$

Rubi [A] time = 0.0783676, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^3d} + \frac{3 \csc^{11}(c+dx)}{11a^3d} - \frac{8 \csc^9(c+dx)}{9a^3d} + \frac{3 \csc^8(c+dx)}{4a^3d} + \frac{6 \csc^7(c+dx)}{7a^3d} - \frac{4 \csc^6(c+dx)}{3a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^13/(a + a*\text{Sin}[c + d*x])^3, x]$

[Out] $-\text{Csc}[c + d*x]^3/(3*a^3*d) + (3*\text{Csc}[c + d*x]^4)/(4*a^3*d) - (4*\text{Csc}[c + d*x]^6)/(3*a^3*d) + (6*\text{Csc}[c + d*x]^7)/(7*a^3*d) + (3*\text{Csc}[c + d*x]^8)/(4*a^3*d) - (8*\text{Csc}[c + d*x]^9)/(9*a^3*d) + (3*\text{Csc}[c + d*x]^11)/(11*a^3*d) - \text{Csc}[c + d*x]^12/(12*a^3*d)$

Rule 2707

$\text{Int}[(a + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)}\tan[(e_*) + (f_*)(x_*)]^{(p_*)}, x_Symbol] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{((p + 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

$\text{Int}[(a + (b_*)(x_*)^{(m_*)})*((c_*) + (d_*)(x_*)^{(n_*)})*((e_*) + (f_*)(x_*)^{(p_*)}), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^{13}(c+dx)}{(a+a \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^3}{x^{13}} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{13}} - \frac{3a^8}{x^{12}} + \frac{8a^6}{x^{10}} - \frac{6a^5}{x^9} - \frac{6a^4}{x^8} + \frac{8a^3}{x^7} - \frac{3a}{x^5} + \frac{1}{x^4}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3 \csc^4(c+dx)}{4a^3d} - \frac{4 \csc^6(c+dx)}{3a^3d} + \frac{6 \csc^7(c+dx)}{7a^3d} + \frac{3 \csc^8(c+dx)}{4a^3d} - \frac{8 \csc^9(c+dx)}{9a^3d} \end{aligned}$$

Mathematica [A] time = 0.117899, size = 88, normalized size = 0.61

$$\frac{\csc^3(c + dx) \left(-231 \csc^9(c + dx) + 756 \csc^8(c + dx) - 2464 \csc^6(c + dx) + 2079 \csc^5(c + dx) + 2376 \csc^4(c + dx) - 3696 \csc^3(c + dx) + 231 \csc^2(c + dx) - 231 \csc(c + dx) + 231 \right)}{2772 a^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]

[Out] (Csc[c + d*x]^3*(-924 + 2079*Csc[c + d*x] - 3696*Csc[c + d*x]^3 + 2376*Csc[c + d*x]^4 + 2079*Csc[c + d*x]^5 - 2464*Csc[c + d*x]^6 + 756*Csc[c + d*x]^8 - 231*Csc[c + d*x]^9))/(2772*a^3*d)

Maple [A] time = 0.174, size = 89, normalized size = 0.6

$$\frac{1}{da^3} \left(\frac{6}{7 (\sin(dx + c))^7} + \frac{3}{11 (\sin(dx + c))^{11}} + \frac{3}{4 (\sin(dx + c))^8} + \frac{3}{4 (\sin(dx + c))^4} - \frac{8}{9 (\sin(dx + c))^9} - \frac{1}{12 (\sin(dx + c))^{12}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x)

[Out] 1/d/a^3*(6/7/sin(d*x+c)^7+3/11/sin(d*x+c)^11+3/4/sin(d*x+c)^8+3/4/sin(d*x+c)^4-8/9/sin(d*x+c)^9-1/12/sin(d*x+c)^12-4/3/sin(d*x+c)^6-1/3/sin(d*x+c)^3)

Maxima [A] time = 2.73298, size = 116, normalized size = 0.8

$$\frac{924 \sin(dx + c)^9 - 2079 \sin(dx + c)^8 + 3696 \sin(dx + c)^6 - 2376 \sin(dx + c)^5 - 2079 \sin(dx + c)^4 + 2464 \sin(dx + c)^3 - 756 \sin(dx + c)^2 + 231 \sin(dx + c) + 231}{2772 a^3 d \sin(dx + c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*x + c)^2 + 231)/(a^3*d*sin(d*x + c)^12)

Fricas [A] time = 1.66886, size = 498, normalized size = 3.43

$$\frac{2079 \cos(dx + c)^8 - 4620 \cos(dx + c)^6 + 3465 \cos(dx + c)^4 - 1386 \cos(dx + c)^2 - 4 \left(231 \cos(dx + c)^8 - 924 \cos(dx + c)^6 + 792 \cos(dx + c)^4 - 352 \cos(dx + c)^2 + 64 \right) \sin(dx + c) + 231}{2772 \left(a^3 d \cos(dx + c)^{12} - 6 a^3 d \cos(dx + c)^{10} + 15 a^3 d \cos(dx + c)^8 - 20 a^3 d \cos(dx + c)^6 + 15 a^3 d \cos(dx + c)^4 - 6 a^3 d \cos(dx + c)^2 + 231 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2772*(2079*cos(d*x + c)^8 - 4620*cos(d*x + c)^6 + 3465*cos(d*x + c)^4 - 1386*cos(d*x + c)^2 - 4*(231*cos(d*x + c)^8 - 924*cos(d*x + c)^6 + 792*cos(d*x + c)^4 - 352*cos(d*x + c)^2 + 64)*sin(d*x + c) + 231)/(a^3*d*cos(d*x + c)^12 - 6*a^3*d*cos(d*x + c)^10 + 15*a^3*d*cos(d*x + c)^8 - 20*a^3*d*cos(d*x + c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + 231)

$$+ c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + a^3*d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.99282, size = 116, normalized size = 0.8

$$\frac{924 \sin(dx + c)^9 - 2079 \sin(dx + c)^8 + 3696 \sin(dx + c)^6 - 2376 \sin(dx + c)^5 - 2079 \sin(dx + c)^4 + 2464 \sin(dx + c)^3 - 756 \sin(dx + c) + 231}{2772 a^3 d \sin(dx + c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*x + c) + 231)/(a^3*d*sin(d*x + c)^12)

$$3.82 \quad \int \frac{\tan^5(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=195

$$\frac{a^2}{48d(a \sin(c+dx)+a)^6} - \frac{3}{256d(a^4 - a^4 \sin(c+dx))} - \frac{1}{256d(a^4 \sin(c+dx)+a^4)} + \frac{1}{256d(a^2 - a^2 \sin(c+dx))^2} - \frac{1}{256d}$$

[Out] -ArcTanh[Sin[c + d*x]]/(128*a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) - (7*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(96*a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(256*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(256*d*(a^4 + a^4*Sin[c + d*x]))

Rubi [A] time = 0.134569, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 88, 206}

$$\frac{a^2}{48d(a \sin(c+dx)+a)^6} - \frac{3}{256d(a^4 - a^4 \sin(c+dx))} - \frac{1}{256d(a^4 \sin(c+dx)+a^4)} + \frac{1}{256d(a^2 - a^2 \sin(c+dx))^2} - \frac{1}{256d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]

[Out] -ArcTanh[Sin[c + d*x]]/(128*a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) - (7*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(96*a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(256*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(256*d*(a^4 + a^4*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^7} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{128a^2(a-x)^3} - \frac{3}{256a^3(a-x)^2} - \frac{a^2}{8(a+x)^7} + \frac{7a}{16(a+x)^6} - \frac{1}{2(a+x)^5} + \frac{5}{32a(a+x)^4} + \frac{5}{128a^2(a+x)^3} + \frac{1}{128a^2(a+x)^3}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{1}{96ad(a+a\sin(c+dx))^3} + \frac{1}{128a^2d(a+a\sin(c+dx))^2} \\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d} + \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{1}{96ad(a+a\sin(c+dx))^3} + \frac{1}{128a^2d(a+a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 1.48884, size = 112, normalized size = 0.57

$$\frac{30 \tanh^{-1}(\sin(c+dx)) - \frac{2(15 \sin^7(c+dx) + 60 \sin^6(c+dx) + 65 \sin^5(c+dx) + 440 \sin^4(c+dx) + 257 \sin^3(c+dx) - 132 \sin^2(c+dx) - 177 \sin(c+dx) - 48)}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^6}}{3840a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4, x]

[Out] -(30*ArcTanh[Sin[c + d*x]] - (2*(-48 - 177*Sin[c + d*x] - 132*Sin[c + d*x]^2 + 257*Sin[c + d*x]^3 + 440*Sin[c + d*x]^4 + 65*Sin[c + d*x]^5 + 60*Sin[c + d*x]^6 + 15*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^6))/(3840*a^4*d)

Maple [A] time = 0.088, size = 180, normalized size = 0.9

$$\frac{1}{256 da^4 (\sin(dx+c)-1)^2} + \frac{3}{256 da^4 (\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{256 da^4} + \frac{1}{48 da^4 (1+\sin(dx+c))^6} - \frac{1}{80 da^4 (1+\sin(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^4, x)

[Out] 1/256/d/a^4/(sin(d*x+c)-1)^2+3/256/d/a^4/(sin(d*x+c)-1)+1/256/d/a^4*ln(sin(d*x+c)-1)+1/48/d/a^4/(1+sin(d*x+c))^6-7/80/d/a^4/(1+sin(d*x+c))^5+1/8/d/a^4/(1+sin(d*x+c))^4-5/96/d/a^4/(1+sin(d*x+c))^3-5/256/d/a^4/(1+sin(d*x+c))^2-1/256/d/a^4/(1+sin(d*x+c))-1/256*ln(1+sin(d*x+c))/a^4/d

Maxima [A] time = 3.47638, size = 288, normalized size = 1.48

$$\frac{2(15 \sin(dx+c)^7 + 60 \sin(dx+c)^6 + 65 \sin(dx+c)^5 + 440 \sin(dx+c)^4 + 257 \sin(dx+c)^3 - 132 \sin(dx+c)^2 - 177 \sin(dx+c) - 48)}{a^4 \sin(dx+c)^8 + 4a^4 \sin(dx+c)^7 + 4a^4 \sin(dx+c)^6 - 4a^4 \sin(dx+c)^5 - 10a^4 \sin(dx+c)^4 - 4a^4 \sin(dx+c)^3 + 4a^4 \sin(dx+c)^2 + 4a^4 \sin(dx+c) + a^4} - \frac{15 \log(\sin(dx+c)-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4, x, algorithm="maxima")

```
[Out] 1/3840*(2*(15*sin(d*x + c)^7 + 60*sin(d*x + c)^6 + 65*sin(d*x + c)^5 + 440*
sin(d*x + c)^4 + 257*sin(d*x + c)^3 - 132*sin(d*x + c)^2 - 177*sin(d*x + c)
- 48)/(a^4*sin(d*x + c)^8 + 4*a^4*sin(d*x + c)^7 + 4*a^4*sin(d*x + c)^6 -
4*a^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^4 - 4*a^4*sin(d*x + c)^3 + 4*a^4
*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 15*log(sin(d*x + c) + 1)/a^4
+ 15*log(sin(d*x + c) - 1)/a^4)/d
```

Fricas [A] time = 1.73448, size = 775, normalized size = 3.97

$$\frac{120 \cos(dx + c)^6 - 1240 \cos(dx + c)^4 + 1856 \cos(dx + c)^2 + 15 (\cos(dx + c)^8 - 8 \cos(dx + c)^6 + 8 \cos(dx + c)^4 - 4 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \log(\sin(dx + c) + 1) - 15 (\cos(dx + c)^8 - 8 \cos(dx + c)^6 + 8 \cos(dx + c)^4 - 4 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \log(\sin(dx + c) - 1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(120*cos(d*x + c)^6 - 1240*cos(d*x + c)^4 + 1856*cos(d*x + c)^2 + 1
5*(cos(d*x + c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6
- 2*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)
^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6 - 2*cos(d*x +
c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 110*cos
(d*x + c)^4 + 432*cos(d*x + c)^2 - 160)*sin(d*x + c) - 640)/(a^4*d*cos(d*x
+ c)^8 - 8*a^4*d*cos(d*x + c)^6 + 8*a^4*d*cos(d*x + c)^4 - 4*(a^4*d*cos(d*x
+ c)^6 - 2*a^4*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 5.53021, size = 197, normalized size = 1.01

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^4} - \frac{60 \log(|\sin(dx+c)-1|)}{a^4} + \frac{30(3 \sin^2(dx+c) - 12 \sin(dx+c) + 7)}{a^4(\sin(dx+c)-1)^2} - \frac{147 \sin^6(dx+c) + 822 \sin^5(dx+c) + 1605 \sin^4(dx+c) + 340 \sin^3(dx+c) - 675 \sin^2(dx+c) - 522 \sin(dx+c) - 117}{a^4(\sin(dx+c)+1)^6}}{15360 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15360*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c) - 1))
/a^4 + 30*(3*sin(d*x + c)^2 - 12*sin(d*x + c) + 7)/(a^4*(sin(d*x + c) - 1)^
2) - (147*sin(d*x + c)^6 + 822*sin(d*x + c)^5 + 1605*sin(d*x + c)^4 + 340*s
in(d*x + c)^3 - 675*sin(d*x + c)^2 - 522*sin(d*x + c) - 117)/(a^4*(sin(d*x
+ c) + 1)^6))/d
```

$$3.83 \quad \int \frac{\tan^3(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=132

$$\frac{1}{64d(a^4 - a^4 \sin(c + dx))} + \frac{1}{64d(a^4 \sin(c + dx) + a^4)} + \frac{1}{32d(a^2 \sin(c + dx) + a^2)^2} + \frac{a}{20d(a \sin(c + dx) + a)^5} - \frac{1}{8d(a \sin(c + dx) + a)^5}$$

[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))

Rubi [A] time = 0.0887882, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$\frac{1}{64d(a^4 - a^4 \sin(c + dx))} + \frac{1}{64d(a^4 \sin(c + dx) + a^4)} + \frac{1}{32d(a^2 \sin(c + dx) + a^2)^2} + \frac{a}{20d(a \sin(c + dx) + a)^5} - \frac{1}{8d(a \sin(c + dx) + a)^5}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]

[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\tan^3(c + dx)}{(a + a \sin(c + dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^6} dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{64a^3(a-x)^2} - \frac{a}{4(a+x)^6} + \frac{1}{2(a+x)^5} - \frac{3}{16a(a+x)^4} - \frac{1}{16a^2(a+x)^3} - \frac{1}{64a^3(a+x)^2}\right) dx, x, a \sin(c + dx)\right)}{d} \\ &= \frac{a}{20d(a + a \sin(c + dx))^5} - \frac{1}{8d(a + a \sin(c + dx))^4} + \frac{1}{16ad(a + a \sin(c + dx))^3} + \frac{1}{32d(a^2 + a^2 \sin(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 0.115064, size = 50, normalized size = 0.38

$$\frac{5 \sin^2(c + dx) + 4 \sin(c + dx) + 1}{20a^4 d (\sin(c + dx) - 1) (\sin(c + dx) + 1)^5}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]

[Out] -(1 + 4*Sin[c + d*x] + 5*Sin[c + d*x]^2)/(20*a^4*d*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^5)

Maple [A] time = 0.088, size = 81, normalized size = 0.6

$$\frac{1}{da^4} \left(-\frac{1}{64 \sin(dx + c) - 64} + \frac{1}{20 (1 + \sin(dx + c))^5} - \frac{1}{8 (1 + \sin(dx + c))^4} + \frac{1}{16 (1 + \sin(dx + c))^3} + \frac{1}{32 (1 + \sin(dx + c))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x)

[Out] 1/d/a^4*(-1/64/(sin(d*x+c)-1)+1/20/(1+sin(d*x+c))^5-1/8/(1+sin(d*x+c))^4+1/16/(1+sin(d*x+c))^3+1/32/(1+sin(d*x+c))^2+1/64/(1+sin(d*x+c)))

Maxima [A] time = 2.93891, size = 128, normalized size = 0.97

$$\frac{5 \sin(dx + c)^2 + 4 \sin(dx + c) + 1}{20 \left(a^4 \sin(dx + c)^6 + 4 a^4 \sin(dx + c)^5 + 5 a^4 \sin(dx + c)^4 - 5 a^4 \sin(dx + c)^2 - 4 a^4 \sin(dx + c) - a^4 \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] -1/20*(5*sin(d*x + c)^2 + 4*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^6 + 4*a^4*sin(d*x + c)^5 + 5*a^4*sin(d*x + c)^4 - 5*a^4*sin(d*x + c)^2 - 4*a^4*sin(d*x + c) - a^4)*d)

Fricas [A] time = 1.35404, size = 250, normalized size = 1.89

$$\frac{5 \cos(dx + c)^2 - 4 \sin(dx + c) - 6}{20 \left(a^4 d \cos(dx + c)^6 - 8 a^4 d \cos(dx + c)^4 + 8 a^4 d \cos(dx + c)^2 - 4 \left(a^4 d \cos(dx + c)^4 - 2 a^4 d \cos(dx + c)^2 \right) \sin(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] -1/20*(5*cos(d*x + c)^2 - 4*sin(d*x + c) - 6)/(a^4*d*cos(d*x + c)^6 - 8*a^4*d*cos(d*x + c)^4 + 8*a^4*d*cos(d*x + c)^2 - 4*(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^2)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1} dx$$

$$\frac{dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**4,x)

[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4

Giac [A] time = 2.77507, size = 103, normalized size = 0.78

$$\frac{\frac{5}{a^4(\sin(dx+c)-1)} - \frac{5 \sin(dx+c)^4 + 30 \sin(dx+c)^3 + 80 \sin(dx+c)^2 + 50 \sin(dx+c) + 11}{a^4(\sin(dx+c)+1)^5}}{320d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] -1/320*(5/(a^4*(sin(d*x + c) - 1)) - (5*sin(d*x + c)^4 + 30*sin(d*x + c)^3 + 80*sin(d*x + c)^2 + 50*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^5))/d

$$3.84 \quad \int \frac{\tan(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=105

$$-\frac{1}{16d(a^4 \sin(c+dx) + a^4)} - \frac{1}{16d(a^2 \sin(c+dx) + a^2)^2} + \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d} - \frac{1}{12ad(a \sin(c+dx) + a)^3} + \frac{1}{8d(a \sin(c+dx) + a)}$$

[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*(a^4 + a^4*Sin[c + d*x]))

Rubi [A] time = 0.0653541, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2707, 77, 206}

$$-\frac{1}{16d(a^4 \sin(c+dx) + a^4)} - \frac{1}{16d(a^2 \sin(c+dx) + a^2)^2} + \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d} - \frac{1}{12ad(a \sin(c+dx) + a)^3} + \frac{1}{8d(a \sin(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]

[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*(a^4 + a^4*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^5} dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^5} + \frac{1}{4a(a+x)^4} + \frac{1}{8a^2(a+x)^3} + \frac{1}{16a^3(a+x)^2} + \frac{1}{16a^3(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{1}{12ad(a+a\sin(c+dx))^3} - \frac{1}{16d(a^2+a^2\sin(c+dx))^2} - \frac{1}{16d(a^2+a^2\sin^2(c+dx))} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{1}{12ad(a+a\sin(c+dx))^3} - \frac{1}{16d(a^2+a^2\sin^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.25388, size = 62, normalized size = 0.59

$$\frac{3 \tanh^{-1}(\sin(c+dx)) - \frac{3 \sin^3(c+dx) + 12 \sin^2(c+dx) + 19 \sin(c+dx) + 4}{(\sin(c+dx)+1)^4}}{48a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^4, x]

[Out] (3*ArcTanh[Sin[c + d*x]] - (4 + 19*Sin[c + d*x] + 12*Sin[c + d*x]^2 + 3*Sin[c + d*x]^3)/(1 + Sin[c + d*x])^4)/(48*a^4*d)

Maple [A] time = 0.092, size = 108, normalized size = 1.

$$-\frac{\ln(\sin(dx+c)-1)}{32da^4} + \frac{1}{8da^4(1+\sin(dx+c))^4} - \frac{1}{12da^4(1+\sin(dx+c))^3} - \frac{1}{16da^4(1+\sin(dx+c))^2} - \frac{1}{16da^4(1+\sin^2(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^4, x)

[Out] -1/32/d/a^4*ln(sin(d*x+c)-1)+1/8/d/a^4/(1+sin(d*x+c))^4-1/12/d/a^4/(1+sin(d*x+c))^3-1/16/d/a^4/(1+sin(d*x+c))^2-1/16/d/a^4/(1+sin(d*x+c))+1/32*ln(1+sin(d*x+c))/a^4/d

Maxima [A] time = 3.1573, size = 163, normalized size = 1.55

$$-\frac{2(3\sin(dx+c)^3+12\sin(dx+c)^2+19\sin(dx+c)+4)}{a^4\sin(dx+c)^4+4a^4\sin(dx+c)^3+6a^4\sin(dx+c)^2+4a^4\sin(dx+c)+a^4} - \frac{3\log(\sin(dx+c)+1)}{a^4} + \frac{3\log(\sin(dx+c)-1)}{a^4}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4, x, algorithm="maxima")

[Out] -1/96*(2*(3*sin(d*x + c)^3 + 12*sin(d*x + c)^2 + 19*sin(d*x + c) + 4)/(a^4*sin(d*x + c)^4 + 4*a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 3*log(sin(d*x + c) + 1)/a^4 + 3*log(sin(d*x + c) - 1)/a^4)/

d

Fricas [B] time = 1.5909, size = 525, normalized size = 5.

$$\frac{24 \cos(dx+c)^2 + 3(\cos(dx+c)^4 - 8 \cos(dx+c)^2 - 4(\cos(dx+c)^2 - 2) \sin(dx+c) + 8) \log(\sin(dx+c) + 1) - 3(\cos(dx+c)^4 - 8 \cos(dx+c)^2 - 4(\cos(dx+c)^2 - 2) \sin(dx+c) + 8) \log(-\sin(dx+c) + 1) + 2(3 \cos(dx+c)^2 - 22) \sin(dx+c) - 32}{96(a^4 d \cos(dx+c)^4 - 8 a^4 d \cos(dx+c)^2 + 8 a^4 d - 4(a^4 d \cos(dx+c)^2 - 2 a^4 d) \sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/96*(24*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(-sin(d*x + c) + 1) + 2*(3*cos(d*x + c)^2 - 22)*sin(d*x + c) - 32)/(a^4*d*cos(d*x + c)^4 - 8*a^4*d*cos(d*x + c)^2 + 8*a^4*d - 4*(a^4*d*cos(d*x + c)^2 - 2*a^4*d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{\frac{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}{a^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**4,x)

[Out] Integral(tan(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4

Giac [A] time = 2.14057, size = 123, normalized size = 1.17

$$\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^4} - \frac{12 \log(|\sin(dx+c)-1|)}{a^4} - \frac{25 \sin(dx+c)^4 + 124 \sin(dx+c)^3 + 246 \sin(dx+c)^2 + 252 \sin(dx+c) + 57}{a^4(\sin(dx+c)+1)^4}}{384 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] 1/384*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c) - 1))/a^4 - (25*sin(d*x + c)^4 + 124*sin(d*x + c)^3 + 246*sin(d*x + c)^2 + 252*sin(d*x + c) + 57)/(a^4*(sin(d*x + c) + 1)^4))/d

$$3.85 \quad \int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=106

$$\frac{5}{d(a^4 \sin(c+dx) + a^4)} + \frac{1}{d(a^2 \sin(c+dx) + a^2)^2} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{4 \csc(c+dx)}{a^4d} + \frac{9 \log(\sin(c+dx))}{a^4d} - \frac{9 \log(\sin(c+dx))}{a^4d}$$

[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x])^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))

Rubi [A] time = 0.082628, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 77}

$$\frac{5}{d(a^4 \sin(c+dx) + a^4)} + \frac{1}{d(a^2 \sin(c+dx) + a^2)^2} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{4 \csc(c+dx)}{a^4d} + \frac{9 \log(\sin(c+dx))}{a^4d} - \frac{9 \log(\sin(c+dx))}{a^4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]

[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x])^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{(a+a \sin(c+dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^3} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{4}{a^3x^2} + \frac{9}{a^4x} - \frac{2}{a^2(a+x)^3} - \frac{5}{a^3(a+x)^2} - \frac{9}{a^4(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{4 \csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{9 \log(\sin(c+dx))}{a^4d} - \frac{9 \log(1 + \sin(c+dx))}{a^4d} + \frac{5}{d(a^2 + a^2 \sin^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.838887, size = 73, normalized size = 0.69

$$\frac{\frac{10}{\sin(c+dx)+1} + \frac{2}{(\sin(c+dx)+1)^2} - \csc^2(c+dx) + 8 \csc(c+dx) + 18 \log(\sin(c+dx)) - 18 \log(\sin(c+dx)+1)}{2a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]

[Out] (8*Csc[c + d*x] - Csc[c + d*x]^2 + 18*Log[Sin[c + d*x]] - 18*Log[1 + Sin[c + d*x]]) + 2/(1 + Sin[c + d*x])^2 + 10/(1 + Sin[c + d*x]))/(2*a^4*d)

Maple [A] time = 0.139, size = 101, normalized size = 1.

$$\frac{1}{da^4(1 + \sin(dx + c))^2} + 5 \frac{1}{da^4(1 + \sin(dx + c))} - 9 \frac{\ln(1 + \sin(dx + c))}{da^4} - \frac{1}{2da^4(\sin(dx + c))^2} + 4 \frac{1}{da^4 \sin(dx + c)} + 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x)

[Out] 1/d/a^4/(1+sin(d*x+c))^2+5/d/a^4/(1+sin(d*x+c))-9*ln(1+sin(d*x+c))/a^4/d-1/2/d/a^4/sin(d*x+c)^2+4/d/a^4/sin(d*x+c)+9*ln(sin(d*x+c))/a^4/d

Maxima [A] time = 2.61648, size = 139, normalized size = 1.31

$$\frac{\frac{18 \sin(dx+c)^3+27 \sin(dx+c)^2+6 \sin(dx+c)-1}{a^4 \sin(dx+c)^4+2a^4 \sin(dx+c)^3+a^4 \sin(dx+c)^2} - \frac{18 \log(\sin(dx+c)+1)}{a^4} + \frac{18 \log(\sin(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] 1/2*((18*sin(d*x + c)^3 + 27*sin(d*x + c)^2 + 6*sin(d*x + c) - 1)/(a^4*sin(d*x + c)^4 + 2*a^4*sin(d*x + c)^3 + a^4*sin(d*x + c)^2) - 18*log(sin(d*x + c) + 1)/a^4 + 18*log(sin(d*x + c))/a^4)/d

Fricas [A] time = 1.58272, size = 522, normalized size = 4.92

$$\frac{27 \cos(dx + c)^2 - 18 (\cos(dx + c)^4 - 3 \cos(dx + c)^2 - 2 (\cos(dx + c)^2 - 1) \sin(dx + c) + 2) \log\left(\frac{1}{2} \sin(dx + c)\right) + 18}{2(a^4d \cos(dx + c)^4 - 3a^4d \cos(dx + c)^2 - 2a^4d \cos(dx + c)^2 - 1) \sin(dx + c) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] -1/2*(27*cos(d*x + c)^2 - 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x + c) + 1) + 6*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 26)/(a^4*d*cos(d*x + c)^4 - 3*a^4*d*cos(d*x + c)^2 - 2*a^4*d*cos(d*x + c)^2 - 1)

$$4 - 3a^4 d \cos(dx + c)^2 + 2a^4 d - 2(a^4 d \cos(dx + c)^2 - a^4 d) \sin(dx + c)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**3/(a+a*sin(dx+c))**4,x)

[Out] Integral(cot(c + dx)**3/(sin(c + dx)**4 + 4*sin(c + dx)**3 + 6*sin(c + dx)**2 + 4*sin(c + dx) + 1), x)/a**4

Giac [A] time = 1.81917, size = 250, normalized size = 2.36

$$\frac{144 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^4} - \frac{72 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^4} + \frac{108 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 16 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1}{a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2} + \frac{a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 16 a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^8}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^3/(a+a*sin(dx+c))^4,x, algorithm="giac")

[Out] $-1/8*(144*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^4 - 72*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))))/a^4 + (108*\tan(1/2*d*x + 1/2*c)^2 - 16*\tan(1/2*d*x + 1/2*c) + 1)/(a^4*\tan(1/2*d*x + 1/2*c)^2) + (a^4*\tan(1/2*d*x + 1/2*c)^2 - 16*a^4*\tan(1/2*d*x + 1/2*c))/a^8 - 4*(75*\tan(1/2*d*x + 1/2*c)^4 + 272*\tan(1/2*d*x + 1/2*c)^3 + 402*\tan(1/2*d*x + 1/2*c)^2 + 272*\tan(1/2*d*x + 1/2*c) + 75)/(a^4*(\tan(1/2*d*x + 1/2*c) + 1)^4)/d$

$$3.86 \quad \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=135

$$-\frac{\csc^6(c+dx)}{6a^4d} + \frac{4 \csc^5(c+dx)}{5a^4d} - \frac{7 \csc^4(c+dx)}{4a^4d} + \frac{8 \csc^3(c+dx)}{3a^4d} - \frac{4 \csc^2(c+dx)}{a^4d} + \frac{8 \csc(c+dx)}{a^4d} + \frac{8 \log(\sin(c+dx))}{a^4d}$$

[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - Csc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c + d*x]])/(a^4*d)

Rubi [A] time = 0.0856235, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2707, 88}

$$-\frac{\csc^6(c+dx)}{6a^4d} + \frac{4 \csc^5(c+dx)}{5a^4d} - \frac{7 \csc^4(c+dx)}{4a^4d} + \frac{8 \csc^3(c+dx)}{3a^4d} - \frac{4 \csc^2(c+dx)}{a^4d} + \frac{8 \csc(c+dx)}{a^4d} + \frac{8 \log(\sin(c+dx))}{a^4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]

[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - Csc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c + d*x]])/(a^4*d)

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^7(c+dx)}{(a+a \sin(c+dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7(a+x)} dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^7} - \frac{4a}{x^6} + \frac{7}{x^5} - \frac{8}{ax^4} + \frac{8}{a^2x^3} - \frac{8}{a^3x^2} + \frac{8}{a^4x} - \frac{8}{a^4(a+x)}\right) dx, x, a \sin(c+dx)\right)}{d} \\ &= \frac{8 \csc(c+dx)}{a^4d} - \frac{4 \csc^2(c+dx)}{a^4d} + \frac{8 \csc^3(c+dx)}{3a^4d} - \frac{7 \csc^4(c+dx)}{4a^4d} + \frac{4 \csc^5(c+dx)}{5a^4d} - \frac{\csc^6(c+dx)}{6a^4d} \end{aligned}$$

Mathematica [A] time = 0.170585, size = 89, normalized size = 0.66

$$\frac{-10 \csc^6(c + dx) + 48 \csc^5(c + dx) - 105 \csc^4(c + dx) + 160 \csc^3(c + dx) - 240 \csc^2(c + dx) + 480 \csc(c + dx) + 480}{60a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]

[Out] (480*Csc[c + d*x] - 240*Csc[c + d*x]^2 + 160*Csc[c + d*x]^3 - 105*Csc[c + d*x]^4 + 48*Csc[c + d*x]^5 - 10*Csc[c + d*x]^6 + 480*Log[Sin[c + d*x]] - 480*Log[1 + Sin[c + d*x]])/(60*a^4*d)

Maple [A] time = 0.148, size = 130, normalized size = 1.

$$-8 \frac{\ln(1 + \sin(dx + c))}{a^4d} - \frac{1}{6a^4d(\sin(dx + c))^6} + \frac{4}{5a^4d(\sin(dx + c))^5} - \frac{7}{4a^4d(\sin(dx + c))^4} + \frac{8}{3a^4d(\sin(dx + c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x)

[Out] -8*ln(1+sin(d*x+c))/a^4/d-1/6/d/a^4/sin(d*x+c)^6+4/5/d/a^4/sin(d*x+c)^5-7/4/d/a^4/sin(d*x+c)^4+8/3/d/a^4/sin(d*x+c)^3-4/d/a^4/sin(d*x+c)^2+8/d/a^4/sin(d*x+c)+8*ln(sin(d*x+c))/a^4/d

Maxima [A] time = 2.68077, size = 128, normalized size = 0.95

$$\frac{\frac{480 \log(\sin(dx+c)+1)}{a^4} - \frac{480 \log(\sin(dx+c))}{a^4} - \frac{480 \sin(dx+c)^5 - 240 \sin(dx+c)^4 + 160 \sin(dx+c)^3 - 105 \sin(dx+c)^2 + 48 \sin(dx+c) - 10}{a^4 \sin(dx+c)^6}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] -1/60*(480*log(sin(d*x + c) + 1)/a^4 - 480*log(sin(d*x + c))/a^4 - (480*sin(d*x + c)^5 - 240*sin(d*x + c)^4 + 160*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 48*sin(d*x + c) - 10)/(a^4*sin(d*x + c)^6))/d

Fricas [A] time = 1.59832, size = 502, normalized size = 3.72

$$\frac{240 \cos(dx + c)^4 - 585 \cos(dx + c)^2 + 480 (\cos(dx + c)^6 - 3 \cos(dx + c)^4 + 3 \cos(dx + c)^2 - 1) \log\left(\frac{1}{2} \sin(dx + c)\right) - 480}{60(a^4d \cos(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/60*(240*cos(d*x + c)^4 - 585*cos(d*x + c)^2 + 480*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 480*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/60*a^4*d*cos(dx + c)

$$c)^6 - 3\cos(dx + c)^4 + 3\cos(dx + c)^2 - 1) \cdot \log(\sin(dx + c) + 1) - 16 \cdot (30\cos(dx + c)^4 - 70\cos(dx + c)^2 + 43) \cdot \sin(dx + c) + 355) / (a^4 \cdot d \cdot \cos(dx + c)^6 - 3a^4 \cdot d \cdot \cos(dx + c)^4 + 3a^4 \cdot d \cdot \cos(dx + c)^2 - a^4 \cdot d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**7/(a+a*sin(dx+c))**4,x)

[Out] Timed out

Giac [A] time = 1.93045, size = 313, normalized size = 2.32

$$\frac{30720 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^4} - \frac{15360 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^4} + \frac{37632 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 10080 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 2835 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 880 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 240 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 48 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 5}{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^7/(a+a*sin(dx+c))^4,x, algorithm="giac")

[Out]
$$-1/1920 \cdot (30720 \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) + 1)) / a^4 - 15360 \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c))) / a^4 + (37632 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^6 - 10080 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 2835 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^4 - 880 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 240 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 48 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 5) / (a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^6) + (5 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^6 - 48 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 240 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^4 - 880 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 2835 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 10080 \cdot a^{20} \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / a^{24} / d$$

$$3.87 \quad \int \frac{\tan^2(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=127

$$\frac{8 \tan^9(c+dx)}{9a^4d} + \frac{16 \tan^7(c+dx)}{7a^4d} + \frac{9 \tan^5(c+dx)}{5a^4d} + \frac{\tan^3(c+dx)}{3a^4d} - \frac{8 \sec^9(c+dx)}{9a^4d} + \frac{12 \sec^7(c+dx)}{7a^4d} - \frac{4 \sec^5(c+dx)}{5a^4d}$$

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
+ (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rubi [A] time = 0.309682, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{8 \tan^9(c+dx)}{9a^4d} + \frac{16 \tan^7(c+dx)}{7a^4d} + \frac{9 \tan^5(c+dx)}{5a^4d} + \frac{\tan^3(c+dx)}{3a^4d} - \frac{8 \sec^9(c+dx)}{9a^4d} + \frac{12 \sec^7(c+dx)}{7a^4d} - \frac{4 \sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
+ (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rule 2711

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x
_)])^(p_), x_Symbol] := Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_S
ymbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(
n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4} dx &= \frac{\int (a^4 \sec^8(c+dx) \tan^2(c+dx) - 4a^4 \sec^7(c+dx) \tan^3(c+dx) + 6a^4 \sec^6(c+dx) \tan^4(c+dx) - 4a^4 \sec^5(c+dx) \tan^5(c+dx) + 2a^4 \sec^4(c+dx) \tan^6(c+dx) - 2a^4 \sec^3(c+dx) \tan^7(c+dx) + a^4 \sec^2(c+dx) \tan^8(c+dx)) dx}{a^8} \\ &= \frac{\int \sec^8(c+dx) \tan^2(c+dx) dx}{a^4} + \frac{\int \sec^4(c+dx) \tan^6(c+dx) dx}{a^4} - \frac{4 \int \sec^7(c+dx) \tan^3(c+dx) dx}{a^4} + \frac{6 \int \sec^6(c+dx) \tan^4(c+dx) dx}{a^4} - \frac{4 \int \sec^5(c+dx) \tan^5(c+dx) dx}{a^4} + \frac{2 \int \sec^4(c+dx) \tan^6(c+dx) dx}{a^4} - \frac{2 \int \sec^3(c+dx) \tan^7(c+dx) dx}{a^4} + \frac{\int \sec^2(c+dx) \tan^8(c+dx) dx}{a^4} \\ &= \frac{\text{Subst}\left(\int x^6(1+x^2) dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int x^2(1+x^2)^3 dx, x, \tan(c+dx)\right)}{a^4 d} - \frac{4 \text{Subst}\left(\int x^7(1+x^2) dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{6 \text{Subst}\left(\int x^4(1+x^2)^2 dx, x, \tan(c+dx)\right)}{a^4 d} - \frac{4 \text{Subst}\left(\int x^5(1+x^2) dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{2 \text{Subst}\left(\int x^2(1+x^2)^3 dx, x, \tan(c+dx)\right)}{a^4 d} - \frac{2 \text{Subst}\left(\int x^3(1+x^2) dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int (x^2+3x^4+3x^6+x^8) dx, x, \tan(c+dx)\right)}{a^4 d} \\ &= -\frac{4 \sec^5(c+dx)}{5a^4 d} + \frac{12 \sec^7(c+dx)}{7a^4 d} - \frac{8 \sec^9(c+dx)}{9a^4 d} + \frac{\tan^3(c+dx)}{3a^4 d} + \frac{9 \tan^5(c+dx)}{5a^4 d} + \frac{16 \tan^7(c+dx)}{7a^4 d} - \frac{2 \tan^9(c+dx)}{9a^4 d} + \frac{2 \tan^{11}(c+dx)}{11a^4 d} \end{aligned}$$

Mathematica [A] time = 0.415533, size = 124, normalized size = 0.98

$$\frac{\sec(c+dx)(34944 \sin(c+dx) + 1776 \sin(2(c+dx)) - 9504 \sin(3(c+dx)) - 296 \sin(4(c+dx)) + 352 \sin(5(c+dx)) + 15 \sin(6(c+dx)) - 16 \sin(7(c+dx)) + 8 \sin(8(c+dx)) - 8 \sin(9(c+dx)) + 8 \sin(10(c+dx)) - 8 \sin(11(c+dx)) + 8 \sin(12(c+dx)) - 8 \sin(13(c+dx)) + 8 \sin(14(c+dx)) - 8 \sin(15(c+dx)) + 8 \sin(16(c+dx)) - 8 \sin(17(c+dx)) + 8 \sin(18(c+dx)) - 8 \sin(19(c+dx)) + 8 \sin(20(c+dx)) - 8 \sin(21(c+dx)) + 8 \sin(22(c+dx)) - 8 \sin(23(c+dx)) + 8 \sin(24(c+dx)) - 8 \sin(25(c+dx)) + 8 \sin(26(c+dx)) - 8 \sin(27(c+dx)) + 8 \sin(28(c+dx)) - 8 \sin(29(c+dx)) + 8 \sin(30(c+dx)) - 8 \sin(31(c+dx)) + 8 \sin(32(c+dx)) - 8 \sin(33(c+dx)) + 8 \sin(34(c+dx)) - 8 \sin(35(c+dx)) + 8 \sin(36(c+dx)) - 8 \sin(37(c+dx)) + 8 \sin(38(c+dx)) - 8 \sin(39(c+dx)) + 8 \sin(40(c+dx)) - 8 \sin(41(c+dx)) + 8 \sin(42(c+dx)) - 8 \sin(43(c+dx)) + 8 \sin(44(c+dx)) - 8 \sin(45(c+dx)) + 8 \sin(46(c+dx)) - 8 \sin(47(c+dx)) + 8 \sin(48(c+dx)) - 8 \sin(49(c+dx)) + 8 \sin(50(c+dx)) - 8 \sin(51(c+dx)) + 8 \sin(52(c+dx)) - 8 \sin(53(c+dx)) + 8 \sin(54(c+dx)) - 8 \sin(55(c+dx)) + 8 \sin(56(c+dx)) - 8 \sin(57(c+dx)) + 8 \sin(58(c+dx)) - 8 \sin(59(c+dx)) + 8 \sin(60(c+dx)) - 8 \sin(61(c+dx)) + 8 \sin(62(c+dx)) - 8 \sin(63(c+dx)) + 8 \sin(64(c+dx)) - 8 \sin(65(c+dx)) + 8 \sin(66(c+dx)) - 8 \sin(67(c+dx)) + 8 \sin(68(c+dx)) - 8 \sin(69(c+dx)) + 8 \sin(70(c+dx)) - 8 \sin(71(c+dx)) + 8 \sin(72(c+dx)) - 8 \sin(73(c+dx)) + 8 \sin(74(c+dx)) - 8 \sin(75(c+dx)) + 8 \sin(76(c+dx)) - 8 \sin(77(c+dx)) + 8 \sin(78(c+dx)) - 8 \sin(79(c+dx)) + 8 \sin(80(c+dx)) - 8 \sin(81(c+dx)) + 8 \sin(82(c+dx)) - 8 \sin(83(c+dx)) + 8 \sin(84(c+dx)) - 8 \sin(85(c+dx)) + 8 \sin(86(c+dx)) - 8 \sin(87(c+dx)) + 8 \sin(88(c+dx)) - 8 \sin(89(c+dx)) + 8 \sin(90(c+dx)) - 8 \sin(91(c+dx)) + 8 \sin(92(c+dx)) - 8 \sin(93(c+dx)) + 8 \sin(94(c+dx)) - 8 \sin(95(c+dx)) + 8 \sin(96(c+dx)) - 8 \sin(97(c+dx)) + 8 \sin(98(c+dx)) - 8 \sin(99(c+dx)) + 8 \sin(100(c+dx))}{80640a^4 d (\sin(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4, x]

[Out] (Sec[c + d*x]*(16128 + 1554*Cos[c + d*x] - 16896*Cos[2*(c + d*x)] - 999*Cos[3*(c + d*x)] + 2816*Cos[4*(c + d*x)] + 37*Cos[5*(c + d*x)] + 34944*Sin[c + d*x] + 1776*Sin[2*(c + d*x)] - 9504*Sin[3*(c + d*x)] - 296*Sin[4*(c + d*x)] + 352*Sin[5*(c + d*x)])/(80640*a^4*d*(1 + Sin[c + d*x])^4)

Maple [A] time = 0.076, size = 158, normalized size = 1.2

$$8 \frac{1}{da^4} \left(-\frac{1}{128 \tan(1/2 dx + c/2) - 128} - \frac{2}{9} (\tan(1/2 dx + c/2) + 1)^{-9} + (\tan(1/2 dx + c/2) + 1)^{-8} - \frac{29}{14 (\tan(1/2 dx + c/2) + 1)^7} + \frac{31}{12 (\tan(1/2 dx + c/2) + 1)^6} - \frac{83}{40 (\tan(1/2 dx + c/2) + 1)^5} + \frac{17}{16 (\tan(1/2 dx + c/2) + 1)^4} - \frac{29}{96 (\tan(1/2 dx + c/2) + 1)^3} + \frac{1}{64 (\tan(1/2 dx + c/2) + 1)^2} + \frac{1}{128 (\tan(1/2 dx + c/2) + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c))^4, x)

[Out] 8/d/a^4*(-1/128/(tan(1/2*d*x+1/2*c)-1)-2/9/(tan(1/2*d*x+1/2*c)+1)^9+1/(tan(1/2*d*x+1/2*c)+1)^8-29/14/(tan(1/2*d*x+1/2*c)+1)^7+31/12/(tan(1/2*d*x+1/2*c)+1)^6-83/40/(tan(1/2*d*x+1/2*c)+1)^5+17/16/(tan(1/2*d*x+1/2*c)+1)^4-29/96/(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] time = 3.08174, size = 481, normalized size = 3.79

$$8 \left(\frac{16 \sin(dx+c)}{\cos(dx+c)+1} + \frac{54 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{201 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{294 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{210 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{105 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{1 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} \right) \frac{1}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out]
$$\frac{8}{315} \cdot \frac{16 \sin(dx+c)}{(\cos(dx+c)+1)} + \frac{54 \sin^2(dx+c)}{(\cos(dx+c)+1)^2} + \frac{201 \sin^3(dx+c)}{(\cos(dx+c)+1)^3} + \frac{294 \sin^4(dx+c)}{(\cos(dx+c)+1)^4} + \frac{378 \sin^5(dx+c)}{(\cos(dx+c)+1)^5} + \frac{210 \sin^6(dx+c)}{(\cos(dx+c)+1)^6} + \frac{105 \sin^7(dx+c)}{(\cos(dx+c)+1)^7} + \frac{2}{((a^4 + 8a^4 \sin(dx+c))/(\cos(dx+c)+1) + 27a^4 \sin^2(dx+c)/(\cos(dx+c)+1)^2 + 48a^4 \sin^3(dx+c)/(\cos(dx+c)+1)^3 + 42a^4 \sin^4(dx+c)/(\cos(dx+c)+1)^4 - 42a^4 \sin^6(dx+c)/(\cos(dx+c)+1)^6 - 48a^4 \sin^7(dx+c)/(\cos(dx+c)+1)^7 - 27a^4 \sin^8(dx+c)/(\cos(dx+c)+1)^8 - 8a^4 \sin^9(dx+c)/(\cos(dx+c)+1)^9 - a^4 \sin^{10}(dx+c)/(\cos(dx+c)+1)^{10}) \cdot d}$$

Fricas [A] time = 1.49638, size = 338, normalized size = 2.66

$$\frac{88 \cos(dx+c)^4 - 220 \cos(dx+c)^2 + (22 \cos(dx+c)^4 - 165 \cos(dx+c)^2 + 175) \sin(dx+c) + 140}{315 (a^4 d \cos(dx+c)^5 - 8a^4 d \cos(dx+c)^3 + 8a^4 d \cos(dx+c) - 4(a^4 d \cos(dx+c)^3 - 2a^4 d \cos(dx+c)) \sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\frac{1}{315} \cdot \frac{(88 \cos^4(dx+c) - 220 \cos^2(dx+c) + (22 \cos^4(dx+c) - 165 \cos^2(dx+c) + 175) \sin(dx+c) + 140) / (a^4 d \cos^5(dx+c) - 8a^4 d \cos^3(dx+c) + 8a^4 d \cos(dx+c) - 4(a^4 d \cos^3(dx+c) - 2a^4 d \cos(dx+c)) \sin(dx+c))}{a^4}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c))**4,x)

[Out]
$$\text{Integral}(\tan(c+dx)**2/(\sin(c+dx)**4+4*\sin(c+dx)**3+6*\sin(c+dx)**2+4*\sin(c+dx)+1),x)/a**4$$

Giac [A] time = 2.53876, size = 197, normalized size = 1.55

$$\frac{315}{a^4 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right)} - \frac{315 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 3150 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 1050 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 630 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 8064 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 6006 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 1008 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1008 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{5040 d a^4 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")

```
[Out] -1/5040*(315/(a^4*(tan(1/2*d*x + 1/2*c) - 1)) - (315*tan(1/2*d*x + 1/2*c)^8
+ 3150*tan(1/2*d*x + 1/2*c)^7 + 1050*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1/2*
d*x + 1/2*c)^5 - 8064*tan(1/2*d*x + 1/2*c)^4 - 6006*tan(1/2*d*x + 1/2*c)^3
- 5274*tan(1/2*d*x + 1/2*c)^2 - 846*tan(1/2*d*x + 1/2*c) - 59)/(a^4*(tan(1/
2*d*x + 1/2*c) + 1)^9))/d
```

$$3.88 \quad \int \frac{\cot^2(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=108

$$-\frac{\cot(c+dx)}{a^4d} + \frac{4 \tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{104 \cot(c+dx)}{15a^4d(\csc(c+dx)+1)} + \frac{31 \cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2} - \frac{2 \cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))

Rubi [A] time = 0.317935, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{\cot(c+dx)}{a^4d} + \frac{4 \tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{104 \cot(c+dx)}{15a^4d(\csc(c+dx)+1)} + \frac{31 \cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2} - \frac{2 \cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]

[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3777

Int[(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege

rQ[2*n]

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c + dx)}{(a + a \sin(c + dx))^4} dx = \frac{\int \left(\frac{9}{a^2} - \frac{4 \csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2} - \frac{16}{a^2(1+\csc(c+dx))} \right) dx}{a^2}$$

$$= \frac{9x}{a^4} + \frac{\int \csc^2(c + dx) dx}{a^4} - \frac{2 \int \frac{1}{(1+\csc(c+dx))^3} dx}{a^4} - \frac{4 \int \csc(c + dx) dx}{a^4} + \frac{9 \int \frac{1}{(1+\csc(c+dx))^2} dx}{a^4} - \frac{16 \int \frac{1}{1+\csc(c+dx)} dx}{a^4}$$

$$= \frac{9x}{a^4} + \frac{4 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{2 \cot(c + dx)}{5a^4 d(1 + \csc(c + dx))^3} + \frac{3 \cot(c + dx)}{a^4 d(1 + \csc(c + dx))^2} - \frac{16 \cot(c + dx)}{a^4 d(1 + \csc(c + dx))}$$

$$= \frac{2x}{a^4} + \frac{4 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{\cot(c + dx)}{a^4 d} - \frac{2 \cot(c + dx)}{5a^4 d(1 + \csc(c + dx))^3} + \frac{31 \cot(c + dx)}{15a^4 d(1 + \csc(c + dx))^2} - \frac{16 \cot(c + dx)}{a^4 d(1 + \csc(c + dx))}$$

$$= \frac{4 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{\cot(c + dx)}{a^4 d} - \frac{2 \cot(c + dx)}{5a^4 d(1 + \csc(c + dx))^3} + \frac{31 \cot(c + dx)}{15a^4 d(1 + \csc(c + dx))^2} - \frac{16 \cot(c + dx)}{a^4 d(1 + \csc(c + dx))}$$

$$= \frac{4 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{\cot(c + dx)}{a^4 d} - \frac{2 \cot(c + dx)}{5a^4 d(1 + \csc(c + dx))^3} + \frac{31 \cot(c + dx)}{15a^4 d(1 + \csc(c + dx))^2} - \frac{16 \cot(c + dx)}{a^4 d(1 + \csc(c + dx))}$$

Mathematica [B] time = 0.41973, size = 315, normalized size = 2.92

$$\frac{\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^3 \left(24 \sin\left(\frac{1}{2}(c + dx)\right) + 316 \sin\left(\frac{1}{2}(c + dx)\right) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)^4 - 38 \right)}{a^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(24*Sin[(c + d*x)/2] - 12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 76*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 38*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 316*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 38)
```

$$\frac{+ d*x)/2]*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^4 - 15*\text{Cot}[(c + d*x)/2]*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^5 + 120*\text{Log}[\text{Cos}[(c + d*x)/2]]*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^5 - 120*\text{Log}[\text{Sin}[(c + d*x)/2]]*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^5 + 15*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^5*\text{Tan}[(c + d*x)/2]))/(30*d*(a + a*\text{Sin}[c + d*x])^4}$$

Maple [A] time = 0.125, size = 161, normalized size = 1.5

$$\frac{1}{2da^4} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{16}{5da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^{-5} + 8 \frac{1}{da^4 (\tan(1/2 dx + c/2) + 1)^4} - \frac{44}{3da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^{-3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x)

[Out] 1/2/d/a^4*tan(1/2*d*x+1/2*c)-16/5/d/a^4/(tan(1/2*d*x+1/2*c)+1)^5+8/d/a^4/(tan(1/2*d*x+1/2*c)+1)^4-44/3/d/a^4/(tan(1/2*d*x+1/2*c)+1)^3+14/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2-18/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4/tan(1/2*d*x+1/2*c)-4/d/a^4*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 3.72867, size = 389, normalized size = 3.6

$$\frac{\frac{491 \sin(dx+c)}{\cos(dx+c)+1} + \frac{1690 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2570 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1815 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{555 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 15}{\frac{a^4 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^4 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{10a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{5a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4} - \frac{15 \sin(dx+c)}{a^4(\cos(dx+c)+1)}$$

30 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] -1/30*((491*sin(d*x + c)/(cos(d*x + c) + 1) + 1690*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2570*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1815*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 555*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15)/(a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 5*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4 - 15*sin(d*x + c)/(a^4*(cos(d*x + c) + 1)))/d

Fricas [B] time = 1.59742, size = 992, normalized size = 9.19

$$94 \cos(dx + c)^4 + 222 \cos(dx + c)^3 - 115 \cos(dx + c)^2 + 30(\cos(dx + c)^4 - 2 \cos(dx + c)^3 - 5 \cos(dx + c)^2 - (\cos(dx + c)^3 + 3 \cos(dx + c)^2 + 3 \cos(dx + c) + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/15*(94*cos(d*x + c)^4 + 222*cos(d*x + c)^3 - 115*cos(d*x + c)^2 + 30*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos(dx + c)^2 + 3*cos(dx + c) + 1)))

$$(d*x + c)^2 - 2*\cos(d*x + c) - 4)*\sin(d*x + c) + 2*\cos(d*x + c) + 4)*\log(1/2*\cos(d*x + c) + 1/2) - 30*(\cos(d*x + c)^4 - 2*\cos(d*x + c)^3 - 5*\cos(d*x + c)^2 - (\cos(d*x + c)^3 + 3*\cos(d*x + c)^2 - 2*\cos(d*x + c) - 4)*\sin(d*x + c) + 2*\cos(d*x + c) + 4)*\log(-1/2*\cos(d*x + c) + 1/2) + (94*\cos(d*x + c)^3 - 128*\cos(d*x + c)^2 - 243*\cos(d*x + c) - 6)*\sin(d*x + c) - 237*\cos(d*x + c) + 6)/(a^4*d*\cos(d*x + c)^4 - 2*a^4*d*\cos(d*x + c)^3 - 5*a^4*d*\cos(d*x + c)^2 + 2*a^4*d*\cos(d*x + c) + 4*a^4*d - (a^4*d*\cos(d*x + c)^3 + 3*a^4*d*\cos(d*x + c)^2 - 2*a^4*d*\cos(d*x + c) - 4*a^4*d)*\sin(d*x + c))$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c))**4,x)

[Out] Integral(cot(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4

Giac [A] time = 1.94505, size = 182, normalized size = 1.69

$$\frac{120 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^4} - \frac{15 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^4} - \frac{15\left(8 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right)}{a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)} + \frac{4\left(135 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 435 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 605 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 385 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 104\right)}{a^4 \left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^5}$$

30 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] -1/30*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 15*tan(1/2*d*x + 1/2*c)/a^4 - 15*(8*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)) + 4*(135*tan(1/2*d*x + 1/2*c)^4 + 435*tan(1/2*d*x + 1/2*c)^3 + 605*tan(1/2*d*x + 1/2*c)^2 + 385*tan(1/2*d*x + 1/2*c) + 104)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

$$3.89 \quad \int \frac{\cot^4(c+dx)}{(a+a \sin(c+dx))^4} dx$$

Optimal. Leaf size=120

$$-\frac{\cot^3(c+dx)}{3a^4d} - \frac{9 \cot(c+dx)}{a^4d} + \frac{14 \tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{2 \cot(c+dx) \csc(c+dx)}{a^4d} - \frac{44 \cot(c+dx)}{3a^4d(\csc(c+dx)+1)} + \frac{1}{3a^4d}$$

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.248752, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3770, 3767, 8, 3768, 3777, 3919, 3794}

$$-\frac{\cot^3(c+dx)}{3a^4d} - \frac{9 \cot(c+dx)}{a^4d} + \frac{14 \tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{2 \cot(c+dx) \csc(c+dx)}{a^4d} - \frac{44 \cot(c+dx)}{3a^4d(\csc(c+dx)+1)} + \frac{1}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]))
```

Rule 2709

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])
```

Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

IntegerQ[2*n]

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c + dx)}{(a + a \sin(c + dx))^4} dx = \frac{\int (16 - 12 \csc(c + dx) + 8 \csc^2(c + dx) - 4 \csc^3(c + dx) + \csc^4(c + dx) + \frac{4}{(1 + \csc(c + dx))^2} - \frac{4}{1 + \csc(c + dx)}) dx}{a^4}$$

$$= \frac{16x}{a^4} + \frac{\int \csc^4(c + dx) dx}{a^4} - \frac{4 \int \csc^3(c + dx) dx}{a^4} + \frac{4 \int \frac{1}{(1 + \csc(c + dx))^2} dx}{a^4} + \frac{8 \int \csc^2(c + dx) dx}{a^4}$$

$$= \frac{16x}{a^4} + \frac{12 \tanh^{-1}(\cos(c + dx))}{a^4 d} + \frac{2 \cot(c + dx) \csc(c + dx)}{a^4 d} + \frac{4 \cot(c + dx)}{3a^4 d (1 + \csc(c + dx))^2} - \frac{2 \cot(c + dx)}{a^4 d}$$

$$= \frac{14 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{9 \cot(c + dx)}{a^4 d} - \frac{\cot^3(c + dx)}{3a^4 d} + \frac{2 \cot(c + dx) \csc(c + dx)}{a^4 d} + \frac{4 \cot(c + dx)}{3a^4 d (1 + \csc(c + dx))^2}$$

$$= \frac{14 \tanh^{-1}(\cos(c + dx))}{a^4 d} - \frac{9 \cot(c + dx)}{a^4 d} - \frac{\cot^3(c + dx)}{3a^4 d} + \frac{2 \cot(c + dx) \csc(c + dx)}{a^4 d} + \frac{4 \cot(c + dx)}{3a^4 d (1 + \csc(c + dx))^2}$$

Mathematica [B] time = 6.0893, size = 589, normalized size = 4.91

$$\frac{80 \sin\left(\frac{1}{2}(c + dx)\right) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^7}{3d(a \sin(c + dx) + a)^4} - \frac{4 \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^6}{3d(a \sin(c + dx) + a)^4} + \frac{8 \sin\left(\frac{1}{2}(c + dx)\right) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^5}{3d(a \sin(c + dx) + a)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]

```
[Out] (8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(3*d*(a + a*Sin[c + d*x])^4) - (4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(3*d*(a + a*Sin[c + d*x])^4) + (80*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(3*d*(a + a*Sin[c + d*x])^4) - (13*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(3*d*(a + a*Sin[c + d*x])^4) + (Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2
```


$$4*d*(a + a*\sin[c + d*x])^4 + (14*\log[\cos[(c + d*x)/2]]*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^8)/(d*(a + a*\sin[c + d*x])^4) - (14*\log[\sin[(c + d*x)/2]]*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^8)/(d*(a + a*\sin[c + d*x])^4) - (\sec[(c + d*x)/2]^2*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^8)/(2*d*(a + a*\sin[c + d*x])^4) + (13*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^8*\tan[(c + d*x)/2])/(3*d*(a + a*\sin[c + d*x])^4) + (\sec[(c + d*x)/2]^2*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^8*\tan[(c + d*x)/2])/(24*d*(a + a*\sin[c + d*x])^4)$$

Maple [A] time = 0.138, size = 195, normalized size = 1.6

$$\frac{1}{24da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 - \frac{1}{2da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{35}{8da^4} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{16}{3da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} + 8 \frac{1}{da^4} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x)

[Out] 1/24/d/a^4*tan(1/2*d*x+1/2*c)^3-1/2/d/a^4*tan(1/2*d*x+1/2*c)^2+35/8/d/a^4*tan(1/2*d*x+1/2*c)-16/3/d/a^4/(tan(1/2*d*x+1/2*c)+1)^3+8/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2-32/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/24/d/a^4/tan(1/2*d*x+1/2*c)^3+1/2/d/a^4/tan(1/2*d*x+1/2*c)^2-35/8/d/a^4/tan(1/2*d*x+1/2*c)-14/d/a^4*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 3.06899, size = 385, normalized size = 3.21

$$\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{72 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{984 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1647 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{873 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1}{\frac{a^4 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{3a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{12 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^4} - \frac{336 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}$$

24d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 72*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 984*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1647*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 873*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (105*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^4 - 336*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4/d

Fricas [B] time = 1.58112, size = 1187, normalized size = 9.89

$$66 \cos(dx + c)^5 - 24 \cos(dx + c)^4 - 147 \cos(dx + c)^3 + 29 \cos(dx + c)^2 - 21 (\cos(dx + c)^5 + 2 \cos(dx + c)^4 - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

```
[Out] -1/3*(66*cos(d*x + c)^5 - 24*cos(d*x + c)^4 - 147*cos(d*x + c)^3 + 29*cos(d*x + c)^2 - 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) - (66*cos(d*x + c)^4 + 90*cos(d*x + c)^3 - 57*cos(d*x + c)^2 - 86*cos(d*x + c) - 4)*sin(d*x + c) + 82*cos(d*x + c) - 4)/(a^4*d*cos(d*x + c)^5 + 2*a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d + (a^4*d*cos(d*x + c)^4 - a^4*d*cos(d*x + c)^3 - 3*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 2.40079, size = 242, normalized size = 2.02

$$\frac{336 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^4} - \frac{308 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 51 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 723 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 676 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 72 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 9 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^3 a^4}$$

24 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/24*(336*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - (308*tan(1/2*d*x + 1/2*c)^6 + 51*tan(1/2*d*x + 1/2*c)^5 - 723*tan(1/2*d*x + 1/2*c)^4 - 676*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^3*a^4) - (a^8*tan(1/2*d*x + 1/2*c)^3 - 12*a^8*tan(1/2*d*x + 1/2*c)^2 + 105*a^8*tan(1/2*d*x + 1/2*c))/a^12)/d
```

3.90 $\int \frac{\cot^6(c+dx)}{(a+a \sin(c+dx))^4} dx$

Optimal. Leaf size=133

$$-\frac{\cot^5(c+dx)}{5a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{16\cot(c+dx)}{a^4d} + \frac{27 \tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\cot(c+dx) \csc^3(c+dx)}{a^4d} + \frac{11\cot(c+dx)}{2a^4d}$$

```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a^4*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.249307, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 3770, 3767, 8, 3768, 3777}

$$-\frac{\cot^5(c+dx)}{5a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{16\cot(c+dx)}{a^4d} + \frac{27 \tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\cot(c+dx) \csc^3(c+dx)}{a^4d} + \frac{11\cot(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a^4*d*(1 + Csc[c + d*x]))
```

Rule 2709

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])
```

Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

IntegerQ[2*n]

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4} dx = \frac{\int (8a^2 - 8a^2 \csc(c+dx) + 8a^2 \csc^2(c+dx) - 8a^2 \csc^3(c+dx) + 7a^2 \csc^4(c+dx) - 4a^2 \csc^5(c+dx)) dx}{a^6}$$

$$= \frac{8x}{a^4} + \frac{\int \csc^6(c+dx) dx}{a^4} - \frac{4 \int \csc^5(c+dx) dx}{a^4} + \frac{7 \int \csc^4(c+dx) dx}{a^4} - \frac{8 \int \csc(c+dx) dx}{a^4} + \dots$$

$$= \frac{8x}{a^4} + \frac{8 \tanh^{-1}(\cos(c+dx))}{a^4 d} + \frac{4 \cot(c+dx) \csc(c+dx)}{a^4 d} + \frac{\cot(c+dx) \csc^3(c+dx)}{a^4 d} - \frac{8}{a^4 d} \dots$$

$$= \frac{12 \tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{16 \cot(c+dx)}{a^4 d} - \frac{3 \cot^3(c+dx)}{a^4 d} - \frac{\cot^5(c+dx)}{5a^4 d} + \frac{11 \cot(c+dx)}{2a^4 d} \dots$$

$$= \frac{27 \tanh^{-1}(\cos(c+dx))}{2a^4 d} - \frac{16 \cot(c+dx)}{a^4 d} - \frac{3 \cot^3(c+dx)}{a^4 d} - \frac{\cot^5(c+dx)}{5a^4 d} + \frac{11 \cot(c+dx)}{2a^4 d} \dots$$

Mathematica [B] time = 6.14507, size = 733, normalized size = 5.51

$$\frac{16 \sin\left(\frac{1}{2}(c+dx)\right) \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)^7}{d(a \sin(c+dx) + a)^4} + \frac{27 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)^8}{2d(a \sin(c+dx) + a)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]

```
[Out] (16*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(d*(a + a*Sin
[c + d*x])^4) - (33*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
8)/(5*d*(a + a*Sin[c + d*x])^4) + (11*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (53*Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*S
in[c + d*x])^4) + (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^8)/(16*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*Sin[c + d*x])^4) + (27
*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a
*Sin[c + d*x])^4) - (27*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (11*Sec[(c + d*x)/2]^2*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(16*d*(a + a*Sin[c + d*x])
^4) + (33*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(5*d*(a
+ a*Sin[c + d*x])^4) + (53*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a
+ a*Sin[c + d*x])^4)
```

Maple [A] time = 0.149, size = 229, normalized size = 1.7

$$\frac{1}{160da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^5 - \frac{1}{16da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 + \frac{11}{32da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 - \frac{3}{2da^4} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{111}{16da^4} \tan\left(\frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x)

[Out] 1/160/d/a^4*tan(1/2*d*x+1/2*c)^5-1/16/d/a^4*tan(1/2*d*x+1/2*c)^4+11/32/d/a^4*tan(1/2*d*x+1/2*c)^3-3/2/d/a^4*tan(1/2*d*x+1/2*c)^2+111/16/d/a^4*tan(1/2*d*x+1/2*c)-16/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/160/d/a^4/tan(1/2*d*x+1/2*c)^5+1/16/d/a^4/tan(1/2*d*x+1/2*c)^4-11/32/d/a^4/tan(1/2*d*x+1/2*c)^3+3/2/d/a^4/tan(1/2*d*x+1/2*c)^2-111/16/d/a^4/tan(1/2*d*x+1/2*c)-27/2/d/a^4*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 3.61884, size = 377, normalized size = 2.83

$$\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{45 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{185 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{870 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{3670 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1}{\frac{a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{1110 \sin(dx+c)}{\cos(dx+c)+1} - \frac{240 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{55 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{10 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^4}$$

160 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] 1/160*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 185*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 870*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3670*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (1110*sin(d*x + c)/(cos(d*x + c) + 1) - 240*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 55*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^4 - 2160*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d

Fricas [B] time = 1.78661, size = 1191, normalized size = 8.95

$$424 \cos(dx+c)^6 + 154 \cos(dx+c)^5 - 1060 \cos(dx+c)^4 - 340 \cos(dx+c)^3 + 800 \cos(dx+c)^2 + 135 (\cos(dx+c) - 1) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 135 (\cos(dx+c) - 1) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 2(212 \cos(dx+c)^5 + 135 \cos(dx+c)^4 - 395 \cos(dx+c)^3 - 225 \cos(dx+c)^2 + 17$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/20*(424*cos(d*x + c)^6 + 154*cos(d*x + c)^5 - 1060*cos(d*x + c)^4 - 340*cos(d*x + c)^3 + 800*cos(d*x + c)^2 + 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) - 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(212*cos(d*x + c)^5 + 135*cos(d*x + c)^4 - 395*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 17

5*cos(d*x + c) + 80)*sin(d*x + c) + 190*cos(d*x + c) - 160)/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*cos(d*x + c)^5 + a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c) + a^4*d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.73458, size = 275, normalized size = 2.07

$$\frac{2160 \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{a^4} + \frac{2560}{a^4\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)} - \frac{4932 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 1110 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 240 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 55 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 10 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1}{a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5}$$

160d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] -1/160*(2160*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 2560/(a^4*(tan(1/2*d*x + 1/2*c) + 1)) - (4932*tan(1/2*d*x + 1/2*c)^5 - 1110*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1/2*c)^3 - 55*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)^5) - (a^16*tan(1/2*d*x + 1/2*c)^5 - 10*a^16*tan(1/2*d*x + 1/2*c)^4 + 55*a^16*tan(1/2*d*x + 1/2*c)^3 - 240*a^16*tan(1/2*d*x + 1/2*c)^2 + 1110*a^16*tan(1/2*d*x + 1/2*c))/a^20)/d

3.91 $\int \sqrt{a + a \sin(e + fx)} \tan^4(e + fx) dx$

Optimal. Leaf size=162

$$\frac{5 \tan^3(e + fx) \sqrt{a(\sin(e + fx) + 1)}}{12f} + \frac{29 \tan(e + fx) \sqrt{a \sin(e + fx) + a}}{12f} - \frac{\sec^3(e + fx) \sqrt{a(\sin(e + fx) + 1)}}{12f} - \frac{27 \sec(e + fx) \sqrt{a \sin(e + fx) + a}}{12f}$$

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) - (27*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])])/(8*f) - (
Sec[e + f*x]^3*Sqrt[a*(1 + Sin[e + f*x])])/(12*f) + (29*Sqrt[a + a*Sin[e +
f*x]]*Tan[e + f*x])/(12*f) + (5*Sqrt[a*(1 + Sin[e + f*x])]*Tan[e + f*x]^3)/
(12*f)
```

Rubi [A] time = 0.923052, antiderivative size = 195, normalized size of antiderivative = 1.2, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2714, 2646, 4401, 2675, 2687, 2650, 2649, 206, 2878, 2855}

$$\frac{11a^2 \cos(e + fx)}{8f(a \sin(e + fx) + a)^{3/2}} - \frac{2a \cos(e + fx)}{f \sqrt{a \sin(e + fx) + a}} + \frac{4 \sec^3(e + fx)(a \sin(e + fx) + a)^{3/2}}{3af} - \frac{7 \sec^3(e + fx) \sqrt{a \sin(e + fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) + (11*a^2*Cos[e + f*x])/(8*f*(a + a*Sin[e + f*x])^(3/2))
- (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (11*a*Sec[e + f*x])/(6
*f*Sqrt[a + a*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/
(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*a*f)
```

Rule 2714

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^4,
x_Symbol] := Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]
```

Rule 2675

```
Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x
_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
```

f, g, x && EqQ[$a^2 - b^2, 0$] && GtQ[$m, 0$] && LeQ[$p, -2*m$] && IntegersQ[$m + 1/2, 2*p$]

Rule 2687

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]], x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)/(a + b*sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[$a^2 - b^2, 0$] && LtQ[$p, -1$] && IntegerQ[$2*p$]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*cos[c + d*x]*(a + b*sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[$a^2 - b^2, 0$] && LtQ[$n, -1$] && IntegerQ[$2*n$]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*cos[c + d*x])/Sqrt[a + b*sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[$a^2 - b^2, 0$]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2878

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_.)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] := -Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m + p + 2)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(b*(m + 1) - a*(p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[$a^2 - b^2, 0$] && NeQ[m + p + 2, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[$a^2 - b^2, 0$] && GtQ[$m, -1$] && LtQ[$p, -1$]

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sin(e + fx)} \tan^4(e + fx) dx &= \int \sqrt{a + a \sin(e + fx)} dx - \int \sec^4(e + fx) \sqrt{a + a \sin(e + fx)} (1 - 2 \sin^2(e + fx)) dx \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \int (\sec^4(e + fx) \sqrt{a(1 + \sin(e + fx))} - 2 \sec^2(e + fx) \sqrt{a(1 + \sin(e + fx))}) dx \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} + 2 \int \sec^2(e + fx) \sqrt{a(1 + \sin(e + fx))} \tan^2(e + fx) dx \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{\sec^3(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} + \frac{4 \sec^3(e + fx) (a + a \sin(e + fx))}{3f} \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{5a \sec(e + fx)}{6f \sqrt{a + a \sin(e + fx)}} - \frac{7 \sec^3(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \\
&= \frac{5a^2 \cos(e + fx)}{8f(a + a \sin(e + fx))^{3/2}} - \frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{11a \sec(e + fx)}{6f \sqrt{a + a \sin(e + fx)}} - \frac{7 \sec^3(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \\
&= \frac{11a^2 \cos(e + fx)}{8f(a + a \sin(e + fx))^{3/2}} - \frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{11a \sec(e + fx)}{6f \sqrt{a + a \sin(e + fx)}} - \frac{7 \sec^3(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \\
&= \frac{5\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2 \cos(e + fx)}{8f(a + a \sin(e + fx))^{3/2}} - \frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} \\
&= \frac{11\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2 \cos(e + fx)}{8f(a + a \sin(e + fx))^{3/2}} - \frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 5.54044, size = 394, normalized size = 2.43

$$\sqrt{a(\sin(e + fx) + 1)} \left(-48 \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right) \cos\left(\frac{fx}{2}\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2 + 48 \left(\sin\left(\frac{e}{2}\right) + \cos\left(\frac{e}{2}\right) \right) \sin\left(\frac{fx}{2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]

[Out] (((6*Sin[(f*x)/2])/(Cos[e/2] + Sin[e/2]) - (3*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(Cos[e/2] + Sin[e/2]) + (33 + 33*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 48*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 48*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (36*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))*Sqrt[a*(1 + Sin[e + f*x])]/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

Maple [A] time = 0.464, size = 172, normalized size = 1.1

$$-\frac{1}{(-48 + 48 \sin(fx + e)) \cos(fx + e) f} \left(96 a^{5/2} \sin(fx + e) (\cos(fx + e))^2 + \left(33 (a - a \sin(fx + e))^{3/2} \sqrt{2} \operatorname{Artanh} \left(\frac{\sqrt{a} \cos(fx + e)}{\sqrt{2} \sqrt{a + a \sin(fx + e)}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x)`

[Out]
$$-1/48/a^{(3/2)}*(96*a^{(5/2)}*\sin(f*x+e)*\cos(f*x+e)^2+(33*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a+20*a^{(5/2)}*\sin(f*x+e)-162*a^{(5/2)}*\cos(f*x+e)^2+33*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a-4*a^{(5/2)})/(-1+\sin(f*x+e))/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)`

Fricas [A] time = 2.00592, size = 548, normalized size = 3.38

$$33 \sqrt{2} \sqrt{a} \cos(fx + e)^3 \log \left(-\frac{a \cos(fx+e)^2 + 2 \sqrt{a \sin(fx+e) + a} (\sqrt{2} \cos(fx+e) - \sqrt{2} \sin(fx+e) + \sqrt{2}) \sqrt{a} + 3 a \cos(fx+e) - (a \cos(fx+e) - 2a) \sin(fx+e) + 2}{\cos(fx+e)^2 - (\cos(fx+e) + 2) \sin(fx+e) - \cos(fx+e) - 2} \right) \frac{1}{96 f \cos(fx + e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")`

[Out]
$$1/96*(33*\sqrt{2}*\sqrt{a}*\cos(f*x + e)^3*\log(-(a*\cos(f*x + e)^2 + 2*\sqrt{a*\sin(f*x + e) + a}*(\sqrt{2}*\cos(f*x + e) - \sqrt{2}*\sin(f*x + e) + \sqrt{2}))*\sqrt{a} + 3*a*\cos(f*x + e) - (a*\cos(f*x + e) - 2*a)*\sin(f*x + e) + 2*a)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \cos(f*x + e) - 2)) - 4*(81*\cos(f*x + e)^2 - 2*(24*\cos(f*x + e)^2 + 5)*\sin(f*x + e) + 2)*\sqrt{a*\sin(f*x + e) + a})/(f*\cos(f*x + e)^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**4,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

3.92 $\int \sqrt{a + a \sin(e + fx)} \tan^2(e + fx) dx$

Optimal. Leaf size=101

$$-\frac{2 \sec(e + fx)(a \sin(e + fx) + a)^{3/2}}{af} + \frac{5 \sec(e + fx)\sqrt{a \sin(e + fx) + a}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{\sqrt{2}f}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rubi [A] time = 0.184754, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2713, 2855, 2649, 206}

$$-\frac{2 \sec(e + fx)(a \sin(e + fx) + a)^{3/2}}{af} + \frac{5 \sec(e + fx)\sqrt{a \sin(e + fx) + a}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 2713

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] + Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sin(e + fx)} \tan^2(e + fx) dx &= -\frac{2 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{af} + \frac{2 \int \sec^2(e + fx) \sqrt{a + a \sin(e + fx)} \left(\frac{3}{2}\right)}{a} \\
&= \frac{5 \sec(e + fx) \sqrt{a + a \sin(e + fx)}}{f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{af} + \frac{1}{2} a \int \sec^2(e + fx) \sqrt{a + a \sin(e + fx)} dx \\
&= \frac{5 \sec(e + fx) \sqrt{a + a \sin(e + fx)}}{f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{af} - \frac{a \operatorname{Su}}{f} \\
&= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{2} f} + \frac{5 \sec(e + fx) \sqrt{a + a \sin(e + fx)}}{f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{af}
\end{aligned}$$

Mathematica [C] time = 0.327891, size = 114, normalized size = 1.13

$$\frac{\sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \left(-2 \sin(e + fx) + (1 - i) \sqrt[4]{-1} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\right) \tanh^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)(-1)\right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]

[Out] (Sec[e + f*x]*(3 + (1 - I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 2*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/f

Maple [A] time = 0.502, size = 89, normalized size = 0.9

$$-\frac{1 + \sin(fx + e)}{2f \cos(fx + e)} \left(\sqrt{a} \sqrt{2} \operatorname{Arctanh} \left(\frac{\sqrt{2}}{2} \sqrt{a - a \sin(fx + e)} \frac{1}{\sqrt{a}} \right) \sqrt{a - a \sin(fx + e)} + 4a \sin(fx + e) - 6a \right) \frac{1}{\sqrt{a + a \sin(fx + e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x)

[Out] -1/2*(1+sin(f*x+e))*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)+4*a*sin(f*x+e)-6*a)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \tan(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)

Fricas [A] time = 1.82154, size = 459, normalized size = 4.54

$$\frac{\sqrt{2}\sqrt{a} \cos(fx + e) \log\left(-\frac{a \cos(fx+e)^2 - 2\sqrt{2}\sqrt{a} \sin(fx+e) + a\sqrt{a}(\cos(fx+e) - \sin(fx+e) + 1) + 3a \cos(fx+e) - (a \cos(fx+e) - 2a) \sin(fx+e) + 2a}{\cos(fx+e)^2 - (\cos(fx+e) + 2) \sin(fx+e) - \cos(fx+e) - 2}\right) - 4\sqrt{2}\sqrt{a} \cos(fx + e)}{4f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")

[Out] 1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a)*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(a*sin(f*x + e) + a)*(2*sin(f*x + e) - 3)/(f*cos(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin(e + fx) + 1)} \tan^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**2,x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \tan(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)

3.93 $\int \cot^2(e + fx) \sqrt{a + a \sin(e + fx)} dx$

Optimal. Leaf size=89

$$\frac{3a \cos(e + fx)}{f \sqrt{a \sin(e + fx) + a}} - \frac{\cot(e + fx) \sqrt{a \sin(e + fx) + a}}{f} - \frac{\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}} \right)}{f}$$

[Out] $-\left(\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a \sin[e + f x] + a}}\right]}{f}\right) + \left(\frac{3 a \cos[e + f x]}{f \sqrt{a \sin[e + f x] + a}} - \frac{\cot[e + f x] \sqrt{a \sin[e + f x] + a}}{f}\right)$

Rubi [A] time = 0.192413, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2716, 2981, 2773, 206}

$$\frac{3a \cos(e + fx)}{f \sqrt{a \sin(e + fx) + a}} - \frac{\cot(e + fx) \sqrt{a \sin(e + fx) + a}}{f} - \frac{\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]

[Out] $-\left(\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a \sin[e + f x] + a}}\right]}{f}\right) + \left(\frac{3 a \cos[e + f x]}{f \sqrt{a \sin[e + f x] + a}} - \frac{\cot[e + f x] \sqrt{a \sin[e + f x] + a}}{f}\right)$

Rule 2716

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a, Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]

Rule 2981

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rule 2773

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \cot^2(e + fx) \sqrt{a + a \sin(e + fx)} dx &= -\frac{\cot(e + fx) \sqrt{a + a \sin(e + fx)}}{f} + \frac{\int \csc(e + fx) \left(\frac{a}{2} - \frac{3}{2}a \sin(e + fx)\right) \sqrt{a + a \sin(e + fx)} dx}{a} \\
 &= \frac{3a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{\cot(e + fx) \sqrt{a + a \sin(e + fx)}}{f} + \frac{1}{2} \int \csc(e + fx) \sqrt{a + a \sin(e + fx)} dx \\
 &= \frac{3a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{\cot(e + fx) \sqrt{a + a \sin(e + fx)}}{f} - \frac{a \operatorname{Subst}\left(\int \frac{1}{a-x^2} dx, x, \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{a}}\right)}{f} \\
 &= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{f} + \frac{3a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{\cot(e + fx) \sqrt{a + a \sin(e + fx)}}{f}
 \end{aligned}$$

Mathematica [B] time = 0.954969, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sin(e + fx) + 1)} \left(4 \sin\left(\frac{1}{2}(e + fx)\right) + 2 \sin\left(\frac{3}{2}(e + fx)\right) - 4 \cos\left(\frac{1}{2}(e + fx)\right) + 2 \cos\left(\frac{3}{2}(e + fx)\right) - \sin(e + fx)\right)}{f \left(\cot\left(\frac{1}{2}(e + fx)\right) + 1\right) \left(\csc\left(\frac{1}{4}(e + fx)\right) - \sec\left(\frac{1}{4}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]

[Out] (Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(-4*Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] + 4*Sin[(e + f*x)/2] - Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 2*Sin[(3*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))

Maple [A] time = 0.55, size = 125, normalized size = 1.4

$$\frac{1 + \sin(fx + e)}{\sin(fx + e) \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(\sin(fx + e) \left(2 \sqrt{a - a \sin(fx + e)} a^{3/2} - \operatorname{Artanh}\left(\sqrt{a - a \sin(fx + e)}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x)

[Out] (1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2)-(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \cot(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^2, x)

Fricas [B] time = 1.93572, size = 749, normalized size = 8.42

$$\left(\cos(fx+e)^2 - (\cos(fx+e)+1)\sin(fx+e) - 1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3 - 7a\cos(fx+e)^2 - 4(\cos(fx+e)^2 + (\cos(fx+e)+3)\sin(fx+e) - \cos(fx+e)^3 + \cos(fx+e))}{\cos(fx+e)^3 + \cos(fx+e)}\right)$$

4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) - 4*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 3)*sin(f*x + e) - cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*sin(f*x + e) - f)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin(e+fx)+1)} \cot^2(e+fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)

Giac [B] time = 2.0909, size = 570, normalized size = 6.4

$$\frac{2a \arctan\left(\frac{\sqrt{a}\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{a\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}}{\sqrt{-a}}\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{\sqrt{-a}} - \sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \sqrt{a\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] 1/2*(2*a*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - sqrt(a)*log(ab

$$\begin{aligned}
& s(-\sqrt{a} \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + \sqrt{a \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + a}) \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \\
& + 2 \cdot a^{3/2} \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) / ((\sqrt{a} \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - \sqrt{a \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + a})^2 - a) \\
& - (2 \cdot \sqrt{2} \cdot a \cdot \arctan((\sqrt{2} \cdot \sqrt{a} + \sqrt{a}) / \sqrt{-a})) - \sqrt{2} \cdot \sqrt{-a} \cdot \sqrt{a} \cdot \log(\sqrt{2} \cdot \sqrt{a} + \sqrt{a}) \\
& + 2 \cdot a \cdot \arctan((\sqrt{2} \cdot \sqrt{a} + \sqrt{a}) / \sqrt{-a})) - \sqrt{-a} \cdot \sqrt{a} \cdot \log(\sqrt{2} \cdot \sqrt{a} + \sqrt{a}) \\
& + 5 \cdot \sqrt{2} \cdot \sqrt{-a} \cdot \sqrt{a} + 11 \cdot \sqrt{-a} \cdot \sqrt{a}) \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) / (\sqrt{2} \cdot \sqrt{-a} + \sqrt{-a}) \\
& + (5 \cdot a \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) + (a \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 4 \cdot a \cdot \operatorname{sgn}(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) / \sqrt{a \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + a}) / f
\end{aligned}$$

3.94 $\int \cot^4(e + fx) \sqrt{a + a \sin(e + fx)} dx$

Optimal. Leaf size=163

$$-\frac{2a \cos(e + fx)}{f \sqrt{a \sin(e + fx) + a}} + \frac{11a \cot(e + fx)}{8f \sqrt{a \sin(e + fx) + a}} + \frac{11\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f} - \frac{\cot(e + fx) \csc^2(e + fx) \sqrt{a \sin(e + fx)}}{3f}$$

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*f)
- (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(
(3*f))
```

Rubi [A] time = 0.37672, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2718, 2646, 3044, 2980, 2772, 2773, 206}

$$-\frac{2a \cos(e + fx)}{f \sqrt{a \sin(e + fx) + a}} + \frac{11a \cot(e + fx)}{8f \sqrt{a \sin(e + fx) + a}} + \frac{11\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f} - \frac{\cot(e + fx) \csc^2(e + fx) \sqrt{a \sin(e + fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*f)
- (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(
(3*f))
```

Rule 2718

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]], x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e + fx) \sqrt{a + a \sin(e + fx)} dx &= \int \sqrt{a + a \sin(e + fx)} dx + \int \csc^4(e + fx) \sqrt{a + a \sin(e + fx)} (1 - 2 \sin^2(e + fx)) \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{\cot(e + fx) \csc^2(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} + \frac{\int \csc^3(e + fx) \sqrt{a + a \sin(e + fx)} dx}{3f} \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} - \frac{a \cot(e + fx) \csc(e + fx)}{12f \sqrt{a + a \sin(e + fx)}} - \frac{\cot(e + fx) \csc^2(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} + \frac{11a \cot(e + fx)}{8f \sqrt{a + a \sin(e + fx)}} - \frac{a \cot(e + fx) \csc(e + fx)}{12f \sqrt{a + a \sin(e + fx)}} - \frac{\int \csc^3(e + fx) \sqrt{a + a \sin(e + fx)} dx}{3f} \\
&= -\frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} + \frac{11a \cot(e + fx)}{8f \sqrt{a + a \sin(e + fx)}} - \frac{a \cot(e + fx) \csc(e + fx)}{12f \sqrt{a + a \sin(e + fx)}} - \frac{\int \csc^3(e + fx) \sqrt{a + a \sin(e + fx)} dx}{3f} \\
&= \frac{11\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{8f} - \frac{2a \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)}} + \frac{11a \cot(e + fx)}{8f \sqrt{a + a \sin(e + fx)}} - \frac{\int \csc^3(e + fx) \sqrt{a + a \sin(e + fx)} dx}{3f}
\end{aligned}$$

Mathematica [A] time = 1.56776, size = 309, normalized size = 1.9

$$\csc^{10}\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sin(e + fx) + 1)} \left(-252 \sin\left(\frac{1}{2}(e + fx)\right) - 250 \sin\left(\frac{3}{2}(e + fx)\right) + 114 \sin\left(\frac{5}{2}(e + fx)\right) + 48 \sin\left(\frac{7}{2}(e + fx)\right) + \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]

[Out] (Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(252*Cos[(e + f*x)/2] - 250*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] + 48*Cos[(7*(e + f*x))/2] - 252*Sin[(e + f*x)/2] + 99*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 99*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 250*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] - 33*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 33*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 48*Sin[(7*(e + f*x))/2]))/(24*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)

Maple [A] time = 0.701, size = 170, normalized size = 1.

$$\frac{1 + \sin(fx + e)}{24 (\sin(fx + e))^3 \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(-48 \sqrt{-a(-1 + \sin(fx + e))} a^{7/2} (\sin(fx + e))^3 + 33 (-a \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)/a^(7/2)*(-48*(-a*(-1+sin(f*x+e)))^(1/2)*a^(7/2)*sin(f*x+e)^3+33*(-a*(-1+sin(f*x+e)))^(5/2)*a^(3/2)+33*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3-56*(-a*(-1+sin(f*x+e)))^(3/2)*a^(5/2)+15*(-a*(-1+sin(f*x+e)))^(1/2)*a^(7/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(fx + e) + a} \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^4, x)

Fricas [B] time = 1.81584, size = 1025, normalized size = 6.29

$$33 \left(\cos(fx + e)^4 - 2 \cos(fx + e)^2 - \left(\cos(fx + e)^3 + \cos(fx + e)^2 - \cos(fx + e) - 1 \right) \sin(fx + e) + 1 \right) \sqrt{a} \log \left(\frac{a \cos(fx + e) + a}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

```
[Out] 1/96*(33*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*(48*cos(f*x + e)^4 - 33*cos(f*x + e)^3 - 139*cos(f*x + e)^2 + (48*cos(f*x + e)^3 + 81*cos(f*x + e)^2 - 58*cos(f*x + e) - 83)*sin(f*x + e) + 25*cos(f*x + e) + 83)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.1154, size = 950, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(66*a*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 33*sqrt(a)*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sgn(tan(1/2*f*x + 1/2*e) + 1) - (330*sqrt(2)*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 165*sqrt(2)*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 462*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 231*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 992*sqrt(2)*sqrt(-a)*sqrt(a) + 1422*sqrt(-a)*sqrt(a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt(-a) + 7*sqrt(-a)) + (130*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - (99*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - (32*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - (2*a*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e) + 3*a*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) - 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - 30*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*sgn(tan(1/2*f*x + 1/2*e) + 1) + 72*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - 34*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) + 1))/((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a^3)/f
```

3.95 $\int (a + a \sin(e + fx))^{3/2} \tan^4(e + fx) dx$

Optimal. Leaf size=167

$$\frac{2a^3 \cos^3(e + fx)}{3f(a \sin(e + fx) + a)^{3/2}} - \frac{4a^2 \cos(e + fx)}{f\sqrt{a \sin(e + fx) + a}} - \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{2\sqrt{2}f} + \frac{\sec^3(e + fx)(a \sin(e + fx) + a)^{3/2}}{3f}$$

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt[2]*f) + (2*a^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^(3/2)) - (4*a^2*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (7*a*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*f) + (Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.975016, antiderivative size = 195, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2647, 2646, 4401, 2675, 2649, 206, 2878, 2855}

$$\frac{8a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{2\sqrt{2}f} - \frac{2a \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f} + \frac{4 \sec^3(e + fx)(a \sin(e + fx) + a)^{3/2}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt[2]*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (a*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*f) - (23*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(a*f)
```

Rule 2714

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2647

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m + 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*cos[c + d*x])/Sqrt[a + b*sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := -Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m + p + 2)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(b*(m + 1) - a*(p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(b*(c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^{3/2} \tan^4(e + fx) dx &= \int (a + a \sin(e + fx))^{3/2} dx - \int \sec^4(e + fx)(a + a \sin(e + fx))^{3/2} (1 - 2 \sin(e + fx)) dx \\
&= -\frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} + \frac{1}{3}(4a) \int \sqrt{a + a \sin(e + fx)} dx - \int (1 - 2 \sin(e + fx)) \sec^4(e + fx) (a + a \sin(e + fx))^{3/2} dx \\
&= -\frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} + 2 \int \sec^2(e + fx) (a + a \sin(e + fx))^{3/2} dx \\
&= -\frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} - \frac{\sec^3(e + fx) (a + a \sin(e + fx))^{3/2}}{3f} \\
&= -\frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} - \frac{a \sec(e + fx) (a + a \sin(e + fx))^{3/2}}{3f} \\
&= -\frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} + \frac{a \sec(e + fx) (a + a \sin(e + fx))^{3/2}}{3f} \\
&= \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \\
&= -\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f}
\end{aligned}$$

Mathematica [C] time = 5.56233, size = 141, normalized size = 0.84

$$\frac{a \sec^3(e + fx) \sqrt{a(\sin(e + fx) + 1)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2 \left(54 \sin(e + fx) + \sin(3(e + fx)) + 6 \cos(2(e + fx)) \right)}{6f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]

[Out] (a*Sec[e + f*x]^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a*(1 + Sin[e + f*x])]*(-45 + 6*Cos[2*(e + f*x)] + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 54*Sin[e + f*x] + Sin[3*(e + f*x)])/(6*f)

Maple [A] time = 0.574, size = 139, normalized size = 0.8

$$\frac{1 + \sin(fx + e)}{12a(-1 + \sin(fx + e)) \cos(fx + e) f} \left(3a^{3/2} \sqrt{2} \operatorname{Arctanh} \left(\frac{1}{2} \frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{\sqrt{a}} \right) (a - a \sin(fx + e))^{3/2} - 8a^3 \sin(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x)

[Out] 1/12*(1+sin(f*x+e))/a/(-1+sin(f*x+e))*(3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)-8*a^3*sin(f*x+e)*cos(f*x+e)^2-24*a^3*cos(f*x+e)^2-106*a^3*sin(f*x+e)+102*a^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 1.90228, size = 648, normalized size = 3.88

$$3\left(\sqrt{2}a\cos(fx+e)\sin(fx+e) - \sqrt{2}a\cos(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2 - 2\sqrt{a\sin(fx+e)+a}(\sqrt{2}\cos(fx+e) - \sqrt{2}\sin(fx+e) + \sqrt{2})\sqrt{a+3a\cos(fx+e)}}{\cos(fx+e)^2 - (\cos(fx+e)+2)\sin(fx+e) - \cos(fx+e) - 2}\right) \\ \frac{1}{24}\left(f\cos(fx+e)\sin(fx+e) - f\cos(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")

[Out] 1/24*(3*(sqrt(2)*a*cos(f*x + e)*sin(f*x + e) - sqrt(2)*a*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a*sin(f*x + e) + a)*(sqrt(2)*cos(f*x + e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sqrt(a) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*a*cos(f*x + e)^2 + (4*a*cos(f*x + e)^2 + 53*a)*sin(f*x + e) - 51*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)*sin(f*x + e) - f*cos(f*x + e))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e) + a)^{\frac{3}{2}} \tan(fx+e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*tan(f*x + e)^4, x)

3.96 $\int (a + a \sin(e + fx))^{3/2} \tan^2(e + fx) dx$

Optimal. Leaf size=88

$$\frac{11a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{2 \sec(e + fx)(a \sin(e + fx) + a)^{5/2}}{3af} + \frac{7 \sec(e + fx)(a \sin(e + fx) + a)^{3/2}}{3f}$$

[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f)

Rubi [A] time = 0.195731, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2713, 2855, 2646}

$$\frac{11a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{2 \sec(e + fx)(a \sin(e + fx) + a)^{5/2}}{3af} + \frac{7 \sec(e + fx)(a \sin(e + fx) + a)^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]

[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f)

Rule 2713

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] + Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x])/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && ! IntegerQ[m] && ! LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{3/2} \tan^2(e + fx) dx &= -\frac{2 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{3af} + \frac{2 \int \sec^2(e + fx)(a + a \sin(e + fx))^{3/2} dx}{3a} \\ &= \frac{7 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{3f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{3af} - \frac{1}{6} \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx \\ &= \frac{11a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} + \frac{7 \sec(e + fx)(a + a \sin(e + fx))^{3/2}}{3f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{3af} \end{aligned}$$

Mathematica [A] time = 3.99156, size = 46, normalized size = 0.52

$$\frac{a \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} (-8 \sin(e + fx) + \cos(2(e + fx)) + 15)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]

[Out] (a*Sec[e + f*x]*(15 + Cos[2*(e + f*x)] - 8*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/(3*f)

Maple [A] time = 0.402, size = 55, normalized size = 0.6

$$\frac{2a^2(1 + \sin(fx + e)) \left((\sin(fx + e))^2 + 4 \sin(fx + e) - 8 \right)}{3f \cos(fx + e)} \frac{1}{\sqrt{a + a \sin(fx + e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x)

[Out] -2/3*a^2*(1+sin(f*x+e))*(sin(f*x+e)^2+4*sin(f*x+e)-8)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [A] time = 1.54691, size = 196, normalized size = 2.23

$$\frac{8 \left(2a^{\frac{3}{2}} - \frac{2a^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^{\frac{3}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} - \frac{2a^{\frac{3}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{2a^{\frac{3}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} \right)}{3f \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1 \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")

[Out] -8/3*(2*a^(3/2) - 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))

Fricas [A] time = 1.7535, size = 123, normalized size = 1.4

$$\frac{2 \left(a \cos (f x + e) \right)^2 - 4 a \sin (f x + e) + 7 a \sqrt{a \sin (f x + e) + a}}{3 f \cos (f x + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")

[Out] 2/3*(a*cos(f*x + e)^2 - 4*a*sin(f*x + e) + 7*a)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (f x + e) + a)^{\frac{3}{2}} \tan (f x + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*tan(f*x + e)^2, x)

3.97 $\int \cot^2(e + fx)(a + a \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=121

$$\frac{11a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{3a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{f} + \frac{5a \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f} - \frac{\cot(e + fx)(a \sin(e + fx) + a)^{3/2}}{f}$$

[Out] $(-3a^{3/2} \text{ArcTanh}[\text{Sqrt}[a] \text{Cos}[e + f*x]]/\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/f + (11a^2 \text{Cos}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (5*a*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(3*f) - (\text{Cot}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2})/f$

Rubi [A] time = 0.320568, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{11a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{3a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{f} + \frac{5a \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f} - \frac{\cot(e + fx)(a \sin(e + fx) + a)^{3/2}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^{3/2}, x]$

[Out] $(-3a^{3/2} \text{ArcTanh}[\text{Sqrt}[a] \text{Cos}[e + f*x]]/\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/f + (11a^2 \text{Cos}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (5*a*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(3*f) - (\text{Cot}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2})/f$

Rule 2716

$\text{Int}[(a + b*\text{Sin}[e + f*x])^m/\text{tan}[(e + f*x)], x]$ Symbol $\rightarrow -\text{Simp}[(a + b*\text{Sin}[e + f*x])^m/(f*\text{Tan}[e + f*x]), x] + \text{Dist}[1/a, \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(b*m - a*(m + 1)*\text{Sin}[e + f*x])/ \text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]

Rule 2976

$\text{Int}[(a + b*\text{Sin}[e + f*x])^m*((A + B*\text{Sin}[e + f*x]) + (C + D*\text{Sin}[e + f*x])^n), x]$ Symbol $\rightarrow -\text{Simp}[(b*B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m-1}*(c + d*\text{Sin}[e + f*x])^{n+1})/(d*f*(m + n + 1)), x] + \text{Dist}[1/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m-1}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*\text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2981

$\text{Int}[\text{Sqrt}[(a + b*\text{Sin}[e + f*x])*(A + B*\text{Sin}[e + f*x]) + (C + D*\text{Sin}[e + f*x])^n], x]$ Symbol $\rightarrow \text{Simp}[(2*b*B*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{n+1})/(d*f*(2*n + 3)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -

$b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{!LtQ}[n, -1]$

Rule 2773

$\text{Int}[\text{Sqrt}[(a_) + (b_.)\sin[(e_) + (f_.)*(x_)]]/((c_) + (d_.)\sin[(e_) + (f_.)*(x_)]), x_Symbol] \ :> \ \text{Dist}[(-2*b)/f, \text{Subst}[\text{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\text{Cos}[e + f*x])/ \text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \cot^2(e + fx)(a + a \sin(e + fx))^{3/2} dx &= -\frac{\cot(e + fx)(a + a \sin(e + fx))^{3/2}}{f} + \frac{\int \csc(e + fx) \left(\frac{3a}{2} - \frac{5}{2}a \sin(e + fx) \right) (a + a \sin(e + fx))^{3/2} dx}{a} \\ &= \frac{5a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} - \frac{\cot(e + fx)(a + a \sin(e + fx))^{3/2}}{f} + \frac{2 \int \csc(e + fx) (a + a \sin(e + fx))^{3/2} dx}{a} \\ &= \frac{11a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} + \frac{5a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} - \frac{\cot(e + fx)(a + a \sin(e + fx))^{3/2}}{f} \\ &= \frac{11a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} + \frac{5a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} - \frac{\cot(e + fx)(a + a \sin(e + fx))^{3/2}}{f} \\ &= -\frac{3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} \right)}{f} + \frac{11a^2 \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} + \frac{5a \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f} \end{aligned}$$

Mathematica [A] time = 0.759996, size = 233, normalized size = 1.93

$$\frac{a \csc^4 \left(\frac{1}{2}(e + fx) \right) \sqrt{a(\sin(e + fx) + 1)} \left(-14 \sin \left(\frac{1}{2}(e + fx) \right) - 9 \sin \left(\frac{3}{2}(e + fx) \right) - \sin \left(\frac{5}{2}(e + fx) \right) + 14 \cos \left(\frac{1}{2}(e + fx) \right) \right)}{3f \left(\cot \left(\frac{1}{2}(e + fx) \right) + 1 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]

[Out] $-(a*\text{Csc}[(e + f*x)/2]^4*\text{Sqrt}[a*(1 + \text{Sin}[e + f*x])]*(14*\text{Cos}[(e + f*x)/2] - 9*\text{Cos}[(3*(e + f*x))/2] + \text{Cos}[(5*(e + f*x))/2] - 14*\text{Sin}[(e + f*x)/2] + 9*\text{Log}[1 + \text{Cos}[(e + f*x)/2] - \text{Sin}[(e + f*x)/2]]*\text{Sin}[e + f*x] - 9*\text{Log}[1 - \text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2]]*\text{Sin}[e + f*x] - 9*\text{Sin}[(3*(e + f*x))/2] - \text{Sin}[(5*(e + f*x))/2]))/(3*f*(1 + \text{Cot}[(e + f*x)/2])*(\text{Csc}[(e + f*x)/4] - \text{Sec}[(e + f*x)/4])*(\text{Csc}[(e + f*x)/4] + \text{Sec}[(e + f*x)/4]))$

Maple [A] time = 0.642, size = 144, normalized size = 1.2

$$-\frac{1 + \sin(fx + e)}{3 \sin(fx + e) \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(\sin(fx + e) \left(-12 \sqrt{a - a \sin(fx + e)} a^{3/2} + 2 \sqrt{a} (a - a \sin(fx + e)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x)`

[Out]
$$-1/3*(1+\sin(f*x+e))*(-a*(-1+\sin(f*x+e)))^{1/2}*(\sin(f*x+e)*(-12*(a-a*\sin(f*x+e))^{1/2})a^{3/2}+2*a^{1/2}*(a-a*\sin(f*x+e))^{3/2}+9*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}/a^{1/2})*a^2)+3*(a-a*\sin(f*x+e))^{1/2}*a^{3/2})/\sin(f*x+e)/a^{1/2}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{3}{2}} \cot(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^2, x)`

Fricas [B] time = 1.84636, size = 833, normalized size = 6.88

$$9 \left(a \cos(fx + e)^2 - (a \cos(fx + e) + a) \sin(fx + e) - a \right) \sqrt{a} \log \left(\frac{a \cos(fx+e)^3 - 7a \cos(fx+e)^2 - 4(\cos(fx+e)^2 + (\cos(fx+e)+3)\sin(fx+e)) \cos(fx+e) + 3 \cos(fx+e) - 1}{\cos(fx+e)^3 + \cos(fx+e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{12} * (9 * (a * \cos(f*x + e)^2 - (a * \cos(f*x + e) + a) * \sin(f*x + e) - a) * \sqrt{a} * \log((a * \cos(f*x + e)^3 - 7 * a * \cos(f*x + e)^2 - 4 * (\cos(f*x + e)^2 + (\cos(f*x + e) + 3) * \sin(f*x + e) - 2 * \cos(f*x + e) - 3) * \sqrt{a * \sin(f*x + e) + a}) * \sqrt{a} - 9 * a * \cos(f*x + e) + (a * \cos(f*x + e)^2 + 8 * a * \cos(f*x + e) - a) * \sin(f*x + e) - a) / (\cos(f*x + e)^3 + \cos(f*x + e)^2 + (\cos(f*x + e)^2 - 1) * \sin(f*x + e) - \cos(f*x + e) - 1) + 4 * (2 * a * \cos(f*x + e)^3 - 8 * a * \cos(f*x + e)^2 + a * \cos(f*x + e) - (2 * a * \cos(f*x + e)^2 + 10 * a * \cos(f*x + e) + 11 * a) * \sin(f*x + e) + 11 * a) * \sqrt{a * \sin(f*x + e) + a}) / (f * \cos(f*x + e)^2 - (f * \cos(f*x + e) + f) * \sin(f*x + e) - f)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(3/2),x)`

[Out] Timed out

Giac [B] time = 2.63422, size = 671, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out]
$$\frac{1}{6} \cdot (18a^2 \arctan(-\sqrt{a} \tan(\frac{1}{2}fx + \frac{1}{2}e) - \sqrt{a \tan^2(\frac{1}{2}fx + \frac{1}{2}e) + a}) / \sqrt{-a}) \cdot \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) / \sqrt{-a} - 9a^{3/2} \cdot \log(\operatorname{abs}(-\sqrt{a} \tan(\frac{1}{2}fx + \frac{1}{2}e) + \sqrt{a \tan^2(\frac{1}{2}fx + \frac{1}{2}e) + a})) \cdot \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) + 6a^{5/2} \cdot \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) / ((\sqrt{a} \tan(\frac{1}{2}fx + \frac{1}{2}e) - \sqrt{a \tan^2(\frac{1}{2}fx + \frac{1}{2}e) + a})^2 - a) - (18\sqrt{2}a^2 \arctan((\sqrt{2}\sqrt{a} + \sqrt{a}) / \sqrt{-a}) - 9\sqrt{2} \sqrt{-a} a^{3/2} \log(\sqrt{2}\sqrt{a} + \sqrt{a}) + 18a^2 \arctan((\sqrt{2}\sqrt{a} + \sqrt{a}) / \sqrt{-a}) - 9\sqrt{-a} a^{3/2} \log(\sqrt{2}\sqrt{a} + \sqrt{a})) + 19\sqrt{2} \sqrt{-a} a^{3/2} + 41\sqrt{-a} a^{3/2}) \cdot \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) / (\sqrt{2}\sqrt{-a} + \sqrt{-a}) + (23a^3 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) - (12a^3 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) - (18a^3 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) + (3a^3 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) \tan(\frac{1}{2}fx + \frac{1}{2}e) - 20a^3 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1)) \tan(\frac{1}{2}fx + \frac{1}{2}e)) \tan(\frac{1}{2}fx + \frac{1}{2}e)) / (a \tan^2(\frac{1}{2}fx + \frac{1}{2}e) + a)^{3/2}) / f$$

3.98 $\int \cot^4(e + fx)(a + a \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=197

$$-\frac{8a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} + \frac{29a^2 \cot(e + fx)}{24f\sqrt{a \sin(e + fx) + a}} + \frac{37a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f} - \frac{2a \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f}$$

[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f) - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)

Rubi [A] time = 0.496362, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2718, 2647, 2646, 3044, 2975, 2980, 2773, 206}

$$-\frac{8a^2 \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} + \frac{29a^2 \cot(e + fx)}{24f\sqrt{a \sin(e + fx) + a}} + \frac{37a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f} - \frac{2a \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2), x]

[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f) - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)

Rule 2718

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^4, x_Symbol] := Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]

Rule 2647

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3044

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)), x] /; FreeQ[{a, b, c, d, A, C}, x] && EqQ[a^2 - b^2, 0]

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2} dx &= \int (a+a\sin(e+fx))^{3/2} dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{3/2} (1-2\sin^2(e+fx)) dx \\
&= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+fx))^{3/2}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+fx))^{3/2}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 1.35636, size = 334, normalized size = 1.7

$$a \csc^{10}\left(\frac{1}{2}(e+fx)\right) \sqrt{a(\sin(e+fx)+1)} \left(276 \sin\left(\frac{1}{2}(e+fx)\right) + 326 \sin\left(\frac{3}{2}(e+fx)\right) - 78 \sin\left(\frac{5}{2}(e+fx)\right) - 72 \sin\left(\frac{7}{2}(e+fx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2), x]

[Out] $-(a*\text{Csc}[(e+f*x)/2]^{10}*\text{Sqrt}[a*(1+\text{Sin}[e+f*x])])*(-276*\text{Cos}[(e+f*x)/2] + 326*\text{Cos}[(3*(e+f*x))/2] + 78*\text{Cos}[(5*(e+f*x))/2] - 72*\text{Cos}[(7*(e+f*x))/2] + 8*\text{Cos}[(9*(e+f*x))/2] + 276*\text{Sin}[(e+f*x)/2] - 333*\text{Log}[1+\text{Cos}[(e+f*x)/2] - \text{Sin}[(e+f*x)/2]]*\text{Sin}[e+f*x] + 333*\text{Log}[1-\text{Cos}[(e+f*x)/2] + \text{Sin}[(e+f*x)/2]]*\text{Sin}[e+f*x] + 326*\text{Sin}[(3*(e+f*x))/2] - 78*\text{Sin}[(5*(e+f*x))/2] + 111*\text{Log}[1+\text{Cos}[(e+f*x)/2] - \text{Sin}[(e+f*x)/2]]*\text{Sin}[3*(e+f*x)] - 111*\text{Log}[1-\text{Cos}[(e+f*x)/2] + \text{Sin}[(e+f*x)/2]]*\text{Sin}[3*(e+f*x)] - 72*\text{Sin}[(7*(e+f*x))/2] - 8*\text{Sin}[(9*(e+f*x))/2]))/(24*f*(1+\text{Cot}[(e+f*x)/2])*(\text{Csc}[(e+f*x)/4]^2 - \text{Sec}[(e+f*x)/4]^2)^3)$

Maple [A] time = 0.651, size = 196, normalized size = 1.

$$-\frac{1+\sin(fx+e)}{24(\sin(fx+e))^3 \cos(fx+e)f} \sqrt{-a(-1+\sin(fx+e))} \left(96 \sqrt{-a(-1+\sin(fx+e))} a^{5/2} (\sin(fx+e))^3 - 16(-a(-1+\sin(fx+e)))^{3/2} a^{5/2} \sin(fx+e)^3 - 16(-a(-1+\sin(fx+e)))^{3/2} a^{5/2} \sin(fx+e)^3 + 16(-a(-1+\sin(fx+e)))^{3/2} a^{5/2} \sin(fx+e)^3 - 16(-a(-1+\sin(fx+e)))^{3/2} a^{5/2} \sin(fx+e)^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2), x)

[Out] $-1/24*(1+\sin(f*x+e))*(-a*(-1+\sin(f*x+e)))^{(1/2)}*(96*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(5/2)}*\sin(f*x+e)^3-16*(-a*(-1+\sin(f*x+e)))^{(3/2)}*\sin(f*x+e)^3*a^{(3/2)}-111*\text{arctanh}((-a*(-1+\sin(f*x+e)))^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^{(3/2)}+15*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(5/2)}+8*(-a*(-1+\sin(f*x+e)))^{(3/2)}*a^{(3/2)}-15*(-a*(-1+\sin(f*x+e)))^{(5/2)}*a^{(1/2)})/a^{(3/2)}/\sin(f*x+e)^3/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(3/2)}$

$e)^{1/2}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{3/2} \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^4, x)

Fricas [B] time = 1.72276, size = 1123, normalized size = 5.7

$$111 \left(a \cos(fx + e)^4 - 2a \cos(fx + e)^2 - \left(a \cos(fx + e)^3 + a \cos(fx + e)^2 - a \cos(fx + e) - a \right) \sin(fx + e) + a \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/96*(111*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a*cos(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) - 4*(16*a*cos(f*x + e)^5 - 64*a*cos(f*x + e)^4 - 17*a*cos(f*x + e)^3 + 165*a*cos(f*x + e)^2 + 9*a*cos(f*x + e) - (16*a*cos(f*x + e)^4 + 80*a*cos(f*x + e)^3 + 63*a*cos(f*x + e)^2 - 102*a*cos(f*x + e) - 93*a)*sin(f*x + e) - 93*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x + e) + f)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(3/2),x)

[Out] Timed out

Giac [B] time = 3.10235, size = 1058, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/48*(222*a^2*\arctan(-(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a}))/\sqrt{-a})*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1)/\sqrt{-a} - 111*a^{3/2}*\log(\operatorname{abs}(-\sqrt{a}*\tan(1/2*f*x + 1/2*e) + \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a}))*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (1110*\sqrt{2})*a^2*\arctan((\sqrt{2})*\sqrt{a} + \sqrt{a})/\sqrt{-a}) - 555*\sqrt{2}*\sqrt{-a}*a^{3/2}*\log(\sqrt{2}*\sqrt{a} + \sqrt{a}) + 1554*a^2*\arctan((\sqrt{2})*\sqrt{a} + \sqrt{a})/\sqrt{-a}) - 777*\sqrt{-a}*a^{3/2}*\log(\sqrt{2}*\sqrt{a} + \sqrt{a}) + 1192*\sqrt{2}*\sqrt{-a}*a^{3/2} + 1706*\sqrt{-a}*a^{3/2})*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1)/(5*\sqrt{2}*\sqrt{-a} + 7*\sqrt{-a}) + (182*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (105*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (138*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (178*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (18*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (2*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))*\tan(1/2*f*x + 1/2*e) + 9*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))*\tan(1/2*f*x + 1/2*e))*\tan(1/2*f*x + 1/2*e))*\tan(1/2*f*x + 1/2*e))/(\sqrt{a}*\tan(1/2*f*x + 1/2*e)^2 + a)^{3/2} - 2*(9*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^5*a^2*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - 18*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^4*a^{5/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + 48*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^2*a^{7/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - 9*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})*a^4*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - 22*a^{9/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))/((\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^2 - a)^3)/f \end{aligned}$$

3.99 $\int (a + a \sin(e + fx))^{5/2} \tan^4(e + fx) dx$

Optimal. Leaf size=151

$$\frac{2a^5 \cos^5(e + fx)}{5f(a \sin(e + fx) + a)^{5/2}} + \frac{8a^4 \cos^3(e + fx)}{3f(a \sin(e + fx) + a)^{3/2}} - \frac{12a^3 \cos(e + fx)}{f\sqrt{a \sin(e + fx) + a}} - \frac{8a^2 \sec(e + fx)\sqrt{a \sin(e + fx) + a}}{f} + \frac{2a \cos(e + fx)}{f}$$

[Out] $(-2*a^5*\text{Cos}[e + f*x]^5)/(5*f*(a + a*\text{Sin}[e + f*x])^{(5/2)}) + (8*a^4*\text{Cos}[e + f*x]^3)/(3*f*(a + a*\text{Sin}[e + f*x])^{(3/2)}) - (12*a^3*\text{Cos}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (8*a^2*\text{Sec}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/f + (2*a*\text{Sec}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(3*f)$

Rubi [A] time = 0.979098, antiderivative size = 208, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2714, 2647, 2646, 4401, 2673, 2878, 2855}

$$\frac{16a^2 \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} - \frac{64a^3 \cos(e + fx)}{15f\sqrt{a \sin(e + fx) + a}} - \frac{46a^2 \sec(e + fx)\sqrt{a \sin(e + fx) + a}}{3f} - \frac{2a \cos(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(5/2)}*\text{Tan}[e + f*x]^4, x]$

[Out] $(-64*a^3*\text{Cos}[e + f*x])/(15*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (16*a^2*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(15*f) - (46*a^2*\text{Sec}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(3*f) - (2*a*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(5*f) - (2*a*\text{Sec}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(3*f) + (26*\text{Sec}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^{(5/2)})/(3*f) - (4*\text{Sec}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^{(7/2)})/(a*f)$

Rule 2714

$\text{Int}[(a + b*\text{Sin}[e + f*x])^m * \text{Tan}[e + f*x]^4, x_Symbol] :> \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x] - \text{Int}[(a + b*\text{Sin}[e + f*x])^{m*(1 - 2*\text{Sin}[e + f*x]^2)} / \text{Cos}[e + f*x]^4, x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]

Rule 2647

$\text{Int}[(a + b*\text{Sin}[c + d*x])^n, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2646

$\text{Int}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]], x_Symbol] :> \text{Simp}[-(2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4401

$\text{Int}[u, x_Symbol] :> \text{With}[\{v = \text{ExpandTrig}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; !\text{InertTrigFreeQ}[u]$

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> -Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m + p + 2)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(b*(m + 1) - a*(p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int (a + a \sin(e + fx))^{5/2} \tan^4(e + fx) dx &= \int (a + a \sin(e + fx))^{5/2} dx - \int \sec^4(e + fx)(a + a \sin(e + fx))^{5/2} (1 - 2 \sin^2(e + fx)) dx \\
 &= -\frac{2a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(8a) \int (a + a \sin(e + fx))^{3/2} dx - \int \sec^4(e + fx)(a + a \sin(e + fx))^{5/2} dx \\
 &= -\frac{16a^2 \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} - \frac{2a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(8a) \int (a + a \sin(e + fx))^{3/2} dx \\
 &= -\frac{64a^3 \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{16a^2 \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} - \frac{2a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(8a) \int (a + a \sin(e + fx))^{3/2} dx \\
 &= -\frac{64a^3 \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{16a^2 \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} - \frac{2a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(8a) \int (a + a \sin(e + fx))^{3/2} dx \\
 &= -\frac{64a^3 \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{16a^2 \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} - \frac{46a^2 \sec(e + fx)}{15f} + \frac{1}{5}(8a) \int (a + a \sin(e + fx))^{3/2} dx
 \end{aligned}$$

Mathematica [A] time = 5.45924, size = 112, normalized size = 0.74

$$\frac{a^2 \sqrt{a(\sin(e + fx) + 1)}(1488 \sin(e + fx) + 16 \sin(3(e + fx)) + 204 \cos(2(e + fx)) - 3 \cos(4(e + fx)) - 1225)}{60f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^3 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sin[e + f*x])]*(-1225 + 204*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 1488*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(60*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```


Maple [A] time = 0.404, size = 87, normalized size = 0.6

$$\frac{2a^3(1 + \sin(fx + e)) \left(3(\sin(fx + e))^4 + 8(\sin(fx + e))^3 + 48(\sin(fx + e))^2 - 192\sin(fx + e) + 128 \right)}{(-15 + 15\sin(fx + e))\cos(fx + e)f} \sqrt{a + a\sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x)

[Out] 2/15*a^3*(1+sin(f*x+e))/(-1+sin(f*x+e))*(3*sin(f*x+e)^4+8*sin(f*x+e)^3+48*sin(f*x+e)^2-192*sin(f*x+e)+128)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [B] time = 1.76375, size = 374, normalized size = 2.48

$$\frac{32 \left(8a^{\frac{5}{2}} - \frac{24a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{44a^{\frac{5}{2}}\sin^2(fx+e)}{(\cos(fx+e)+1)^2} - \frac{68a^{\frac{5}{2}}\sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{75a^{\frac{5}{2}}\sin^4(fx+e)}{(\cos(fx+e)+1)^4} - \frac{68a^{\frac{5}{2}}\sin^5(fx+e)}{(\cos(fx+e)+1)^5} + \frac{44a^{\frac{5}{2}}\sin^6(fx+e)}{(\cos(fx+e)+1)^6} - \frac{24a^{\frac{5}{2}}\sin^7(fx+e)}{(\cos(fx+e)+1)^7} + 8a^{\frac{5}{2}}\sin^8(fx+e) \right)}{15f \left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{\sin^3(fx+e)}{(\cos(fx+e)+1)^3} - 1 \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="maxima")

[Out] 32/15*(8*a^(5/2) - 24*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 44*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 68*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 75*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 68*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 44*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 8*a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)/(f*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))

Fricas [A] time = 1.43246, size = 246, normalized size = 1.63

$$\frac{2 \left(3a^2 \cos^4(fx + e) - 54a^2 \cos^2(fx + e) + 179a^2 - 8 \left(a^2 \cos^2(fx + e) + 23a^2 \right) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a}}{15(f \cos(fx + e) \sin(fx + e) - f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="fricas")

[Out] 2/15*(3*a^2*cos(f*x + e)^4 - 54*a^2*cos(f*x + e)^2 + 179*a^2 - 8*(a^2*cos(f*x + e)^2 + 23*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)*sin(f*x + e) - f*cos(f*x + e))

Sympy [F-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{5}{2}} \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(5/2)*tan(f*x + e)^4, x)

3.100 $\int (a + a \sin(e + fx))^{5/2} \tan^2(e + fx) dx$

Optimal. Leaf size=118

$$\frac{31a^2 \cos(e + fx) \sqrt{a \sin(e + fx) + a}}{15f} + \frac{124a^3 \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}} - \frac{2 \sec(e + fx) (a \sin(e + fx) + a)^{7/2}}{5af} + \frac{9 \sec(e + fx) (a \sin(e + fx) + a)^{5/2}}{5af}$$

[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)

Rubi [A] time = 0.214287, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2713, 2855, 2647, 2646}

$$\frac{31a^2 \cos(e + fx) \sqrt{a \sin(e + fx) + a}}{15f} + \frac{124a^3 \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}} - \frac{2 \sec(e + fx) (a \sin(e + fx) + a)^{7/2}}{5af} + \frac{9 \sec(e + fx) (a \sin(e + fx) + a)^{5/2}}{5af}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]

[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)

Rule 2713

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] + Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2647

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^{5/2} \tan^2(e + fx) dx &= -\frac{2 \sec(e + fx)(a + a \sin(e + fx))^{7/2}}{5af} + \frac{2 \int \sec^2(e + fx)(a + a \sin(e + fx))^{5/2} dx}{5a} \\
&= \frac{9 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{5f} - \frac{2 \sec(e + fx)(a + a \sin(e + fx))^{7/2}}{5af} - \frac{1}{10} \int \frac{2 \sec^2(e + fx)(a + a \sin(e + fx))^{5/2}}{a} dx \\
&= \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} + \frac{9 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{5f} - \frac{2}{10} \int \frac{2 \sec^2(e + fx)(a + a \sin(e + fx))^{5/2}}{a} dx \\
&= \frac{124a^3 \cos(e + fx)}{15f \sqrt{a + a \sin(e + fx)}} + \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} + \frac{9 \sec(e + fx)(a + a \sin(e + fx))^{5/2}}{5f} - \frac{2}{10} \int \frac{2 \sec^2(e + fx)(a + a \sin(e + fx))^{5/2}}{a} dx
\end{aligned}$$

Mathematica [A] time = 5.46323, size = 60, normalized size = 0.51

$$\frac{a^2 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} (-185 \sin(e + fx) + 3 \sin(3(e + fx)) + 22 \cos(2(e + fx)) + 330)}{30f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]

[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(330 + 22*Cos[2*(e + f*x)] - 185*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(30*f)

Maple [A] time = 0.412, size = 67, normalized size = 0.6

$$\frac{2a^3(1 + \sin(fx + e)) \left(3(\sin(fx + e))^3 + 11(\sin(fx + e))^2 + 44\sin(fx + e) - 88 \right)}{15f \cos(fx + e)} \frac{1}{\sqrt{a + a \sin(fx + e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x)

[Out] -2/15*a^3*(1+sin(f*x+e))*(3*sin(f*x+e)^3+11*sin(f*x+e)^2+44*sin(f*x+e)-88)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [A] time = 1.66292, size = 258, normalized size = 2.19

$$\frac{8 \left(22a^{\frac{5}{2}} - \frac{22a^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{55a^{\frac{5}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} - \frac{50a^{\frac{5}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{55a^{\frac{5}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} - \frac{22a^{\frac{5}{2}} \sin^5(fx+e)}{(\cos(fx+e)+1)^5} + \frac{22a^{\frac{5}{2}} \sin^6(fx+e)}{(\cos(fx+e)+1)^6} \right)}{15f \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1 \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="maxima")

```
[Out] -8/15*(22*a^(5/2) - 22*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 50*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 55*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22*a^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.42272, size = 173, normalized size = 1.47

$$\frac{2\left(11a^2\cos^2(fx+e) + 77a^2 + \left(3a^2\cos^2(fx+e) - 47a^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e) + a}}{15f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(11*a^2*cos(f*x + e)^2 + 77*a^2 + (3*a^2*cos(f*x + e)^2 - 47*a^2)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e) + a)^{\frac{5}{2}} \tan^2(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*tan(f*x + e)^2, x)
```

3.101 $\int \cot^2(e + fx)(a + a \sin(e + fx))^{5/2} dx$

Optimal. Leaf size=151

$$\frac{49a^3 \cos(e + fx)}{15f\sqrt{a \sin(e + fx) + a}} + \frac{31a^2 \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} - \frac{5a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{f} + \frac{7a \cos(e + fx)(a \sin(e + fx))^{5/2}}{5f}$$

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rubi [A] time = 0.428696, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{49a^3 \cos(e + fx)}{15f\sqrt{a \sin(e + fx) + a}} + \frac{31a^2 \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} - \frac{5a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{f} + \frac{7a \cos(e + fx)(a \sin(e + fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2), x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rule 2716

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] && !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rule 2773

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \cot^2(e + fx)(a + a \sin(e + fx))^{5/2} dx &= -\frac{\cot(e + fx)(a + a \sin(e + fx))^{5/2}}{f} + \frac{\int \csc(e + fx) \left(\frac{5a}{2} - \frac{7}{2}a \sin(e + fx) \right) (a + a \sin(e + fx))^{3/2} dx}{a} \\ &= \frac{7a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} - \frac{\cot(e + fx)(a + a \sin(e + fx))^{5/2}}{f} + \frac{\int \csc(e + fx) \left(\frac{5a}{2} - \frac{7}{2}a \sin(e + fx) \right) (a + a \sin(e + fx))^{1/2} dx}{a} \\ &= \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} + \frac{7a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{\int \csc(e + fx) \left(\frac{5a}{2} - \frac{7}{2}a \sin(e + fx) \right) dx}{a} \\ &= \frac{49a^3 \cos(e + fx)}{15f \sqrt{a + a \sin(e + fx)}} + \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} + \frac{7a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{\int \csc(e + fx) \left(\frac{5a}{2} - \frac{7}{2}a \sin(e + fx) \right) dx}{a} \\ &= \frac{49a^3 \cos(e + fx)}{15f \sqrt{a + a \sin(e + fx)}} + \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} + \frac{7a \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{5a^{5/2} \tanh^{-1} \left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} \right)}{f} + \frac{49a^3 \cos(e + fx)}{15f \sqrt{a + a \sin(e + fx)}} + \frac{31a^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15f} \end{aligned}$$

Mathematica [A] time = 1.19308, size = 261, normalized size = 1.73

$$\frac{a^2 \csc^4 \left(\frac{1}{2}(e + fx) \right) \sqrt{a(\sin(e + fx) + 1)} \left(-125 \sin \left(\frac{1}{2}(e + fx) \right) - 93 \sin \left(\frac{3}{2}(e + fx) \right) - 25 \sin \left(\frac{5}{2}(e + fx) \right) + 3 \sin \left(\frac{7}{2}(e + fx) \right) \right)}{(30*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2), x]

[Out] -(a^2*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(125*Cos[(e + f*x)/2] - 93*Cos[(3*(e + f*x))/2] + 25*Cos[(5*(e + f*x))/2] + 3*Cos[(7*(e + f*x))/2] - 125*Sin[(e + f*x)/2] + 150*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 150*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 93*Sin[(3*(e + f*x))/2] - 25*Sin[(5*(e + f*x))/2] + 3*Sin[(7*(e + f*x))/2]))/(30*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))

Maple [A] time = 0.733, size = 162, normalized size = 1.1

$$\frac{1 + \sin(fx + e)}{15 \sin(fx + e) \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(\sin(fx + e) \left(90 \sqrt{a - a \sin(fx + e)} a^{5/2} - 40 a^{3/2} (a - a \sin(fx + e)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x)

[Out] 1/15*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*(90*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-40*a^(3/2)*(a-a*sin(f*x+e))^(3/2)+6*a^(1/2)*(a-a*sin(f*x+e))^(5/2)-75*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^3)-15*(a-a*sin(f*x+e))^(1/2)*a^(5/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{5}{2}} \cot(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^2, x)

Fricas [B] time = 1.57082, size = 934, normalized size = 6.19

$$75 \left(a^2 \cos(fx + e)^2 - a^2 - (a^2 \cos(fx + e) + a^2) \sin(fx + e) \right) \sqrt{a} \log \left(\frac{a \cos(fx+e)^3 - 7a \cos(fx+e)^2 - 4(\cos(fx+e)^2 + (\cos(fx+e)+3) \sin(fx+e))}{\cos(fx+e)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/60*(75*(a^2*cos(f*x + e)^2 - a^2 - (a^2*cos(f*x + e) + a^2)*sin(f*x + e))*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*(6*a^2*cos(f*x + e)^4 + 28*a^2*cos(f*x + e)^3 - 40*a^2*cos(f*x + e)^2 - 13*a^2*cos(f*x + e) + 49*a^2 + (6*a^2*cos(f*x + e)^3 - 22*a^2*cos(f*x + e)^2 - 62*a^2*cos(f*x + e) - 49*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*sin(f*x + e) - f)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.90585, size = 714, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/30*(150*a^3*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 75*a^(5/2)*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)))*sgn(tan(1/2*f*x + 1/2*e) + 1) + 30*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) + 1)/((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a) - (150*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 75*sqrt(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 150*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 75*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 83*sqrt(2)*sqrt(-a)*a^(5/2) + 181*sqrt(-a)*a^(5/2))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(2)*sqrt(-a) + sqrt(-a)) + (127*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + (205*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) - (160*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) - (45*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + (15*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e) - 112*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))^2)/(a*tan(1/2*f*x + 1/2*e)^2 + a)^(5/2))/f
```

3.102 $\int \cot^4(e + fx)(a + a \sin(e + fx))^{5/2} dx$

Optimal. Leaf size=227

$$\frac{9a^3 \cos(e + fx)}{40f\sqrt{a \sin(e + fx) + a}} - \frac{16a^2 \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} + \frac{17a^2 \cot(e + fx)\sqrt{a \sin(e + fx) + a}}{24f} + \frac{55a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f}$$

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*f)
- (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rubi [A] time = 0.627151, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2718, 2647, 2646, 3044, 2975, 2981, 2773, 206}

$$\frac{9a^3 \cos(e + fx)}{40f\sqrt{a \sin(e + fx) + a}} - \frac{16a^2 \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} + \frac{17a^2 \cot(e + fx)\sqrt{a \sin(e + fx) + a}}{24f} + \frac{55a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*f)
- (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rule 2718

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)^4,
x_Symbol] := Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]
```

Rule 2647

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
- b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] :=
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])

```

Rule 2975

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rule 2981

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

```

Rule 2773

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 206

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2} dx &= \int (a+a\sin(e+fx))^{5/2} dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{5/2} (1-2\sin^2(e+fx)) dx \\
&= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+fx))^{5/2}}{3f} \\
&= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} \\
&= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e+fx)}{15f} \\
&= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e+fx)}{15f} \\
&= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e+fx)}{15f} \\
&= \frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}
\end{aligned}$$

Mathematica [A] time = 1.89914, size = 360, normalized size = 1.59

$$a^2 \csc^{10}\left(\frac{1}{2}(e+fx)\right) \sqrt{a(\sin(e+fx)+1)} \left(-108 \sin\left(\frac{1}{2}(e+fx)\right) + 706 \sin\left(\frac{3}{2}(e+fx)\right) + 450 \sin\left(\frac{5}{2}(e+fx)\right) - 156 \sin\left(\frac{7}{2}(e+fx)\right) + 100 \cos\left(\frac{9}{2}(e+fx)\right) + 12 \cos\left(\frac{11}{2}(e+fx)\right) - 108 \sin\left[\frac{(e+fx)}{2}\right] - 2475 \operatorname{Log}[1 + \cos\left[\frac{(e+fx)}{2}\right] - \sin\left[\frac{(e+fx)}{2}\right]] \sin[e+fx] + 2475 \operatorname{Log}[1 - \cos\left[\frac{(e+fx)}{2}\right] + \sin\left[\frac{(e+fx)}{2}\right]] \sin[e+fx] + 706 \sin\left[\frac{3}{2}(e+fx)\right] + 450 \sin\left[\frac{5}{2}(e+fx)\right] + 825 \operatorname{Log}[1 + \cos\left[\frac{(e+fx)}{2}\right] - \sin\left[\frac{(e+fx)}{2}\right]] \sin[3(e+fx)] - 825 \operatorname{Log}[1 - \cos\left[\frac{(e+fx)}{2}\right] + \sin\left[\frac{(e+fx)}{2}\right]] \sin[3(e+fx)] - 156 \sin\left[\frac{7}{2}(e+fx)\right] - 100 \sin\left[\frac{9}{2}(e+fx)\right] + 12 \sin\left[\frac{11}{2}(e+fx)\right]\right) / (120 f (1 + \cot\left[\frac{(e+fx)}{2}\right]) (\csc\left[\frac{(e+fx)}{4}\right]^2 - \sec\left[\frac{(e+fx)}{4}\right]^2)^3)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2), x]

[Out] $-(a^2 \csc^{10}((e+fx)/2) \sqrt{a(1+\sin(e+fx))} (108 \cos((e+fx)/2) + 706 \cos(3(e+fx)/2) - 450 \cos(5(e+fx)/2) - 156 \cos(7(e+fx)/2) + 100 \cos(9(e+fx)/2) + 12 \cos(11(e+fx)/2) - 108 \sin((e+fx)/2) - 2475 \operatorname{Log}[1 + \cos((e+fx)/2) - \sin((e+fx)/2)] \sin[e+fx] + 2475 \operatorname{Log}[1 - \cos((e+fx)/2) + \sin((e+fx)/2)] \sin[e+fx] + 706 \sin(3(e+fx)/2) + 450 \sin(5(e+fx)/2) + 825 \operatorname{Log}[1 + \cos((e+fx)/2) - \sin((e+fx)/2)] \sin[3(e+fx)] - 825 \operatorname{Log}[1 - \cos((e+fx)/2) + \sin((e+fx)/2)] \sin[3(e+fx)] - 156 \sin(7(e+fx)/2) - 100 \sin(9(e+fx)/2) + 12 \sin(11(e+fx)/2)) / (120 f (1 + \cot((e+fx)/2)) (\csc((e+fx)/4)^2 - \sec((e+fx)/4)^2)^3)$

Maple [A] time = 0.694, size = 222, normalized size = 1.

$$-\frac{1 + \sin(fx + e)}{120 (\sin(fx + e))^3 \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(48 (-a(-1 + \sin(fx + e)))^{5/2} (\sin(fx + e))^3 \sqrt{a} - 320 (-a(-1 + \sin(fx + e)))^{3/2} \sin(fx + e)^3 a^{3/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2), x)

[Out] $-1/120*(1+\sin(f*x+e))*(-a*(-1+\sin(f*x+e)))^{(1/2)}*(48*(-a*(-1+\sin(f*x+e)))^{(5/2)}*\sin(f*x+e)^3*a^{(1/2)}-320*(-a*(-1+\sin(f*x+e)))^{(3/2)}*\sin(f*x+e)^3*a^{(3/2)})$

$$2)+480*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(5/2)}*\sin(f*x+e)^3-825*\operatorname{arctanh}((-a*(-1+\sin(f*x+e)))^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^3+135*(-a*(-1+\sin(f*x+e)))^{(5/2)}*a^{(1/2)}-440*(-a*(-1+\sin(f*x+e)))^{(3/2)}*a^{(3/2)}+345*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(5/2)})/\sin(f*x+e)^3/a^{(1/2)}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 1.73164, size = 1249, normalized size = 5.5

$$825 \left(a^2 \cos^4(fx + e) - 2a^2 \cos^2(fx + e) + a^2 - \left(a^2 \cos^3(fx + e) + a^2 \cos^2(fx + e) - a^2 \cos(fx + e) - a^2 \right) \sin(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] $\frac{1}{480} \cdot (825 \cdot (a^2 \cos^4(fx + e) - 2a^2 \cos^2(fx + e) + a^2 - (a^2 \cos^3(fx + e) + a^2 \cos^2(fx + e) - a^2) \sin(fx + e)) \sqrt{a} \cdot \log((a \cos^3(fx + e) - 7a \cos^2(fx + e) + 4(\cos^2(fx + e) + (\cos(fx + e) + 3) \sin(fx + e) - 2 \cos(fx + e) - 3) \sqrt{a \sin(fx + e) + a}) \sqrt{a} - 9a \cos(fx + e) + (a \cos^2(fx + e) + 8a \cos(fx + e) - a) \sin(fx + e) - a) / (\cos^3(fx + e) + \cos^2(fx + e) + (\cos^2(fx + e) - 1) \sin(fx + e) - \cos(fx + e) - 1)) - 4 \cdot (48a^2 \cos^6(fx + e) + 224a^2 \cos^5(fx + e) - 128a^2 \cos^4(fx + e) - 583a^2 \cos^3(fx + e) + 147a^2 \cos^2(fx + e) + 399a^2 \cos(fx + e) - 27a^2 + (48a^2 \cos^5(fx + e) - 176a^2 \cos^4(fx + e) - 304a^2 \cos^3(fx + e) + 279a^2 \cos^2(fx + e) + 426a^2 \cos(fx + e) + 27a^2) \sin(fx + e)) \sqrt{a \sin(fx + e) + a}) / (f \cos^4(fx + e) - 2f \cos^2(fx + e) - (f \cos^3(fx + e) + f \cos^2(fx + e) - f \cos(fx + e) - f) \sin(fx + e) + f)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(5/2),x)

[Out] Timed out

Giac [B] time = 3.1945, size = 1135, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out]
$$-1/240*(1650*a^3*\arctan(-(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})/\sqrt{-a})*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1)/\sqrt{-a} - 825*a^{5/2}*\log(\operatorname{abs}(-\sqrt{a}*\tan(1/2*f*x + 1/2*e) + \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a}))*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (8250*\sqrt{2})*a^3*\arctan((\sqrt{2})*\sqrt{a} + \sqrt{a})/\sqrt{-a}) - 4125*\sqrt{2}*\sqrt{-a}*a^{5/2}*\log(\sqrt{2}*\sqrt{a} + \sqrt{a}) + 11550*a^3*\arctan((\sqrt{2})*\sqrt{a} + \sqrt{a})/\sqrt{-a}) - 5775*\sqrt{-a}*a^{5/2}*\log(\sqrt{2}*\sqrt{a} + \sqrt{a}) + 728*\sqrt{2}*\sqrt{-a}*a^{5/2} + 1030*\sqrt{-a}*a^{5/2})*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1)/(5*\sqrt{2}*\sqrt{-a} + 7*\sqrt{-a}) + (346*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + (405*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + (100*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - (545*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + (720*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + (641*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + 5*(20*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + (2*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))*\tan(1/2*f*x + 1/2*e) + 15*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))*\tan(1/2*f*x + 1/2*e))*\tan(1/2*f*x + 1/2*e))*\tan(1/2*f*x + 1/2*e))*\tan(1/2*f*x + 1/2*e))/(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^{5/2} - 10*(15*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^5*a^3*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + 18*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^4*a^{7/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - 24*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^2*a^{9/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) - 15*(\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})*a^5*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1) + 14*a^{11/2}*\operatorname{sgn}(\tan(1/2*f*x + 1/2*e) + 1))/((\sqrt{a}*\tan(1/2*f*x + 1/2*e) - \sqrt{a*\tan(1/2*f*x + 1/2*e)^2 + a})^2 - a)^3)/f$$

$$3.103 \quad \int \frac{\tan^4(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=150

$$\frac{\tan^3(e+fx)}{3f\sqrt{a \sin(e+fx)+a}} + \frac{a \sin(e+fx) \tan(e+fx)}{24f(a \sin(e+fx)+a)^{3/2}} - \frac{(127 \sin(e+fx)+53) \sec(e+fx)}{192f\sqrt{a \sin(e+fx)+a}} - \frac{67 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{64\sqrt{2}\sqrt{a}f}$$

[Out] (-67*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(64*Sqrt[2]*Sqrt[a]*f) - (Sec[e + f*x]*(53 + 127*Sin[e + f*x]))/(192*f*Sqrt[a + a*Sin[e + f*x]]) + (a*Sin[e + f*x]*Tan[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2)) + Tan[e + f*x]^3/(3*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.933351, antiderivative size = 241, normalized size of antiderivative = 1.61, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2649, 206, 4401, 2687, 2681, 2650, 2877, 2855}

$$\frac{61a \cos(e+fx)}{64f(a \sin(e+fx)+a)^{3/2}} + \frac{7 \sec^3(e+fx) \sqrt{a \sin(e+fx)+a}}{12af} - \frac{5 \sec^3(e+fx)}{6f\sqrt{a \sin(e+fx)+a}} - \frac{61 \sec(e+fx)}{48f\sqrt{a \sin(e+fx)+a}} + \frac{67 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{64\sqrt{2}\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]

[Out] (61*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(64*Sqrt[2]*Sqrt[a]*f) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*f) + (61*a*Cos[e + f*x])/(64*f*(a + a*Sin[e + f*x])^(3/2)) + (7*a*Sec[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2)) - (61*Sec[e + f*x])/(48*f*Sqrt[a + a*Sin[e + f*x]]) - (5*Sec[e + f*x]^3)/(6*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/(12*a*f)

Rule 2714

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4401

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqr
t[a + b*sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*cos
[e + f*x])^(p + 2)/(a + b*sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] := Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*cos[c
+ d*x]*(a + b*sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(b*(g*cos[e + f*x])^
(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*
c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x
])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx &= \int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx - \int \frac{\sec^4(e+fx)(1-2\sin^2(e+fx))}{\sqrt{a+a\sin(e+fx)}} dx \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} - \int \left(\frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}} - \frac{2\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(1+\sin(e+fx))}} \right) dx \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + 2 \int \frac{\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(1+\sin(e+fx))}} dx - \int \frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}} dx \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec^4(e+fx)\sqrt{a+a\sin(e+fx)} dx}{2a^2} \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{7\sec^5(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{61\sec(e+fx)}{48f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{7\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} \\
&= -\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{7\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} \\
&= \frac{61 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\sqrt{af}} - \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{af}} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.673361, size = 118, normalized size = 0.79

$$\frac{-\sec^3(e+fx)(-41\sin(e+fx)+183\sin(3(e+fx))+122\cos(2(e+fx))+90)+(804+804i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{768f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]], x]

[Out] ((804 + 804*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - Sec[e + f*x]^3*(90 + 122*Cos[2*(e + f*x)] - 41*Sin[e + f*x] + 183*Sin[3*(e + f*x)])/(768*f*Sqrt[a*(1 + Sin[e + f*x])])

Maple [A] time = 0.766, size = 231, normalized size = 1.5

$$\frac{1}{(-384 + 384 \sin(fx + e))(1 + \sin(fx + e)) \cos(fx + e)} f \left(366 a^{7/2} \sin(fx + e) (\cos(fx + e))^2 + \left(402 \sqrt{2} \operatorname{Artanh} \left(\frac{\sqrt{a} \cos(fx + e)}{\sqrt{2} \sqrt{a + a \sin(fx + e)}} \right) \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)`

[Out] $\frac{1}{384} \cdot (366 \cdot a^{7/2} \cdot \sin(fx+e) \cdot \cos(fx+e)^2 + (402 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cdot (a-a \cdot \sin(fx+e))^{1/2}) \cdot 2^{1/2} / a^{1/2}) \cdot a^2 \cdot (a-a \cdot \sin(fx+e))^{3/2} - 112 \cdot a^{7/2}) \cdot \sin(fx+e) + (-201 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cdot (a-a \cdot \sin(fx+e))^{1/2}) \cdot 2^{1/2} / a^{1/2}) \cdot a^2 \cdot (a-a \cdot \sin(fx+e))^{3/2} + 122 \cdot a^{7/2}) \cdot \cos(fx+e)^2 + 402 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cdot (a-a \cdot \sin(fx+e))^{1/2}) \cdot 2^{1/2} / a^{1/2}) \cdot a^2 \cdot (a-a \cdot \sin(fx+e))^{3/2} - 16 \cdot a^{7/2}) / a^{7/2} / (-1 + \sin(fx+e)) / (1 + \sin(fx+e)) / \cos(fx+e) / (a + a \cdot \sin(fx+e))^{1/2} / f$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] Timed out

Fricas [A] time = 1.63851, size = 624, normalized size = 4.16

$$\frac{201 \sqrt{2} \left(\cos(fx+e)^3 \sin(fx+e) + \cos(fx+e)^3 \right) \sqrt{a} \log \left(-\frac{a \cos(fx+e)^2 - 2 \sqrt{2} \sqrt{a \sin(fx+e) + a} \sqrt{a} (\cos(fx+e) - \sin(fx+e) + 1) + 3 a \cos(fx+e)}{\cos(fx+e)^2 - (\cos(fx+e) + 2) \sin(fx+e) - \cos(fx+e) - 2} \right)}{768 \left(a f \cos(fx+e)^3 \sin(fx+e) + a f \cos(fx+e)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{768} \cdot (201 \cdot \sqrt{2} \cdot (\cos(fx+e)^3 \cdot \sin(fx+e) + \cos(fx+e)^3) \cdot \sqrt{a} \cdot \log(- (a \cdot \cos(fx+e)^2 - 2 \cdot \sqrt{2} \cdot \sqrt{a \cdot \sin(fx+e) + a} \cdot \sqrt{a} \cdot (\cos(fx+e) - \sin(fx+e) + 1) + 3 \cdot a \cdot \cos(fx+e) - (a \cdot \cos(fx+e) - 2 \cdot a) \cdot \sin(fx+e) + 2 \cdot a) / (\cos(fx+e)^2 - (\cos(fx+e) + 2) \cdot \sin(fx+e) - \cos(fx+e) - 2)) - 4 \cdot (61 \cdot \cos(fx+e)^2 + (183 \cdot \cos(fx+e)^2 - 56) \cdot \sin(fx+e) - 8) \cdot \sqrt{a \cdot \sin(fx+e) + a}) / (a \cdot f \cdot \cos(fx+e)^3 \cdot \sin(fx+e) + a \cdot f \cdot \cos(fx+e)^3))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e + fx)}{\sqrt{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)`

[Out] `Integral(tan(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)`

Giac [B] time = 5.88912, size = 1056, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out]
$$\frac{1}{192} \cdot (201 \cdot \sqrt{2} \cdot \arctan(-\frac{1}{2} \sqrt{2} \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a}) / \sqrt{-a})) / (\sqrt{-a} \cdot \operatorname{sgn}(\tan(\frac{1}{2} f x + \frac{1}{2} e) + 1)) - 16 \cdot (9 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^5 - 39 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^4 \cdot \sqrt{a} - 26 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^3 \cdot a + 78 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^2 \cdot a^{3/2} + 69 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a}) \cdot a^2 + 13 \cdot a^{5/2})) / (((\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^2 - 2 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})) \cdot \sqrt{a} - a)^3 \cdot \operatorname{sgn}(\tan(\frac{1}{2} f x + \frac{1}{2} e) + 1)) - 6 \cdot (43 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^7 + 237 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^6 \cdot \sqrt{a} + 161 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^5 \cdot a - 221 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^4 \cdot a^{3/2} + 25 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^3 \cdot a^2 + 103 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^2 \cdot a^{5/2} - 93 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a}) \cdot a^3 + 17 \cdot a^{7/2})) / (((\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})^2 + 2 \cdot (\sqrt{a} \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e) - \sqrt{a \cdot \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + a})) \cdot \sqrt{a} - a)^4 \cdot \operatorname{sgn}(\tan(\frac{1}{2} f x + \frac{1}{2} e) + 1))) / f$$

3.104 $\int \frac{\tan^2(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$

Optimal. Leaf size=107

$$\frac{3 \sec(e+fx) \sqrt{a \sin(e+fx)+a}}{4af} - \frac{\sec(e+fx)}{2f \sqrt{a \sin(e+fx)+a}} + \frac{5 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{4\sqrt{2} \sqrt{af}}$$

[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)

Rubi [A] time = 0.195246, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2712, 2855, 2649, 206}

$$\frac{3 \sec(e+fx) \sqrt{a \sin(e+fx)+a}}{4af} - \frac{\sec(e+fx)}{2f \sqrt{a \sin(e+fx)+a}} + \frac{5 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{4\sqrt{2} \sqrt{af}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]

[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)

Rule 2712

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]), x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec^2(e+fx)\sqrt{a+a\sin(e+fx)}\left(-\frac{a}{2}+2a\sin(e+fx)\right) dx}{2a^2} \\ &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af} - \frac{5}{8} \int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx \\ &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af} + \frac{5 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{4f} \\ &= \frac{5 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2\sqrt{a+a\sin(e+fx)}}}\right)}{4\sqrt{2}\sqrt{af}} - \frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af} \end{aligned}$$

Mathematica [C] time = 0.269125, size = 118, normalized size = 1.1

$$\frac{\sec(e+fx)\left(-3\sin(e+fx) + (5+5i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{4f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]], x]

[Out] -(Sec[e + f*x]*(-1 + (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 3*Sin[e + f*x]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])])

Maple [A] time = 0.462, size = 130, normalized size = 1.2

$$\frac{1}{8f\cos(fx+e)}\left(\sin(fx+e)\left(5\sqrt{2}\text{Artanh}\left(\frac{1}{2}\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{\sqrt{a}}\right)a\sqrt{a-a\sin(fx+e)}+6a^{3/2}\right)+5\sqrt{2}\text{Artan}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2), x)

[Out] 1/8*(sin(f*x+e)*(5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(f*x+e))^(1/2)+6*a^(3/2))+5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(f*x+e))^(1/2)+2*a^(3/2))/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(fx+e)}{\sqrt{a\sin(fx+e)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)

Fricas [B] time = 1.54293, size = 549, normalized size = 5.13

$$5\sqrt{2}(\cos(fx+e)\sin(fx+e) + \cos(fx+e))\sqrt{a}\log\left(\frac{-\frac{a\cos(fx+e)^2+2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2))}{16(af\cos(fx+e)\sin(fx+e) + af\cos(fx+e))}}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/16*(5*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*sqrt(a*sin(f*x + e) + a)*(3*sin(f*x + e) + 1))/(a*f*cos(f*x + e)*sin(f*x + e) + a*f*cos(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e + fx)}{\sqrt{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)

[Out] Integral(tan(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)

Giac [B] time = 2.82308, size = 594, normalized size = 5.55

$$\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+\sqrt{a}}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} - \frac{4\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+\sqrt{a}}\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] -1/4*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x

$$\begin{aligned}
& + 1/2*e) + 1)) - 4*(\text{sqrt}(a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2 \\
& *e)^2 + a) + \text{sqrt}(a))/(((\text{sqrt}(a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x \\
& + 1/2*e)^2 + a))^2 - 2*(\text{sqrt}(a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + \\
& 1/2*e)^2 + a))*\text{sqrt}(a) - a)*\text{sgn}(\tan(1/2*f*x + 1/2*e) + 1)) - 2*(3*(\text{sqrt}(a) \\
& *\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2*e)^2 + a))^3 + (\text{sqrt}(a)*\tan \\
& (1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2*e)^2 + a))^2*\text{sqrt}(a) - (\text{sqrt}(\\
& a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2*e)^2 + a))*a + a^{(3/2)})/ \\
& (((\text{sqrt}(a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2 \\
& *(\text{sqrt}(a)*\tan(1/2*f*x + 1/2*e) - \text{sqrt}(a*\tan(1/2*f*x + 1/2*e)^2 + a))*\text{sqrt}(a \\
&) - a)^2*\text{sgn}(\tan(1/2*f*x + 1/2*e) + 1)))/f
\end{aligned}$$

$$3.105 \quad \int \frac{\cot^2(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{af}} - \frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.110095, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2716, 21, 2773, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{af}} - \frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]

[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])

Rule 2716

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^2,
  x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
  Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x],
  x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
  2] && !LtQ[m, -1]
```

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplrQ[c + d*x,
  a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
  f_)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
  ], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
  e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx &= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{a} \\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)} dx}{2a} \\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 0.307822, size = 138, normalized size = 2.23

$$\frac{\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)\csc\left(\frac{1}{4}(e+fx)\right)\sec\left(\frac{1}{4}(e+fx)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)-2\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\log\left(-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]], x]

[Out] (Csc[(e + f*x)/4]*Sec[(e + f*x)/4]*(-2*Cos[(e + f*x)/2] + 2*Sin[(e + f*x)/2] + (Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]*(1 + Tan[(e + f*x)/2]))/(8*f*Sqrt[a*(1 + Sin[e + f*x])])

Maple [A] time = 0.59, size = 103, normalized size = 1.7

$$-\frac{1+\sin(fx+e)}{\sin(fx+e)\cos(fx+e)}f\sqrt{-a(-1+\sin(fx+e))}\left(-\text{Arctanh}\left(\sqrt{a-a\sin(fx+e)}\frac{1}{\sqrt{a}}\right)\sin(fx+e)a+\sqrt{a-a\sin(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2), x)

[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*sin(f*x+e)*a+(a-a*sin(f*x+e))^(1/2)*a^(1/2))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(fx+e)}{\sqrt{a\sin(fx+e)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2), x, algorithm="maxima")

[Out] integrate(cot(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)

Fricas [B] time = 1.60944, size = 706, normalized size = 11.39

$$\frac{\left(\cos(fx + e)^2 - (\cos(fx + e) + 1)\sin(fx + e) - 1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3 - 7a\cos(fx+e)^2 + 4(\cos(fx+e)^2 + (\cos(fx+e)+3)\sin(fx+e) - 2)\cos(fx+e) + a}{\cos(fx+e)^3 + \cos(fx+e)^2 + (\cos(fx+e)^2 - 1)\sin(fx+e) - \cos(fx+e) - 1}\right)}{4\left(af\cos(fx + e)^2 - af - (af\cos(fx + e) + af)\sin(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1))/(a*f*cos(f*x + e)^2 - a*f - (a*f*cos(f*x + e) + a*f)*sin(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e + fx)}{\sqrt{a}(\sin(e + fx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)

[Out] Integral(cot(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)

Giac [B] time = 2.21903, size = 502, normalized size = 8.1

$$\frac{\left(2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}\sqrt{-a}\log(\sqrt{2}\sqrt{a}+\sqrt{a})+2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\log(\sqrt{2}\sqrt{a}+\sqrt{a})-\sqrt{2}\sqrt{-a}-3\sqrt{-a}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\sqrt{-a}\sqrt{a}+\sqrt{-a}\sqrt{a}} - \frac{2\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - sqrt(2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(2)*sqrt(-a)*sqrt(a) + sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(2)*sqrt(a))/sqrt(-a))

$$\frac{(a \tan(1/2 f x + 1/2 e)^2 + a) / \sqrt{-a}}{(\sqrt{-a} \operatorname{sgn}(\tan(1/2 f x + 1/2 e) + 1)) + \log(\operatorname{abs}(-\sqrt{a} \tan(1/2 f x + 1/2 e) + \sqrt{a \tan(1/2 f x + 1/2 e)^2 + a})) / (\sqrt{a} \operatorname{sgn}(\tan(1/2 f x + 1/2 e) + 1)) + \sqrt{a \tan(1/2 f x + 1/2 e)^2 + a} / (a \operatorname{sgn}(\tan(1/2 f x + 1/2 e) + 1)) + 2 \sqrt{a} / (((\sqrt{a} \tan(1/2 f x + 1/2 e) - \sqrt{a \tan(1/2 f x + 1/2 e)^2 + a})^2 - a) \operatorname{sgn}(\tan(1/2 f x + 1/2 e) + 1))} / f$$

$$3.106 \quad \int \frac{\cot^4(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=135

$$\frac{9 \cot(e+fx)}{8f\sqrt{a \sin(e+fx)+a}} - \frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{8\sqrt{af}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3f\sqrt{a \sin(e+fx)+a}} + \frac{\cot(e+fx) \csc(e+fx)}{12f\sqrt{a \sin(e+fx)+a}}$$

[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*Sqrt[a]*f) + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.621613, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2718, 2649, 206, 3044, 2984, 2985, 2773}

$$\frac{9 \cot(e+fx)}{8f\sqrt{a \sin(e+fx)+a}} - \frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{8\sqrt{af}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3f\sqrt{a \sin(e+fx)+a}} + \frac{\cot(e+fx) \csc(e+fx)}{12f\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]

[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*Sqrt[a]*f) + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2718

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^4, x_Symbol] := Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && !LtQ[m, -1]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3044

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*

$m + b*c*(n + 1) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1))) * \text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])

Rule 2984

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x))^m) \cdot (A + (B \cdot \sin(e + f \cdot x) + (f \cdot x)) \cdot (c + (d \cdot \sin(e + f \cdot x))^n)), x_Symbol] \rightarrow \text{Simp}[(B \cdot c - A \cdot d) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \text{Sin}[e + f \cdot x])^m \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^{n+1} / (f \cdot (n + 1) \cdot (c^2 - d^2)), x] + \text{Dist}[1 / (b \cdot (n + 1) \cdot (c^2 - d^2)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^m \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^{n+1} \cdot \text{Simp}[A \cdot (a \cdot d \cdot m + b \cdot c \cdot (n + 1)) - B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n + 1)) + b \cdot (B \cdot c - A \cdot d) \cdot (m + n + 2) \cdot \text{Sin}[e + f \cdot x], x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 2985

$\text{Int}[(A + (B \cdot \sin(e + f \cdot x)) / (\text{Sqrt}[a + b \cdot \text{Sin}[e + f \cdot x]]) \cdot (c + (d \cdot \sin(e + f \cdot x))^n)), x_Symbol] \rightarrow \text{Dist}[(A \cdot b - a \cdot B) / (b \cdot c - a \cdot d), \text{Int}[1 / \text{Sqrt}[a + b \cdot \text{Sin}[e + f \cdot x]], x], x] + \text{Dist}[(B \cdot c - A \cdot d) / (b \cdot c - a \cdot d), \text{Int}[\text{Sqrt}[a + b \cdot \text{Sin}[e + f \cdot x]] / (c + d \cdot \text{Sin}[e + f \cdot x]), x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2773

$\text{Int}[\text{Sqrt}[a + (b \cdot \sin(e + f \cdot x))] / ((c + (d \cdot \sin(e + f \cdot x) + (f \cdot x)) \cdot (c + (d \cdot \sin(e + f \cdot x))^n)), x_Symbol] \rightarrow \text{Dist}[(-2 \cdot b) / f, \text{Subst}[\text{Int}[1 / (b \cdot c + a \cdot d - d \cdot x^2), x], x, (b \cdot \text{Cos}[e + f \cdot x]) / \text{Sqrt}[a + b \cdot \text{Sin}[e + f \cdot x]]], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx &= \int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx + \int \frac{\csc^4(e+fx)(1-2\sin^2(e+fx))}{\sqrt{a+a\sin(e+fx)}} dx \\
&= -\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a} - \frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a} \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a} \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a} \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a} \\
&= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a}
\end{aligned}$$

Mathematica [B] time = 0.579591, size = 292, normalized size = 2.16

$$\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-36\sin\left(\frac{1}{2}(e+fx)\right)-46\sin\left(\frac{3}{2}(e+fx)\right)+54\sin\left(\frac{5}{2}(e+fx)\right)+36\sin\left(\frac{7}{2}(e+fx)\right)+36\sin\left(\frac{9}{2}(e+fx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]

[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(36*Cos[(e + f*x)/2] - 46*Cos[(3*(e + f*x))/2] - 54*Cos[(5*(e + f*x))/2] - 36*Sin[(e + f*x)/2] - 63*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 63*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 46*Sin[(3*(e + f*x))/2] + 54*Sin[(5*(e + f*x))/2] + 21*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 21*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)])/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*sqrt[a*(1 + Sin[e + f*x])])

Maple [A] time = 0.638, size = 144, normalized size = 1.1

$$\frac{1 + \sin(fx + e)}{24 (\sin(fx + e))^3 \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(-21 \operatorname{Arctanh}\left(\frac{\sqrt{-a(-1 + \sin(fx + e))}}{\sqrt{a}}\right) \right) (\sin(fx + e))^3 a^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-21*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+27*(-a*(-1+sin(f*x+e)))^(5/2)*a^(1/2)-56*(-a*(-1+sin(f*x+e)))^(3/2)*a^(3/2)+21*(-a*(-1+sin(f*x+e)))^(1/2)*a^(5/2))/sin(f*x+e)^3/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.46294, size = 987, normalized size = 7.31

$$21 \left(\cos(fx + e)^4 - 2 \cos(fx + e)^2 - \left(\cos(fx + e)^3 + \cos(fx + e)^2 - \cos(fx + e) - 1 \right) \sin(fx + e) + 1 \right) \sqrt{a} \log \left(\frac{a \cos(fx + e)^3 - 7 a \cos(fx + e)^2 - 4(\cos(fx + e)^2 + (\cos(fx + e) + 3) \sin(fx + e) - 2 \cos(fx + e) - 3) \sqrt{a \sin(fx + e) + a} \sqrt{a} - 9 a \cos(fx + e) + (a \cos(fx + e)^2 + 8 a \cos(fx + e) - a) \sin(fx + e) - a}{\cos(fx + e)^3 + \cos(fx + e)^2 + (\cos(fx + e)^2 - 1) \sin(fx + e) - \cos(fx + e) - 1} \right) - 4(27 \cos(fx + e)^3 + 25 \cos(fx + e)^2 - (27 \cos(fx + e)^2 + 2 \cos(fx + e) - 17) \sin(fx + e) - 19 \cos(fx + e) - 17) \sqrt{a \sin(fx + e) + a} / (a f \cos(fx + e)^4 - 2 a f \cos(fx + e)^2 + a f - (a f \cos(fx + e)^3 + a f \cos(fx + e)^2 - a f \cos(fx + e) - a f) \sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) - 4*(27*cos(f*x + e)^3 + 25*cos(f*x + e)^2 - (27*cos(f*x + e)^2 + 2*cos(f*x + e) - 17)*sin(f*x + e) - 19*cos(f*x + e) - 17)*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f - (a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2 - a*f*cos(f*x + e) - a*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(e + fx)}{\sqrt{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.37322, size = 819, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out]
$$\frac{1}{48} \left(\sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \left(\frac{2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{a \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} - \frac{3}{a \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} \right) \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \frac{22}{a \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} - \left(210 \sqrt{2} \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 105 \sqrt{2} \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right) + 294 \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 147 \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right) - 128 \sqrt{2} \sqrt{-a} - 186 \sqrt{-a} \right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) / \left(5 \sqrt{2} \sqrt{-a} \sqrt{a} + 7 \sqrt{-a} \sqrt{a} \right) + 42 \arctan\left(-\frac{\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}}{\sqrt{-a}}\right) / \left(\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) \right) - 21 \log\left(\frac{\operatorname{abs}\left(-\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)}{\sqrt{a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} - 2 \left(3 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^5 + 18 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^4 \sqrt{a} - 48 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 a^{3/2} - 3 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right) a^2 + 22 a^{5/2}}{\left(\left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 - a \right)^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} \right) / f$$

$$3.107 \quad \int \frac{\tan^4(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=177

$$\frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{\tan^3(e+fx)}{3f(a \sin(e+fx)+a)^{3/2}} + \frac{a \sin(e+fx) \tan(e+fx)}{12f(a \sin(e+fx)+a)^{5/2}} + \frac{7 \cos(e+fx)}{256f(a \sin(e+fx)+a)^{3/2}} - \frac{(87 \cos(e+fx) \tan(e+fx) - 65 \sec(e+fx))}{192f(a \sin(e+fx)+a)^{3/2}} + \frac{a \sin(e+fx) \tan(e+fx)}{12f(a \sin(e+fx)+a)^{5/2}} + \frac{7 \cos(e+fx)}{256f(a \sin(e+fx)+a)^{3/2}} - \frac{(87 \cos(e+fx) \tan(e+fx) - 65 \sec(e+fx))}{192f(a \sin(e+fx)+a)^{3/2}}$$

[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) - (Sec[e + f*x]*(65 + 87*Sin[e + f*x]))/(192*f*(a + a*Sin[e + f*x])^(3/2)) + (a*Sin[e + f*x]*Tan[e + f*x])/(12*f*(a + a*Sin[e + f*x])^(5/2)) + Tan[e + f*x]^3/(3*f*(a + a*Sin[e + f*x])^(3/2))

Rubi [A] time = 1.19927, antiderivative size = 195, normalized size of antiderivative = 1.1, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2855}

$$\frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7 \cos(e+fx)}{256f(a \sin(e+fx)+a)^{3/2}} + \frac{\sec^3(e+fx)}{4af\sqrt{a \sin(e+fx)+a}} - \frac{\sec^3(e+fx)}{6f(a \sin(e+fx)+a)^{3/2}} - \frac{4 \sec^3(e+fx)}{64af}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2), x]

[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) + (9*Sec[e + f*x])/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]^3/(6*f*(a + a*Sin[e + f*x])^(3/2)) - (45*Sec[e + f*x])/(64*a*f*Sqrt[a + a*Sin[e + f*x]]) + Sec[e + f*x]^3/(4*a*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2714

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 4401

Int[u_, x_Symbol] :=> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]

Rule 2681

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :=> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2687

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]], x_Symbol] :=> -Simp[(b*(g*cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)/(a + b*sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2877

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_.)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :=> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1)*(a*m - b*(2*m + p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :=> -Simp[(b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}} dx &= \int \frac{1}{(a+a\sin(e+fx))^{3/2}} dx - \int \frac{\sec^4(e+fx)(1-2\sin^2(e+fx))}{(a+a\sin(e+fx))^{3/2}} dx \\
&= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{4a} - \int \left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{3/2}} - \frac{2\sec^2(e+fx)}{(a(1+\sin(e+fx)))^{3/2}} \right) dx \\
&= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + 2 \int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{3/2}} dx - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af} \\
&= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sec^2(e+fx)}{2a-x^2} dx}{2af}
\end{aligned}$$

Mathematica [C] time = 0.355786, size = 334, normalized size = 1.89

$$-\frac{192\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{32\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - 171\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2 + 342\sin\left(\frac{1}{2}(e+fx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2), x]

[Out] (124 + (64*Sin[(e + f*x)/2]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 32/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (248*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 342*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 171*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (21 + 21*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (192*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])/(768*f*(a*(1 + Sin[e + f*x]))^(3/2))

Maple [A] time = 0.619, size = 289, normalized size = 1.6

$$\frac{1}{(-1536 + 1536 \sin(fx + e))(1 + \sin(fx + e))^2 \cos(fx + e) f} \left(\left(-1080 a^{9/2} - 21 (a - a \sin(fx + e))^{3/2} \sqrt{2} \operatorname{Artanh} \left(\frac{1}{2} \sqrt{a \sin(fx + e) + a} \sqrt{a (\cos(fx + e) - \sin(fx + e))} \right) \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)`

[Out] `-1/1536/a^(11/2)*((-1080*a^(9/2)-21*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(384*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+42*a^(9/2)*cos(f*x+e)^4+(-648*a^(9/2)-63*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(f*x+e)^2+128*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(-1+sin(f*x+e))/(1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f`

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] Timed out

Fricas [A] time = 1.62036, size = 726, normalized size = 4.1

$$\frac{21 \sqrt{2} \left(\cos(fx + e)^5 - 2 \cos(fx + e)^3 \sin(fx + e) - 2 \cos(fx + e)^3 \right) \sqrt{a} \log \left(-\frac{a \cos(fx + e)^2 + 2 \sqrt{2} \sqrt{a \sin(fx + e) + a} \sqrt{a (\cos(fx + e) - \sin(fx + e))}}{\cos(fx + e)^2 - (\cos(fx + e) + 2) \sin(fx + e)} \right)}{3072 \left(a^2 f \cos(fx + e)^5 - 2 a^2 f \cos(fx + e)^3 \sin(fx + e) - 2 a^2 f \cos(fx + e)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")`

[Out] `1/3072*(21*sqrt(2)*(cos(f*x + e)^5 - 2*cos(f*x + e)^3*sin(f*x + e) - 2*cos(f*x + e)^3)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(21*cos(f*x + e)^4 - 324*cos(f*x + e)^2 - 12*(45*cos(f*x + e)^2 - 16)*sin(f*x + e) + 64)*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^5 - 2*a^2*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2*f*cos(f*x + e)^3)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

$$3.108 \quad \int \frac{\tan^2(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{32\sqrt{2}a^{3/2}f} + \frac{\cos(e+fx)}{32f(a \sin(e+fx)+a)^{3/2}} + \frac{5 \sec(e+fx)}{8af\sqrt{a \sin(e+fx)+a}} - \frac{\sec(e+fx)}{4f(a \sin(e+fx)+a)^{3/2}}$$

[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.223266, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2712, 2855, 2650, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{32\sqrt{2}a^{3/2}f} + \frac{\cos(e+fx)}{32f(a \sin(e+fx)+a)^{3/2}} + \frac{5 \sec(e+fx)}{8af\sqrt{a \sin(e+fx)+a}} - \frac{\sec(e+fx)}{4f(a \sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]

[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2712

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*tan[(e_) + (f_.)*(x_)]^2, x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]), x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_.)*(x_)]*(g_.)^(p_))*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2650

Int[((a_) + (b_.)*sin[(c_) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}} dx &= -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{3a}{2}+4a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{4a^2} \\ &= -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{1}{16} \int \frac{1}{(a+a\sin(e+fx))^{3/2}} dx \\ &= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{16} \\ &= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{\text{Subst}\left[\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx\right]}{16} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}a^{3/2}f} + \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{8af} \end{aligned}$$

Mathematica [C] time = 0.443707, size = 128, normalized size = 0.96

$$\frac{\sec(e+fx)\left(-40\sin(e+fx) - \cos(2(e+fx)) + (2+2i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{64f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2), x]
```

```
[Out] -(Sec[e + f*x]*(-25 - Cos[2*(e + f*x)] + (2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2
+ I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 40*Sin[e + f*x]))/(64*f*(a
(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.607, size = 202, normalized size = 1.5

$$\frac{1}{(64 + 64 \sin(fx + e)) \cos(fx + e) f} \left(\sin(fx + e) \left(2 \sqrt{a - a \sin(fx + e)} \sqrt{2} \operatorname{Artanh} \left(1/2 \frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{\sqrt{a}} \right) \right) a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2), x)
```

```
[Out] 1/64/a^(7/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+40*a^(5/2))+(-(a-a*sin(f*x+e))^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+2*a^(5/2)
)*cos(f*x+e)^2+2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+24*a^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin
(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [B] time = 1.63547, size = 644, normalized size = 4.81

$$\frac{\sqrt{2}\left(\cos(fx+e)^3 - 2\cos(fx+e)\sin(fx+e) - 2\cos(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2 + 2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e))}{\cos(fx+e)^2 - (\cos(fx+e)+2)}\right)}{128\left(a^2f\cos(fx+e)^3 - 2a^2f\cos(fx+e)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt
(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
- 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) - 4*(cos(f*x + e)^2 + 20*sin(f*x + e) + 12)*sqrt(a*
sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^3 - 2*a^2*f*cos(f*x + e)*sin(f*x + e)
) - 2*a^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

$$3.109 \quad \int \frac{\cot^2(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=113

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{a^{3/2} f} - \frac{2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2\sqrt{a} \sin(e+fx)+a}}\right)}{a^{3/2} f} - \frac{\cot(e+fx)}{af\sqrt{a \sin(e+fx)+a}}$$

[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.228806, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2715, 2985, 2649, 206, 2773}

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{a^{3/2} f} - \frac{2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2\sqrt{a} \sin(e+fx)+a}}\right)}{a^{3/2} f} - \frac{\cot(e+fx)}{af\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2), x]

[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2715

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^2, x_Symbol] := -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Dist[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && LtQ[m, -1]

Rule 2985

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$)

Rule 2773

$\text{Int}[\text{Sqrt}[(a_) + (b_)\sin[(e_) + (f_)(x_)]]/((c_) + (d_)\sin[(e_) + (f_)(x_)]) , x_Symbol] \rightarrow \text{Dist}[(-2*b)/f, \text{Subst}[\text{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\text{Cos}[e + f*x])/ \text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}} dx &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{3a}{2} + \frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{a^2} \\ &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{3 \int \csc(e+fx)\sqrt{a+a\sin(e+fx)} dx}{2a^2} + \frac{2 \int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{a} \\ &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \frac{3 \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{af} - \frac{4 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{af} \\ &= \frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}} \end{aligned}$$

Mathematica [C] time = 2.1984, size = 206, normalized size = 1.82

$$\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^3 \left(-\cot\left(\frac{1}{4}(e+fx)\right) + (16+16i)(-1)^{3/4} \tanh^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - Cot[(e + f*x)/4] + 2*(3*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sec[(e + f*x)/2] + Csc[e + f*x]*Sin[(e + f*x)/4]^2 - Csc[e + f*x]*Sin[(e + f*x)/4]*Sin[(3*(e + f*x))/4]))/(4*f*(a*(1 + Sin[e + f*x]))^(3/2))

Maple [A] time = 0.591, size = 134, normalized size = 1.2

$$-\frac{1 + \sin(fx + e)}{\sin(fx + e) \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(\sin(fx + e) a^2 \left(2\sqrt{2} \text{Artanh} \left(\frac{1}{2} \frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{\sqrt{a}} \right) - 3 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2), x)

[Out] -1/a^(7/2)*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*a^2*(2*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*sin

$$\frac{(f*x+e)^{(1/2)/a^{(1/2)}}+(a-a*\sin(f*x+e))^{(1/2)*a^{(3/2)}}/\sin(f*x+e)/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)/f}}$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 1.76445, size = 1133, normalized size = 10.03

$$3\left(\cos(fx+e)^2 - (\cos(fx+e) + 1)\sin(fx+e) - 1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3 - 7a\cos(fx+e)^2 + 4(\cos(fx+e)^2 + (\cos(fx+e) + 3)\sin(fx+e) - 2\cos(fx+e) - 3)\sqrt{a}\cos(fx+e) + (a\cos(fx+e)^2 + 8a\cos(fx+e) - a)\sin(fx+e) - a}{\cos(fx+e)^3 + \cos(fx+e)^2 + (\cos(fx+e)^2 - 1)\sin(fx+e) - \cos(fx+e) - 1} + 4\sqrt{2}(a\cos(fx+e)^2 - (a\cos(fx+e) + a)\sin(fx+e) - a)\log(-(\cos(fx+e)^2 - (\cos(fx+e) - 2)\sin(fx+e) - 2\sqrt{2}\sqrt{a}\sin(fx+e) + a)(\cos(fx+e) - \sin(fx+e) + 1)/\sqrt{a} + 3\cos(fx+e) + 2)/(\cos(fx+e)^2 - (\cos(fx+e) + 2)\sin(fx+e) - \cos(fx+e) - 2)/\sqrt{a} + 4\sqrt{2}\sqrt{a}\sin(fx+e) + a)(\cos(fx+e) - \sin(fx+e) + 1))/(a^2f\cos(fx+e)^2 - a^2f - (a^2f\cos(fx+e) + a^2f)\sin(fx+e))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/4*(3*(cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*sqrt(2)*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(f*x + e) - a)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*sqrt(2)*sqrt(a)*sin(f*x + e) + a*(cos(f*x + e) - sin(f*x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f - (a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)

[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)

Giac [B] time = 2.48466, size = 656, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] $\frac{1}{2} \left((6\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 8\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) - 3\sqrt{2}\sqrt{-a}\log(\sqrt{2}\sqrt{a} + \sqrt{a}) + 6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 16\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) - 3\sqrt{-a}\log(\sqrt{2}\sqrt{a} + \sqrt{a}) - \sqrt{2}\sqrt{-a} - 3\sqrt{-a}\right) \operatorname{sgn}(\tan(1/2fx + 1/2e) + 1) / (\sqrt{2}\sqrt{-a}a^{3/2} + \sqrt{-a}a^{3/2}) + 8\sqrt{2}\arctan(-1/2\sqrt{2}\sqrt{a}\tan(1/2fx + 1/2e) - \sqrt{a\tan(1/2fx + 1/2e)^2 + a} + \sqrt{a}) / \sqrt{-a} / (\sqrt{-a}a\operatorname{sgn}(\tan(1/2fx + 1/2e) + 1)) - 6\arctan(-(\sqrt{a}\tan(1/2fx + 1/2e) - \sqrt{a\tan(1/2fx + 1/2e)^2 + a}) / \sqrt{-a}) / (\sqrt{-a}a\operatorname{sgn}(\tan(1/2fx + 1/2e) + 1)) + 3\log(\operatorname{abs}(-\sqrt{a}\tan(1/2fx + 1/2e) + \sqrt{a\tan(1/2fx + 1/2e)^2 + a})) / (a^{3/2}\operatorname{sgn}(\tan(1/2fx + 1/2e) + 1)) + \sqrt{a\tan(1/2fx + 1/2e)^2 + a} / (a^2\operatorname{sgn}(\tan(1/2fx + 1/2e) + 1)) + 2 / (((\sqrt{a}\tan(1/2fx + 1/2e) - \sqrt{a\tan(1/2fx + 1/2e)^2 + a})^2 - a)\sqrt{a}\operatorname{sgn}(\tan(1/2fx + 1/2e) + 1))) / f$

$$3.110 \quad \int \frac{\cot^4(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f} - \frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a\sin(e+fx)}}$$

[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*a^(3/2)*f) - Cot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f*x])/((12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(3*a^2*f))

Rubi [A] time = 0.553107, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2717, 2772, 2773, 206, 3044, 2980}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f} - \frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]

[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*a^(3/2)*f) - Cot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f*x])/((12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(3*a^2*f))

Rule 2717

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && LtQ[m, -1]

Rule 2772

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 2773

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3044

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])

Rule 2980

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}} dx &= \frac{\int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}(1+\sin^2(e+fx)) dx}{a^2} - \frac{2 \int \csc^3(e+fx)\sqrt{a+a\sin(e+fx)} dx}{a^2} \\ &= \frac{\cot(e+fx)\csc(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} + \frac{\int \csc^3(e+fx)\sqrt{a+a\sin(e+fx)} dx}{3a^2f} \\ &= \frac{3\cot(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} \\ &= -\frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} \\ &= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} \end{aligned}$$

Mathematica [B] time = 0.760534, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)-\csc^7\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)+\csc^5\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)-\csc^3\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)+\csc\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)}{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2), x]

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-132*Cos[(e + f*x)/2] + 62*Cos[(3*(e + f*x))/2] + 6*Cos[(5*(e + f*x))/2] + 132*Sin[(e + f*x)/2] - 9*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 9*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 62*Sin[(3*(e + f*x))/2] - 6*Sin[(5*(e + f*x))/2] + 3*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.621, size = 144, normalized size = 1.

$$\frac{1 + \sin(fx + e)}{24 (\sin(fx + e))^3 \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(3 \sqrt{-a(-1 + \sin(fx + e))} a^{7/2} - 8 (-a(-1 + \sin(fx + e)))^{3/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*(-a*(-1+sin(f*x+e)))^(1/2)*a^(7/2)-8*(-a*(-1+sin(f*x+e)))^(3/2)*a^(5/2)-3*(-a*(-1+sin(f*x+e)))^(5/2)*a^(3/2)-3*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3/a^(11/2)/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.57176, size = 1003, normalized size = 6.97

$$3 \left(\cos(fx + e)^4 - 2 \cos(fx + e)^2 - \left(\cos(fx + e)^3 + \cos(fx + e)^2 - \cos(fx + e) - 1 \right) \sin(fx + e) + 1 \right) \sqrt{a} \log \left(\frac{a \cos(fx + e)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4
```



```

*(3*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - (3*cos(f*x + e)^2 - 14*cos(f*x + e)
) - 25)*sin(f*x + e) - 11*cos(f*x + e) - 25)*sqrt(a*sin(f*x + e) + a))/(a^2
*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f - (a^2*f*cos(f*x + e)^3
+ a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - a^2*f)*sin(f*x + e))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 3.13146, size = 828, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)*((2*tan(1/2*f*x + 1/2*e)/(a^2*sgn(
tan(1/2*f*x + 1/2*e) + 1)) - 9/(a^2*sgn(tan(1/2*f*x + 1/2*e) + 1)))*tan(1/2
*f*x + 1/2*e) + 14/(a^2*sgn(tan(1/2*f*x + 1/2*e) + 1))) - (30*sqrt(2)*sqrt(
a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*log(s
qrt(2)*sqrt(a) + sqrt(a)) + 42*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/s
qrt(-a)) - 21*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 280*sqrt(2)*sqrt(-a
) + 402*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2)
+ 7*sqrt(-a)*a^(3/2)) + 6*arctan(-sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*t
an(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*f*x + 1/2*e)
+ 1)) - 3*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*
e)^2 + a)))/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 2*(9*(sqrt(a)*tan(1/2
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^5 - 18*(sqrt(a)*tan(1/2
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a) + 24*(sqrt(a)
*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) - 9*(
sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - 14
*a^(5/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 +
a))^2 - a)^3*a*sgn(tan(1/2*f*x + 1/2*e) + 1))/f

```

$$3.111 \quad \int \frac{\tan^4(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=207

$$\frac{317 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{4096\sqrt{2}a^{5/2}f} + \frac{\tan^3(e+fx)}{3f(a \sin(e+fx)+a)^{5/2}} + \frac{5a \sin(e+fx) \tan(e+fx)}{48f(a \sin(e+fx)+a)^{7/2}} + \frac{317 \cos(e+fx)}{4096af(a \sin(e+fx)+a)^{3/2}} +$$

[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4096*Sqrt[2]*a^(5/2)*f) + (317*Cos[e + f*x])/(3072*f*(a + a*Sin[e + f*x])^(5/2)) - (Sec[e + f*x]*(115 + 129*Sin[e + f*x]))/(384*f*(a + a*Sin[e + f*x])^(5/2)) + (317*Cos[e + f*x])/(4096*a*f*(a + a*Sin[e + f*x])^(3/2)) + (5*a*Sin[e + f*x]*Tan[e + f*x])/(48*f*(a + a*Sin[e + f*x])^(7/2)) + Tan[e + f*x]^3/(3*f*(a + a*Sin[e + f*x])^(5/2))

Rubi [A] time = 1.43374, antiderivative size = 260, normalized size of antiderivative = 1.26, number of steps used = 23, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2859}

$$-\frac{31 \sec^3(e+fx)}{192a^2f\sqrt{a \sin(e+fx)+a}} - \frac{1085 \sec(e+fx)}{3072a^2f\sqrt{a \sin(e+fx)+a}} + \frac{317 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{4096\sqrt{2}a^{5/2}f} + \frac{317 \cos(e+fx)}{4096af(a \sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4096*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - Sec[e + f*x]^3/(8*f*(a + a*Sin[e + f*x])^(5/2)) + (317*Cos[e + f*x])/(4096*a*f*(a + a*Sin[e + f*x])^(3/2)) + (217*Sec[e + f*x])/(1536*a*f*(a + a*Sin[e + f*x])^(3/2)) + (53*Sec[e + f*x]^3)/(96*a*f*(a + a*Sin[e + f*x])^(3/2)) - (1085*Sec[e + f*x])/(3072*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (31*Sec[e + f*x]^3)/(192*a^2*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2714

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^4, x_Symbol] := Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4401

Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rule 2681

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*SIN[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2687

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*SIN[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)/(a + b*SIN[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2877

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*SIN[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*SIN[e + f*x])^(m + 1)*(a*m - b*(2*m + p + 1)*SIN[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*SIN[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}} dx &= \int \frac{1}{(a+a\sin(e+fx))^{5/2}} dx - \int \frac{\sec^4(e+fx)(1-2\sin^2(e+fx))}{(a+a\sin(e+fx))^{5/2}} dx \\
&= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3 \int \frac{1}{(a+a\sin(e+fx))^{3/2}} dx}{8a} - \int \left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} - \frac{2\sec^2(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} \right) dx \\
&= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + 2 \int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} dx \\
&= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \int \frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} dx \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^4(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^4(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^4(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^4(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{\sec^4(e+fx)}{4096\sqrt{2}a^{5/2}f} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{\sec^4(e+fx)}{4096\sqrt{2}a^{5/2}f} \\
&= \frac{317 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{\sec^4(e+fx)}{4096\sqrt{2}a^{5/2}f}
\end{aligned}$$

Mathematica [C] time = 0.546479, size = 394, normalized size = 1.9

$$\frac{1152\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{256\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - 201\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 + 402\sin\left(\frac{1}{2}(e+fx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (1312 + (768*Sin[(e + f*x)/2]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 384/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (2624*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2584*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1292*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 402*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 201*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (951 + 951*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + (256*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])

$\frac{1}{(-24576 + 24576 \sin(fx + e))(1 + \sin(fx + e))^3 \cos(fx + e) f} \left(1902 a^{11/2} \sin(fx + e) (\cos(fx + e))^4 + (-13888 \dots \right)$

Maple [A] time = 0.843, size = 353, normalized size = 1.7

$$\frac{1}{(-24576 + 24576 \sin(fx + e))(1 + \sin(fx + e))^3 \cos(fx + e) f} \left(1902 a^{11/2} \sin(fx + e) (\cos(fx + e))^4 + (-13888 \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)

[Out]
$$-1/24576/a^{(15/2)}*(1902*a^{(11/2)}*\sin(f*x+e)*\cos(f*x+e)^4+(-13888*a^{(11/2)}-3804*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^4*\cos(f*x+e)^2*\sin(f*x+e)+(5632*a^{(11/2)}+7608*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^4*\sin(f*x+e)+(4438*a^{(11/2)}+951*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^4*\cos(f*x+e)^4+(-9920*a^{(11/2)}-7608*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^4*\cos(f*x+e)^2+2560*a^{(11/2)}+7608*(a-a*\sin(f*x+e))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^4)/(-1+\sin(f*x+e))/(1+\sin(f*x+e))^3/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 1.90076, size = 830, normalized size = 4.01

$$951 \sqrt{2} \left(3 \cos(fx + e)^5 - 4 \cos(fx + e)^3 + (\cos(fx + e)^5 - 4 \cos(fx + e)^3) \sin(fx + e) \right) \sqrt{a} \log \left(-\frac{a \cos(fx + e)^2 + 2 \sqrt{2} \dots}{\dots} \right)$$

4915

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out]
$$1/49152*(951*\sqrt{2}*(3*\cos(f*x + e)^5 - 4*\cos(f*x + e)^3 + (\cos(f*x + e)^5 - 4*\cos(f*x + e)^3)*\sin(f*x + e))*\sqrt{a}*\log(-(a*\cos(f*x + e)^2 + 2*\sqrt{2}*(2)*\sqrt{a}*\sin(f*x + e) + a)*\sqrt{a}*(\cos(f*x + e) - \sin(f*x + e) + 1) + 3*a*\cos(f*x + e) - (a*\cos(f*x + e) - 2*a)*\sin(f*x + e) + 2*a)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \cos(f*x + e) - 2)) - 4*(2219*\cos(f*x + e)^4 - 4960*\cos(f*x + e)^2 + (951*\cos(f*x + e)^4 - 6944*\cos(f*x + e)^2 + 28$$

```
16)*sin(f*x + e) + 1280)*sqrt(a*sin(f*x + e) + a))/(3*a^3*f*cos(f*x + e)^5
- 4*a^3*f*cos(f*x + e)^3 + (a^3*f*cos(f*x + e)^5 - 4*a^3*f*cos(f*x + e)^3)*
sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

$$3.112 \quad \int \frac{\tan^2(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=167

$$\frac{11 \sec(e+fx)}{96a^2 f \sqrt{a \sin(e+fx)+a}} - \frac{11 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{128 \sqrt{2} a^{5/2} f} - \frac{11 \cos(e+fx)}{128 a f (a \sin(e+fx)+a)^{3/2}} + \frac{17 \sec(e+fx)}{48 a f (a \sin(e+fx)+a)^{3/2}}$$

[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(128*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.30282, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2712, 2859, 2687, 2650, 2649, 206}

$$\frac{11 \sec(e+fx)}{96a^2 f \sqrt{a \sin(e+fx)+a}} - \frac{11 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{128 \sqrt{2} a^{5/2} f} - \frac{11 \cos(e+fx)}{128 a f (a \sin(e+fx)+a)^{3/2}} + \frac{17 \sec(e+fx)}{48 a f (a \sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]

[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(128*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2712

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)^2, x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]), x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && LtQ[m, 0]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2687

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co

$s[e + f*x]^{(p + 2)/(a + b*\sin[e + f*x]^{(3/2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntegerQ}[2*p]$

Rule 2650

$\text{Int}[(a_ + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] \ :> \ \text{Simp}[(b*\cos[c + d*x]*(a + b*\sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + \text{Dist}[(n + 1)/(a*(2*n + 1)), \text{Int}[(a + b*\sin[c + d*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2649

$\text{Int}[1/\text{Sqrt}[(a_ + (b_.)*\sin[(c_.) + (d_.)*(x_)]), x_Symbol] \ :> \ \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, (b*\cos[c + d*x])/ \text{Sqrt}[a + b*\sin[c + d*x]]], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx &= -\frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} + \frac{\int \frac{\sec^2(e + fx) \left(-\frac{5a}{2} + 6a \sin(e + fx)\right)}{(a + a \sin(e + fx))^{3/2}} dx}{6a^2} \\ &= -\frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} + \frac{17 \sec(e + fx)}{48af(a + a \sin(e + fx))^{3/2}} + \frac{11 \int \frac{\sec^2(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx}{96a^2} \\ &= -\frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} + \frac{17 \sec(e + fx)}{48af(a + a \sin(e + fx))^{3/2}} + \frac{11 \sec(e + fx)}{96a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{11 \int}{96} \\ &= -\frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} - \frac{11 \cos(e + fx)}{128af(a + a \sin(e + fx))^{3/2}} + \frac{17 \sec(e + fx)}{48af(a + a \sin(e + fx))^{3/2}} + \frac{11 \int}{96} \\ &= -\frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} - \frac{11 \cos(e + fx)}{128af(a + a \sin(e + fx))^{3/2}} + \frac{17 \sec(e + fx)}{48af(a + a \sin(e + fx))^{3/2}} + \frac{11 \int}{96} \\ &= -\frac{11 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{128\sqrt{2}a^{5/2}f} - \frac{\sec(e + fx)}{6f(a + a \sin(e + fx))^{5/2}} - \frac{11 \cos(e + fx)}{128af(a + a \sin(e + fx))^{3/2}} + \frac{11 \int}{96} \end{aligned}$$

Mathematica [C] time = 0.408413, size = 284, normalized size = 1.7

$$\frac{48 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^5}{\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)} + 15 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^4 - 30 \sin\left(\frac{1}{2}(e + fx)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (-32 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 104*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 52*(Cos[(e + f*x)/2])

$$+ \sin[(e + f*x)/2]^2 - 30*\sin[(e + f*x)/2]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^3 + 15*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^4 + (33 + 33*I)*(-1)^(3/4)*\text{ArcTanh}[(1/2 + I/2)*(-1)^(3/4)*(-1 + \tan[(e + f*x)/4])]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^5 + (48*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^5)/(\cos[(e + f*x)/2] - \sin[(e + f*x)/2])/(384*f*(a*(1 + \sin[e + f*x]))^(5/2))$$

Maple [A] time = 0.648, size = 266, normalized size = 1.6

$$\frac{1}{768 (1 + \sin(fx + e))^2 \cos(fx + e) f} \left(\left(66 a^{7/2} - 33 \sqrt{a - a \sin(fx + e)} \sqrt{2} \text{Arctanh} \left(1/2 \frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{\sqrt{a}} \right) \right) a^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)

[Out] -1/768/a^(11/2)*((66*a^(7/2)-33*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(-448*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+(154*a^(7/2)-99*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(f*x+e)^2-320*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 1.69739, size = 751, normalized size = 4.5

$$\frac{33 \sqrt{2} \left(3 \cos(fx + e)^3 + (\cos(fx + e)^3 - 4 \cos(fx + e)) \sin(fx + e) - 4 \cos(fx + e) \right) \sqrt{a} \log \left(-\frac{a \cos(fx + e)^2 - 2 \sqrt{2} \sqrt{a} \sin(fx + e)}{\dots} \right)}{1536 \left(3 a^3 f \cos(fx + e)^3 - 4 \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/1536*(33*sqrt(2)*(3*cos(f*x + e)^3 + (cos(f*x + e)^3 - 4*cos(f*x + e))*sin(f*x + e) - 4*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a)*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(77*cos(f*x + e)^2 + (

$$33*\cos(f*x + e)^2 - 224*\sin(f*x + e) - 160)*\sqrt{a*\sin(f*x + e) + a})/(3*a^3*f*\cos(f*x + e)^3 - 4*a^3*f*\cos(f*x + e) + (a^3*f*\cos(f*x + e)^3 - 4*a^3*f*\cos(f*x + e))*\sin(f*x + e))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] sage2

$$3.113 \quad \int \frac{\cot^2(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=141

$$\frac{5 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f} - \frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{2} a^{5/2} f} - \frac{2 \cos(e+fx)}{af(a \sin(e+fx)+a)^{3/2}} - \frac{\cot(e+fx)}{af(a \sin(e+fx)+a)^{3/2}}$$

[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(a^(5/2)*f) - (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sqrt[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))

Rubi [A] time = 0.346435, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2715, 2978, 2985, 2649, 206, 2773}

$$\frac{5 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f} - \frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{2} a^{5/2} f} - \frac{2 \cos(e+fx)}{af(a \sin(e+fx)+a)^{3/2}} - \frac{\cot(e+fx)}{af(a \sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(a^(5/2)*f) - (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sqrt[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))

Rule 2715

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Dist[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && LtQ[m, -1]

Rule 2978

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2985

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -

$A*d)/(b*c - a*d)$, $\text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/(c + d*\text{Sin}[e + f*x]), x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, A, B, x\}$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$

Rule 2649

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]]]$, $x_Symbol]$ \rightarrow $\text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, (b*\text{Cos}[c + d*x])/ \text{Sqrt}[a + b*\text{Sin}[c + d*x]]], x]$ /; $\text{FreeQ}\{a, b, c, d, x\}$ && $\text{EqQ}[a^2 - b^2, 0]$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2]^{-1}$, $x_Symbol]$ \rightarrow $\text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2])]$, $x]$ /; $\text{FreeQ}\{a, b, x\}$ && $\text{NegQ}[a/b]$ && $(\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 2773

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]/((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])]$, $x_Symbol]$ \rightarrow $\text{Dist}[(-2*b)/f, \text{Subst}[\text{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\text{Cos}[e + f*x])/ \text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, x\}$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}} dx &= -\frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{5a}{2} + \frac{3}{2}a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}} dx}{a^2} \\ &= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-5a^2+2a^2\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{2a^4} \\ &= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \csc(e+fx)\sqrt{a+a\sin(e+fx)} dx}{2a^3} \\ &= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{5\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^2f} \\ &= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} - \frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.71848, size = 451, normalized size = 3.2

$$\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^3 \left(8\sin\left(\frac{1}{2}(e+fx)\right) + \frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^2}{\cos\left(\frac{1}{4}(e+fx)\right) - \sin\left(\frac{1}{4}(e+fx)\right)} - \frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sin\left(\frac{1}{4}(e+fx)\right) + \cos\left(\frac{1}{4}(e+fx)\right)}\right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cot}[e + f*x]^2/(a + a*\text{Sin}[e + f*x])^{(5/2)}, x]$

[Out] $((\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2])^3*(8*\text{Sin}[(e + f*x)/2] - 4*(\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2]) + 2*(\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2])^2 + (28 + 28*I)*(-1)^{(3/4)}*\text{ArcTanh}[(1/2 + I/2)*(-1)^{(3/4)}*(-1 + \text{Tan}[(e + f*x)/4])$

$$\begin{aligned} &]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2 - \cot[(e + f*x)/4]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2 + 10*\log[1 + \cos[(e + f*x)/2] - \sin[(e + f*x)/2]]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2 - 10*\log[1 - \cos[(e + f*x)/2] + \sin[(e + f*x)/2]]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2 + (2*\sin[(e + f*x)/4]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2)/(\cos[(e + f*x)/4] - \sin[(e + f*x)/4]) - (2*\sin[(e + f*x)/4]*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2)/(\cos[(e + f*x)/4] + \sin[(e + f*x)/4]) - (\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^2 * \tan[(e + f*x)/4])/((4*f*(a*(1 + \sin[e + f*x]))^(5/2)) \end{aligned}$$

Maple [A] time = 0.57, size = 219, normalized size = 1.6

$$-\frac{1}{2 \sin(fx + e) \cos(fx + e) f} \left(7\sqrt{2} \operatorname{Arctanh} \left(\frac{1}{2} \frac{\sqrt{-a(-1 + \sin(fx + e))\sqrt{2}}}{\sqrt{a}} \right) (\sin(fx + e))^2 a - 10 \operatorname{Arctanh} \left(\frac{\sqrt{-a}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)

[Out]
$$-1/2/a^{(7/2)}*(7*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(-1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a-10*\operatorname{arctanh}((-a*(-1+\sin(f*x+e)))^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a+7*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(-1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a*\sin(f*x+e)+4*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(1/2)}*\sin(f*x+e)-10*\operatorname{arctanh}((-a*(-1+\sin(f*x+e)))^{(1/2)}/a^{(1/2)})*\sin(f*x+e)*a+2*(-a*(-1+\sin(f*x+e)))^{(1/2)}*a^{(1/2)}*(-a*(-1+\sin(f*x+e)))^{(1/2)}/\sin(f*x+e)/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 1.8593, size = 1436, normalized size = 10.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out]
$$1/4*(5*(\cos(f*x + e))^3 + 2*\cos(f*x + e)^2 + (\cos(f*x + e))^2 - \cos(f*x + e) - 2)*\sin(f*x + e) - \cos(f*x + e) - 2)*\sqrt{a}*\log((a*\cos(f*x + e))^3 - 7*a*\cos(f*x + e)^2 + 4*(\cos(f*x + e))^2 + (\cos(f*x + e) + 3)*\sin(f*x + e) - 2*\cos(f*x + e) - 3)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{a} - 9*a*\cos(f*x + e) + (a*\cos(f*x + e))^2 + 8*a*\cos(f*x + e) - a)*\sin(f*x + e) - a)/(\cos(f*x + e))^3 + \cos(f*x + e)^2 + (\cos(f*x + e))^2 - 1)*\sin(f*x + e) - \cos(f*x + e) - 1)) + 7*\sqrt{2}*(a*\cos(f*x + e))^3 + 2*a*\cos(f*x + e)^2 - a*\cos(f*x + e) + (a*\cos(f*x$$

```
+ e)^2 - a*cos(f*x + e) - 2*a)*sin(f*x + e) - 2*a)*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(2*cos(f*x
+ e)^2 + (2*cos(f*x + e) + 1)*sin(f*x + e) + cos(f*x + e) - 1)*sqrt(a*sin(
f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 2*a^3*f*cos(f*x + e)^2 - a^3*f*cos(f
*x + e) - 2*a^3*f + (a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - 2*a^3*f)*s
in(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage₂

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

$$3.114 \quad \int \frac{\cot^4(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=191

$$\frac{19 \cot(e+fx)}{8a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{45 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{8a^{5/2} f} - \frac{4\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a \sin(e+fx)+a}} +$$

[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*a^(5/2)*f) - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])

Rubi [A] time = 0.958734, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2717, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$\frac{19 \cot(e+fx)}{8a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{45 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a}}\right)}{8a^{5/2} f} - \frac{4\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a \sin(e+fx)+a}} +$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]

[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(8*a^(5/2)*f) - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2717

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && LtQ[m, -1]

Rule 2779

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]], x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2984

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n

```
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] := Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_.)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_.)]^(n_.))*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :=
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}} dx &= \frac{\int \frac{\csc^4(e+fx)(1+\sin^2(e+fx))}{\sqrt{a+a\sin(e+fx)}} dx}{a^2} - \frac{2 \int \frac{\csc^3(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx}{a^2} \\
&= \frac{\cot(e+fx) \csc(e+fx)}{a^2 f \sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx) \left(-\frac{a}{2} + \frac{11}{2} a \sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}} dx}{3a^3} + \dots \\
&= -\frac{\cot(e+fx)}{2a^2 f \sqrt{a+a\sin(e+fx)}} + \frac{13 \cot(e+fx) \csc(e+fx)}{12a^2 f \sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a+a\sin(e+fx)}} + \dots \\
&= -\frac{19 \cot(e+fx)}{8a^2 f \sqrt{a+a\sin(e+fx)}} + \frac{13 \cot(e+fx) \csc(e+fx)}{12a^2 f \sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a+a\sin(e+fx)}} + \dots \\
&= -\frac{19 \cot(e+fx)}{8a^2 f \sqrt{a+a\sin(e+fx)}} + \frac{13 \cot(e+fx) \csc(e+fx)}{12a^2 f \sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx) \csc^2(e+fx)}{3a^2 f \sqrt{a+a\sin(e+fx)}} - \dots \\
&= \frac{7 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{5/2} f} - \frac{2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2} f} - \frac{19 \cot(e+fx)}{8a^2 f \sqrt{a+a\sin(e+fx)}} + \dots \\
&= \frac{45 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2} f} - \frac{4\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2} f} - \frac{19 \cot(e+fx)}{8a^2 f \sqrt{a+a\sin(e+fx)}} + \dots
\end{aligned}$$

Mathematica [C] time = 2.38875, size = 332, normalized size = 1.74

$$\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^5 \left(-\frac{8 \csc^9\left(\frac{1}{2}(e+fx)\right) \left(-396 \sin\left(\frac{1}{2}(e+fx)\right) - 218 \sin\left(\frac{3}{2}(e+fx)\right) + 114 \sin\left(\frac{5}{2}(e+fx)\right) + 396 \cos\left(\frac{1}{2}(e+fx)\right) - 218 \cos\left(\frac{3}{2}(e+fx)\right) + 114 \cos\left(\frac{5}{2}(e+fx)\right) - 396\right)}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - (8*Csc[(e + f*x)/2]^9*(396*Cos[(e + f*x)/2] - 218*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] - 396*Sin[(e + f*x)/2] - 405*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 405*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 218*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] + 135*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 135*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)/(192*f*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [A] time = 0.722, size = 182, normalized size = 1.

$$-\frac{1 + \sin(fx + e)}{24 (\sin(fx + e))^3 \cos(fx + e) f} \sqrt{-a(-1 + \sin(fx + e))} \left(-135 a^5 \operatorname{Artanh}\left(\frac{\sqrt{-a(-1 + \sin(fx + e))}}{\sqrt{a}}\right) \right) (\sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)`

[Out]
$$-1/24/a^{15/2}*(1+\sin(f*x+e))*(-a*(-1+\sin(f*x+e)))^{1/2}*(-135*a^5*\operatorname{arctanh}((-a*(-1+\sin(f*x+e)))^{1/2}/a^{1/2})*\sin(f*x+e)^3+57*(-a*(-1+\sin(f*x+e)))^{5/2}*a^{5/2}+96*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(-1+\sin(f*x+e)))^{1/2}*2^{1/2}/a^{1/2}))*a^5*\sin(f*x+e)^3-88*(-a*(-1+\sin(f*x+e)))^{3/2}*a^{7/2}+39*(-a*(-1+\sin(f*x+e)))^{1/2}*a^{9/2})/\sin(f*x+e)^3/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")`

[Out] Timed out

Fricas [B] time = 1.74451, size = 1504, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{96}*(135*(\cos(f*x + e)^4 - 2*\cos(f*x + e)^2 - (\cos(f*x + e)^3 + \cos(f*x + e)^2 - \cos(f*x + e) - 1)*\sin(f*x + e) + 1)*\sqrt{a}*\log((a*\cos(f*x + e)^3 - 7*a*\cos(f*x + e)^2 + 4*(\cos(f*x + e)^2 + (\cos(f*x + e) + 3)*\sin(f*x + e) - 2*\cos(f*x + e) - 3)*\sqrt{a*\sin(f*x + e) + a})*\sqrt{a} - 9*a*\cos(f*x + e) + (a*\cos(f*x + e)^2 + 8*a*\cos(f*x + e) - a)*\sin(f*x + e) - a)/(\cos(f*x + e)^3 + \cos(f*x + e)^2 + (\cos(f*x + e)^2 - 1)*\sin(f*x + e) - \cos(f*x + e) - 1)) + 192*\sqrt{2}*(a*\cos(f*x + e)^4 - 2*a*\cos(f*x + e)^2 - (a*\cos(f*x + e)^3 + a*\cos(f*x + e)^2 - a*\cos(f*x + e) - a)*\sin(f*x + e) + a)*\log(-(\cos(f*x + e)^2 - (\cos(f*x + e) - 2)*\sin(f*x + e) - 2*\sqrt{2})*\sqrt{a*\sin(f*x + e) + a}*(\cos(f*x + e) - \sin(f*x + e) + 1)/\sqrt{a} + 3*\cos(f*x + e) + 2)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \cos(f*x + e) - 2))/\sqrt{a} + 4*(57*\cos(f*x + e)^3 + 83*\cos(f*x + e)^2 - (57*\cos(f*x + e)^2 - 26*\cos(f*x + e) - 91)*\sin(f*x + e) - 65*\cos(f*x + e) - 91)*\sqrt{a*\sin(f*x + e) + a})/(a^3*f*\cos(f*x + e)^4 - 2*a^3*f*\cos(f*x + e)^2 + a^3*f - (a^3*f*\cos(f*x + e)^3 + a^3*f*\cos(f*x + e)^2 - a^3*f*\cos(f*x + e) - a^3*f)*\sin(f*x + e))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)`

[Out] Timed out

Giac [B] time = 3.15976, size = 975, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out]
$$\frac{1}{48} \left(\sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \left(\frac{2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} - \frac{15}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} \right) \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + \frac{74}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)} \right) + (1890 \sqrt{2} \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 3840 \sqrt{2} \sqrt{a} \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) - 945 \sqrt{2} \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right) + 2700 \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 5376 \sqrt{a} \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) - 1350 \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right) - 1302 \sqrt{2} \sqrt{-a} - 1808 \sqrt{-a}\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) / (7 \sqrt{2} \sqrt{-a} a^{5/2} + 10 \sqrt{-a} a^{5/2}) + 384 \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} + \sqrt{a}\right) / \sqrt{-a} \right) / \left(\sqrt{-a} a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) - 270 \arctan\left(-\left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right) / \sqrt{-a}\right) / \left(\sqrt{-a} a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) + 135 \log\left(\operatorname{abs}\left(-\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)\right) / \left(a^{5/2} \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) - 2 \left(15 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)^5 - 78 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)^4 \sqrt{a} + 144 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)^2 a^{3/2} - 15 \left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right) a^2 - 74 a^{5/2}\right) / \left(\left(\sqrt{a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a}\right)^2 - a\right)^3 a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right) \right) / f$$

3.115 $\int \sqrt[3]{a + a \sin(e + fx)} \tan^4(e + fx) dx$

Optimal. Leaf size=982

result too large to display

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f
x])^(1/3)]], (2 + Sqrt[3])/4)*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a
^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(21*3^(3/4)*a^(2/3)*f
*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])
^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)])
- (361*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a +
a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x
])^(1/3)]], (2 + Sqrt[3])/4)*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/
3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a
^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(63*2^(2/3)*3^(1/4)*a
^(2/3)*f*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e
+ f*x])^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3
))^2)]) + (3*a^2*Sin[e + f*x]*Tan[e + f*x])/(2*f*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(2/3)) - (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e
+ f*x])*(a + a*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 1.2706, antiderivative size = 982, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2719, 100, 153, 144, 51, 63, 308, 225, 1881}

$$\frac{3 \sin^2(e + fx) \tan(e + fx) a^2}{f(a - a \sin(e + fx))(\sin(e + fx)a + a)^{2/3}} + \frac{3 \sin(e + fx) \tan(e + fx) a^2}{2f(a - a \sin(e + fx))(\sin(e + fx)a + a)^{2/3}} - \frac{\sec(e + fx) (65a^2 - 142a^2 \sin(e + fx))}{42f(a - a \sin(e + fx))(\sin(e + fx)a + a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f
x])^(1/3)]], (2 + Sqrt[3])/4)*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a
^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(21*3^(3/4)*a^(2/3)*f
```

$$\begin{aligned} & * \text{Sqrt}[-((a + a \sin[e + f x])^{1/3} (2^{1/3} a^{1/3} - (a + a \sin[e + f x])^{1/3})) / (2^{1/3} a^{1/3} - (1 + \text{Sqrt}[3]) (a + a \sin[e + f x])^{1/3})^2)] \\ & - (361 (1 - \text{Sqrt}[3]) \text{EllipticF}[\text{ArcCos}[(2^{1/3} a^{1/3} - (1 - \text{Sqrt}[3]) (a + a \sin[e + f x])^{1/3}) / (2^{1/3} a^{1/3} - (1 + \text{Sqrt}[3]) (a + a \sin[e + f x])^{1/3})], (2 + \text{Sqrt}[3]) / 4] \text{Sec}[e + f x] (a + a \sin[e + f x])^{2/3} (2^{1/3} a^{1/3} - (a + a \sin[e + f x])^{1/3}) \text{Sqrt}[(2^{2/3} a^{2/3} + 2^{1/3} a^{1/3} (a + a \sin[e + f x])^{1/3} + (a + a \sin[e + f x])^{2/3}) / (2^{1/3} a^{1/3} - (1 + \text{Sqrt}[3]) (a + a \sin[e + f x])^{1/3})^2]) / (63 \cdot 2^{2/3} \cdot 3^{1/4} a^{2/3} f \text{Sqrt}[-((a + a \sin[e + f x])^{1/3} (2^{1/3} a^{1/3} - (a + a \sin[e + f x])^{1/3})) / (2^{1/3} a^{1/3} - (1 + \text{Sqrt}[3]) (a + a \sin[e + f x])^{1/3})^2]) + (3 a^2 \sin[e + f x] \text{Tan}[e + f x]) / (2 f (a - a \sin[e + f x]) (a + a \sin[e + f x])^{2/3}) - (3 a^2 \sin[e + f x]^2 \text{Tan}[e + f x]) / (f (a - a \sin[e + f x]) (a + a \sin[e + f x])^{2/3}) \end{aligned}$$

Rule 2719

$$\text{Int}[(a + b \sin[e + f x])^m \tan[e + f x]^{p-1}, x] \text{Symbol} \rightarrow \text{Dist}[\text{Sqrt}[a + b \sin[e + f x]] \text{Sqrt}[a - b \sin[e + f x]] / (b f \cos[e + f x]), \text{Subst}[\text{Int}[(x^p (a + x)^{m - (p + 1)/2}) / (a - x)^{(p + 1)/2}], x], x, b \sin[e + f x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p/2]$$

Rule 100

$$\text{Int}[(a + b x)^m (c + d x)^n (e + f x)^p, x] \text{Symbol} \rightarrow \text{Simp}[(b (a + b x)^{m-1} (c + d x)^{n+1} (e + f x)^{p+1}) / (d f (m + n + p + 1)), x] + \text{Dist}[1 / (d f (m + n + p + 1)), \text{Int}[(a + b x)^{m-2} (c + d x)^n (e + f x)^p \text{Simp}[a^2 d f (m + n + p + 1) - b (b c e (m - 1) + a (d e (n + 1) + c f (p + 1))) + b (a d f (2 m + n + p) - b (d e (m + n) + c f (m + p))) x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[m + n + p + 1, 0] \&\& \text{IntegerQ}[m]$$

Rule 153

$$\text{Int}[(a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^q, x] \text{Symbol} \rightarrow \text{Simp}[(h (a + b x)^m (c + d x)^{n+1} (e + f x)^{p+1}) / (d f (m + n + p + 2)), x] + \text{Dist}[1 / (d f (m + n + p + 2)), \text{Int}[(a + b x)^{m-1} (c + d x)^n (e + f x)^p \text{Simp}[a d f g (m + n + p + 2) - h (b c e m + a (d e (n + 1) + c f (p + 1))) + (b d f g (m + n + p + 2) + h (a d f m - b (d e (m + n + 1) + c f (m + p + 1))) x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m + n + p + 2, 0] \&\& \text{IntegerQ}[m]$$

Rule 144

$$\text{Int}[(a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^q, x] \text{Symbol} \rightarrow \text{Simp}[(b^2 c d e g (n + 1) + a^2 c d f h (n + 1) + a b (d^2 e g (m + 1) + c^2 f h (m + 1) - c d (f g + e h) (m + 2)) + (a^2 d^2 f h (n + 1) - a b d^2 (f g + e h) (n + 1) + b^2 (c^2 f h (m + 1) - c d (f g + e h) (m + 1) + d^2 e g (m + n + 2))) x] (a + b x)^{m+1} (c + d x)^{n+1} / (b d (b c - a d)^2 (m + 1) (n + 1)), x] - \text{Dist}[(a^2 d^2 f h (2 + 3 n + n^2) + a b d (n + 1) (2 c f h (m + 1) - d (f g + e h) (m + n + 3)) + b^2 (c^2 f h (2 + 3 m + m^2) - c d (f g + e h) (m + 1) (m + n + 3) + d^2 e g (6 + m^2 + 5 n + n^2 + m (2 n + 5)))] / (b d (b c - a d)^2 (m + 1) (n + 1)), \text{Int}[(a + b x)^{m+1} (c + d x)^{n+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x \&\& \text{LtQ}[m, -1] \&\& \text{LtQ}[n, -1]$$

Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
+ b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r =
Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{a + a \sin(e + fx)} \tan^4(e + fx) dx &= \frac{(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}) \operatorname{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x)^{13/6}} dx\right)}{af} \\
&= -\frac{3a^2 \sin^2(e + fx) \tan(e + fx)}{f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} - \frac{(3 \sec(e + fx) \sqrt{a - a \sin(e + fx)})}{f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} \\
&= \frac{3a^2 \sin(e + fx) \tan(e + fx)}{2f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} - \frac{3a^2 \sin^2(e + fx) \tan(e + fx)}{f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} \\
&= -\frac{\sec(e + fx) (65a^2 - 142a^2 \sin(e + fx))}{42f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} + \frac{3a^2 \sin(e + fx) \tan(e + fx)}{2f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} \\
&= -\frac{361 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{126f} - \frac{\sec(e + fx) (65a^2 - 142a^2 \sin(e + fx))}{42f(a - a \sin(e + fx))(a + a \sin(e + fx))^{2/3}} \\
&= -\frac{361 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{126f} + \frac{361 \sec(e + fx) (1 - \sin(e + fx)) \sqrt[3]{a + a \sin(e + fx)}}{63f} \\
&= -\frac{361 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{126f} + \frac{361 \sec(e + fx) (1 - \sin(e + fx)) \sqrt[3]{a + a \sin(e + fx)}}{63f} \\
&= -\frac{361 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{126f} + \frac{361 \sec(e + fx) (1 - \sin(e + fx)) \sqrt[3]{a + a \sin(e + fx)}}{63f} \\
&= -\frac{361 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{126f} + \frac{361 \sec(e + fx) (1 - \sin(e + fx)) \sqrt[3]{a + a \sin(e + fx)}}{63f}
\end{aligned}$$

Mathematica [C] time = 3.273, size = 318, normalized size = 0.32

$$\sqrt[3]{a(\sin(e + fx) + 1)} \left(3(-172 \tan(e + fx) - 3 \sec^3(e + fx) + 86 \sec(e + fx) + 24 \tan(e + fx) \sec^2(e + fx) + 361) + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]

[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((1083/10 + (1083*I)/10)*(-1)^(3/4)*(20*E^(I*(e + f*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-I)/E^(I*(e + f*x))] - 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*Sin[e + f*x]]))/Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[(I*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) + 3*(361 + 86*Sec[e + f*x] - 3*Sec[e + f*x]^3 - 172*Tan[e + f*x] + 24*Sec[e + f*x]^2*Tan[e + f*x]))/(189*f)

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a \sin(fx + e)} (\tan(fx + e))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)

[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^{\frac{1}{3}} \tan(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

3.116 $\int \sqrt[3]{a + a \sin(e + fx)} \tan^2(e + fx) dx$

Optimal. Leaf size=123

$$\frac{5a\sqrt[6]{\sin(e+fx)+1} \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}f(a\sin(e+fx)+a)^{2/3}} - \frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af} + \frac{7\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{af}$$

[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x])^(4/3))/(a*f)

Rubi [A] time = 0.19337, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2713, 2855, 2652, 2651}

$$\frac{5a\sqrt[6]{\sin(e+fx)+1} \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}f(a\sin(e+fx)+a)^{2/3}} - \frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af} + \frac{7\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{af}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]

[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x])^(4/3))/(a*f)

Rule 2713

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] := -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] + Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && ! IntegerQ[m] && ! LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && ! IntegerQ[2*n] && ! GtQ[a, 0]

Rule 2651

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1

- (b*Sin[c + d*x])/a))/2)]/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + a \sin(e + fx)} \tan^2(e + fx) dx &= -\frac{3 \sec(e + fx)(a + a \sin(e + fx))^{4/3}}{af} + \frac{3 \int \sec^2(e + fx) \sqrt[3]{a + a \sin(e + fx)} dx}{a} \\ &= \frac{7 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{f} - \frac{3 \sec(e + fx)(a + a \sin(e + fx))^{4/3}}{af} + \frac{1}{3} (5a) \\ &= \frac{7 \sec(e + fx) \sqrt[3]{a + a \sin(e + fx)}}{f} - \frac{3 \sec(e + fx)(a + a \sin(e + fx))^{4/3}}{af} + \frac{(5a)}{3} \\ &= -\frac{5a \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) \sqrt[6]{1 + \sin(e + fx)}}{3 \sqrt[6]{2} f (a + a \sin(e + fx))^{2/3}} + \frac{7 \sec(e + fx)}{3} \end{aligned}$$

Mathematica [C] time = 2.746, size = 290, normalized size = 2.36

$$\sqrt[3]{a(\sin(e + fx) + 1)} \left(-3(-2 \tan(e + fx) + \sec(e + fx) + 5) + \frac{\left(\frac{3}{2} + \frac{3i}{2}\right) (-1)^{3/4} e^{-i(e+fx)} \left(2(1+ie^{-i(e+fx)})^{2/3} (1+e^{2i(e+fx)}) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \sin^2\left(\frac{2e+fx}{4}\right)\right) - (5i) \sqrt{2-2\sin\left(\frac{2e+fx}{4}\right)}\right) \sqrt{2} E^{i(e+fx)} (1 + I/E^{i(e+fx)})^{2/3} \sqrt{(I(-I + E^{i(e+fx)})^2)/E^{i(e+fx)}} - 3(5 + \sec[e + fx] - 2\tan[e + fx])\right)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]

[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((3/2 + (3*I)/2)*(-1)^(3/4)*(-20*E^(I*(e + f*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-I)/E^(I*(e + f*x))] + 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*(e + f*x))))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2] - (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*Sin[e + f*x]]))/Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[(I*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))] - 3*(5 + Sec[e + f*x] - 2*Tan[e + f*x])))/(3*f)

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a \sin(fx + e)} (\tan(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)

[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \tan(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin (f x+e)+a\right)^{\frac{1}{3}} \tan (f x+e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a(\sin(e+fx)+1)} \tan^2(e+fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**2,x)

[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin (f x+e)+a\right)^{\frac{1}{3}} \tan (f x+e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)

3.117 $\int \cot^2(e + fx) \sqrt[3]{a + a \sin(e + fx)} dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx) (a \sin(e + fx) + a)^{7/3} F_1\left(\frac{11}{6}; -\frac{1}{2}, 2; \frac{17}{6}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{11a^2 f}$$

[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a^2*f)

Rubi [A] time = 0.101394, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx) (a \sin(e + fx) + a)^{7/3} F_1\left(\frac{11}{6}; -\frac{1}{2}, 2; \frac{17}{6}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{11a^2 f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]

[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a^2*f)

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*(c + d*x)/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \cot^2(e + fx) \sqrt[3]{a + a \sin(e + fx)} dx = \frac{(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{\sqrt{a-x}(a+x)^{5/6}}{x^2} dx, x, a \right)}{af}$$

$$= \frac{(\sqrt{2} \sec(e + fx) (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{(a+x)^{5/6} \sqrt{\frac{1}{2} - \frac{x}{2a}}}{x^2} dx \right)}{af \sqrt{\frac{a - a \sin(e + fx)}{a}}}$$

$$= \frac{6\sqrt{2} F_1 \left(\frac{11}{6}; -\frac{1}{2}, 2; \frac{17}{6}; \frac{1}{2}(1 + \sin(e + fx)), 1 + \sin(e + fx) \right) \sec(e + fx) \sqrt{1 - \sin(e + fx)}}{11a^2 f}$$

Mathematica [C] time = 23.8982, size = 2692, normalized size = 33.65

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]

[Out] ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/ (f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]) + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])])*(1 + Cot[(e + f*x)/2])) + ((-4 - Cot[e + f*x])*(a*(1 + Sin[e + f*x]))^(1/3))/f + ((5/2 + (5*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/ (f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]) + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2])) + (Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])])*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + Tan[(e + f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])])*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(4*(1 + Tan[(e + f*x)/2])^2 + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])])*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(2*Sqrt[Sec[(e + f*x)/2]^2]))/(1 + Tan[(e + f*x)/2])*(((1 + Tan[(e + f*x)/2])

)/Sqrt[Sec[(e + f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(-(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]])*(2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2])^2 - ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]))) *Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]])*(2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/((-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]])*(2 + 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]])*Csc[(e + f*x)/2]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]])*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2]))*(-((Sec[(e + f*x)/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[(e + f*x)/2])^2) + ((1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]))*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]) + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]))^(-1/3))/((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2*(1 + Tan[(e + f*x)/2]))))

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (\cot (fx + e))^2 \sqrt[3]{a + a \sin (fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)

[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (fx + e) + a)^{\frac{1}{3}} \cot (fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a(\sin(e+fx)+1)} \cot^2(e+fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/3),x)

[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \cot(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)

3.118 $\int \cot^4(e + fx) \sqrt[3]{a + a \sin(e + fx)} dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{10/3} F_1\left(\frac{17}{6}; -\frac{3}{2}, 4; \frac{23}{6}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{17a^3 f}$$

[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17*a^3*f)

Rubi [A] time = 0.0938648, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{10/3} F_1\left(\frac{17}{6}; -\frac{3}{2}, 4; \frac{23}{6}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{17a^3 f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]

[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17*a^3*f)

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \cot^4(e + fx) \sqrt[3]{a + a \sin(e + fx)} dx = \frac{(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{(a-x)^{3/2} (a+x)^{11/6}}{x^4} dx, x \right)}{af}$$

$$= \frac{(2\sqrt{2} \sec(e + fx) (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{(a+x)^{11/6} \left(\frac{1-x}{2-2a}\right)^{3/2}}{x^4} dx \right)}{f \sqrt{\frac{a - a \sin(e + fx)}{a}}}$$

$$= \frac{12\sqrt{2} F_1 \left(\frac{17}{6}; -\frac{3}{2}, 4; \frac{23}{6}; \frac{1}{2} (1 + \sin(e + fx)), 1 + \sin(e + fx) \right) \sec(e + fx) \sqrt{1 - \sin(e + fx)}}{17a^3 f}$$

Mathematica [C] time = 22.5988, size = 2796, normalized size = 34.95

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3), x]

[Out] ((239/54 + (77*Cot[e + f*x])/54 - (Cot[e + f*x]*Csc[e + f*x])/18 - (Cot[e + f*x]*Csc[e + f*x]^2)/3)*(a*(1 + Sin[e + f*x]))^(1/3))/f - ((70/9 + (70*I)/9)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Sec[(e + f*x)/2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - ((355/108 + (355*I)/108)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]) + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2])) - (239*Cos[(3*(e + f*x))/2]*Csc[e + f*x]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(216*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + Tan[(e + f*x)/2])*(-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))/(8*(1 + Tan[(e + f*x)/2])^2 + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -

$$I*(I + \cot[(e + f*x)/2])^{1/3}*(1 + \tan[(e + f*x)/2])*(\sqrt{\sec[(e + f*x)/2]^2}/2 - (\tan[(e + f*x)/2]*(1 + \tan[(e + f*x)/2]))/(2*\sqrt{\sec[(e + f*x)/2]^2}))/((2*(1 + \tan[(e + f*x)/2])*((1 + \tan[(e + f*x)/2])/(\sqrt{\sec[(e + f*x)/2]^2}))^{1/3}) + (3*((1 + \tan[(e + f*x)/2])/(\sqrt{\sec[(e + f*x)/2]^2}))^{2/3}*(-\text{AppellF1}[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + \cot[(e + f*x)/2]), (1/2 - I/2)*(1 + \cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*\cot[(e + f*x)/2])^{1/3}*((-1 - I)*(I + \cot[(e + f*x)/2]))^{1/3}*\sec[(e + f*x)/2]^2/2 + ((1 + I)*(((1 - I)*(I + \cot[(e + f*x)/2]))/(1 + \cot[(e + f*x)/2]))^{1/3}*\text{Hypergeometric2F1}[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*\tan[(e + f*x)/2])/(2 + 2*\tan[(e + f*x)/2])]*\sec[(e + f*x)/2]^2/2^{1/3} + ((1/3 + I/3)*2^{2/3}*(((1/2 - I/2)*(I + \cot[(e + f*x)/2])*\csc[(e + f*x)/2]^2)/(1 + \cot[(e + f*x)/2])^2 - (1/2 - I/2)*\csc[(e + f*x)/2]^2)/(1 + \cot[(e + f*x)/2]))*\text{Hypergeometric2F1}[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*\tan[(e + f*x)/2])/(2 + 2*\tan[(e + f*x)/2])]*(I + \tan[(e + f*x)/2]))/(((1 - I)*(I + \cot[(e + f*x)/2]))/(1 + \cot[(e + f*x)/2]))^{2/3} - ((1/6 + I/6)*\text{AppellF1}[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + \cot[(e + f*x)/2]), (1/2 - I/2)*(1 + \cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*\cot[(e + f*x)/2])^{1/3}*\csc[(e + f*x)/2]^2*(1 + \tan[(e + f*x)/2]))/((-1 - I)*(I + \cot[(e + f*x)/2]))^{2/3} - ((1/3 - I/3)*\text{AppellF1}[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + \cot[(e + f*x)/2]), (1/2 - I/2)*(1 + \cot[(e + f*x)/2])]*((-1 - I)*(I + \cot[(e + f*x)/2]))^{1/3}*\csc[(e + f*x)/2]^2*(1 + \tan[(e + f*x)/2]))/((2 + 2*I) - (2 - 2*I)*\cot[(e + f*x)/2])^{2/3} - ((2 + 2*I) - (2 - 2*I)*\cot[(e + f*x)/2])^{1/3}*((-1 - I)*(I + \cot[(e + f*x)/2]))^{1/3}*((-1/30 + I/30)*\text{AppellF1}[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + \cot[(e + f*x)/2]), (1/2 - I/2)*(1 + \cot[(e + f*x)/2])]*\csc[(e + f*x)/2]^2 - (1/30 + I/30)*\text{AppellF1}[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + \cot[(e + f*x)/2]), (1/2 - I/2)*(1 + \cot[(e + f*x)/2])]*\csc[(e + f*x)/2]^2*(1 + \tan[(e + f*x)/2]) + ((2/3 + (2*I)/3)*2^{2/3}*(((1 - I)*(I + \cot[(e + f*x)/2]))/(1 + \cot[(e + f*x)/2]))^{1/3}*(I + \tan[(e + f*x)/2])*(2 + 2*\tan[(e + f*x)/2])*(-((\sec[(e + f*x)/2]^2*((1 + I) + (1 - I)*\tan[(e + f*x)/2]))/(2 + 2*\tan[(e + f*x)/2])^2) + ((1/2 - I/2)*\sec[(e + f*x)/2]^2)/(2 + 2*\tan[(e + f*x)/2]))*(-\text{Hypergeometric2F1}[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*\tan[(e + f*x)/2])/(2 + 2*\tan[(e + f*x)/2])]) + (1 - ((1 + I) + (1 - I)*\tan[(e + f*x)/2])/(2 + 2*\tan[(e + f*x)/2]))^{(-1/3)))/((1 + I) + (1 - I)*\tan[(e + f*x)/2]))/(4*(1 + \tan[(e + f*x)/2]))$$

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\cot(fx + e))^4 \sqrt[3]{a + a \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)

[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/3),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^{\frac{1}{3}} \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")`

[Out] `integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)`

$$3.119 \quad \int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=551

$$\frac{3a^2 \sin^2(e+fx) \tan(e+fx)}{f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}} + \frac{3a^2 \sin(e+fx) \tan(e+fx)}{4f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}} + \frac{973 \sec(e+fx)(a \sin(e+fx))}{f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}}$$

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
+ 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)]],
(2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rubi [A] time = 0.493573, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2719, 100, 153, 144, 51, 63, 225}

$$\frac{3a^2 \sin^2(e+fx) \tan(e+fx)}{f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}} + \frac{3a^2 \sin(e+fx) \tan(e+fx)}{4f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}} + \frac{973 \sec(e+fx)(a \sin(e+fx))}{f(a-a \sin(e+fx))(a \sin(e+fx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
+ 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)]],
(2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rule 2719

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_
), x_Symbol] := Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && !IntegerQ[m] && IntegerQ[p/2]
```

Rule 100

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_
))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))] + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_
))^(p_)*((g_) + (h_)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
 + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
 + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
 p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))] + (b*d*f*g*(m + n + p
 + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 144

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_
))*((g_) + (h_)*(x_)), x_Symbol] := Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b*d*(b*c - a*d)^2*(m + 1)*(n + 1)), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
 + 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5)))/(b*d*(b*c - a*d)^2*(m
 + 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 51

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx = \frac{(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}) \operatorname{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x)^{17/6}} dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{3a^2 \sin^2(e+fx) \tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{(3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)})}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

$$= \frac{3a^2 \sin(e+fx) \tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2 \sin^2(e+fx) \tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} +$$

$$= \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2 \sin(e+fx) \tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

$$= \frac{973 \sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2 \sin(e+fx)}{4f(a-a\sin(e+fx))^{4/3}}$$

$$= \frac{973 \sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973 \sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

$$= \frac{973 \sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973 \sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

$$= \frac{973 \sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973 \sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

Mathematica [C] time = 0.796308, size = 128, normalized size = 0.23

$$\frac{973\sqrt{2} \cos(e+fx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right) + \sqrt{1-\sin(e+fx)} \sec^3(e+fx)(22\sin(e+fx) - 128\sin(3e+3fx))}{495f\sqrt{1-\sin(e+fx)}\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]

[Out] (973*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/4]^2] + Sec[e + f*x]^3*Sqrt[1 - Sin[e + f*x]]*(-49 - 64*Cos[2*(e + f*x)] + 22*Sin[e + f*x] - 128*Sin[3*(e + f*x)]))/(495*f*Sqrt[1 - Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (\tan(fx + e))^4 \frac{1}{\sqrt[3]{a + a \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)

[Out] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\tan(fx + e)^4}{(a \sin(fx + e) + a)^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")

[Out] integral(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e + fx)}{\sqrt[3]{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)

[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx + e)^4}{(a \sin(fx + e) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

$$3.120 \quad \int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=126

$$\frac{11\sqrt[6]{2} \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx) + 1}\sqrt[3]{a \sin(e+fx) + a}} + \frac{4 \sec(e+fx)(a \sin(e+fx) + a)^{2/3}}{5af} - \frac{3 \sec(e+fx)}{5f\sqrt[3]{a \sin(e+fx) + a}}$$

[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3))/(5*a*f)

Rubi [A] time = 0.216494, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2712, 2855, 2652, 2651}

$$\frac{11\sqrt[6]{2} \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx) + 1}\sqrt[3]{a \sin(e+fx) + a}} + \frac{4 \sec(e+fx)(a \sin(e+fx) + a)^{2/3}}{5af} - \frac{3 \sec(e+fx)}{5f\sqrt[3]{a \sin(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]

[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3))/(5*a*f)

Rule 2712

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]), x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && LtQ[m, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n +
1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx &= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{3\int \sec^2(e+fx)(a+a\sin(e+fx))^{2/3} \left(-\frac{a}{3} + \frac{5}{3}a\sin(e+fx)\right) dx}{5a^2} \\ &= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{11}{15} \int \frac{1}{\sqrt[3]{a+a\sin(e+fx)}} dx \\ &= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{(11\sqrt[3]{1+\sin(e+fx)}) \int \frac{1}{\sqrt[3]{a+a\sin(e+fx)}} dx}{15\sqrt[3]{a+a\sin(e+fx)}} \\ &= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{11\sqrt[6]{2}\cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{1+\sin(e+fx)}\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} \end{aligned}$$

Mathematica [A] time = 0.513245, size = 100, normalized size = 0.79

$$\frac{\sqrt{2-2\sin(e+fx)}(4\tan(e+fx)+\sec(e+fx))-22\cos(e+fx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{5f\sqrt{2-2\sin(e+fx)}\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]

[Out] (-22*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/4]^2] + Sqrt[2 - 2*Sin[e + f*x]]*(Sec[e + f*x] + 4*Tan[e + f*x]))/(5*f*Sqrt[2 - 2*Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (\tan(fx+e))^2 \frac{1}{\sqrt[3]{a+a\sin(fx+e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3), x)

[Out] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan^2(fx + e)}{(a \sin(fx + e) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")

[Out] integral(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e + fx)}{\sqrt[3]{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)

[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(fx + e)}{(a \sin(fx + e) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)

$$3.121 \quad \int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2*f)

Rubi [A] time = 0.0949979, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]

[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2*f)

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx = \frac{(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}) \operatorname{Subst}\left(\int \frac{\sqrt{a-x}\sqrt[6]{a+x}}{x^2} dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}) \operatorname{Subst}\left(\int \frac{\sqrt[6]{a+x}\sqrt{\frac{1-x}{2-2a}}}{x^2} dx, x, a\sin(e+fx)\right)}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}F_1\left(\frac{7}{6}; -\frac{1}{2}, 2; \frac{13}{6}; \frac{1}{2}(1+\sin(e+fx)), 1+\sin(e+fx)\right) \sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a\sin(e+fx))}{7a^2f}$$

Mathematica [F] time = 12.7663, size = 0, normalized size = 0.

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]

[Out] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (\cot(fx+e))^2 \frac{1}{\sqrt[3]{a+a\sin(fx+e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3), x)

[Out] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3), x, algorithm="maxima")

[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e + fx)}{\sqrt[3]{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)`

[Out] `Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(fx + e)}{(a \sin(fx + e) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")`

[Out] `integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)`

$$3.122 \quad \int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*a^3*f)

Rubi [A] time = 0.0935749, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]

[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*a^3*f)

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx = \frac{(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}) \operatorname{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{7/6}}{x^4} dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}) \operatorname{Subst}\left(\int \frac{(a+x)^{7/6}\left(\frac{1-x}{2-2a}\right)^{3/2}}{x^4} dx, x, a\sin(e+fx)\right)}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}F_1\left(\frac{13}{6}; -\frac{3}{2}, 4; \frac{19}{6}; \frac{1}{2}(1+\sin(e+fx)), 1+\sin(e+fx)\right) \sec(e+fx)\sqrt{1-\sin(e+fx)}(a-a\sin(e+fx))}{13a^3f}$$

Mathematica [F] time = 9.12866, size = 0, normalized size = 0.

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]

[Out] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (\cot(fx+e))^4 \frac{1}{\sqrt[3]{a+a\sin(fx+e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3), x)

[Out] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3), x, algorithm="maxima")

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(e + fx)}{\sqrt[3]{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(fx + e)}{(a \sin(fx + e) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.123 $\int (a + a \sin(e + fx))^3 (g \tan(e + fx))^p dx$

Optimal. Leaf size=269

$$\frac{3a^3(g \tan(e + fx))^{p+3} {}_2F_1\left(2, \frac{p+3}{2}; \frac{p+5}{2}; -\tan^2(e + fx)\right)}{fg^3(p+3)} + \frac{a^3(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \dots$$

[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rubi [A] time = 0.345108, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^3(g \tan(e + fx))^{p+3} {}_2F_1\left(2, \frac{p+3}{2}; \frac{p+5}{2}; -\tan^2(e + fx)\right)}{fg^3(p+3)} + \frac{a^3(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]

[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a*cos[e + f*x]^(n + 1)*(b*tan[e + f*x])^(n + 1))/(b*(a*sin[e + f*x])^(n + 1)), Int[(a*sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*cos[e + f*x])^(2*FracPart[(n - 1)/2])*(a*sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (g \tan(e + fx))^p dx &= \int (a^3 (g \tan(e + fx))^p + 3a^3 \sin(e + fx) (g \tan(e + fx))^p + 3a^3 \sin^2(e + fx) (g \tan(e + fx))^p + a^3 \sin^3(e + fx) (g \tan(e + fx))^p) dx \\ &= a^3 \int (g \tan(e + fx))^p dx + a^3 \int \sin^3(e + fx) (g \tan(e + fx))^p dx + (3a^3) \int \sin^2(e + fx) (g \tan(e + fx))^p dx + a^3 \int \sin^3(e + fx) (g \tan(e + fx))^p dx \\ &= \frac{(a^3 g) \text{Subst}\left(\int \frac{x^p}{g^2 + x^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(3a^3 g) \text{Subst}\left(\int \frac{x^{2+p}}{(g^2 + x^2)^2} dx, x, g \tan(e + fx)\right)}{f} \\ &= \frac{a^3 {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{3a^3 \cos^2(e + fx) \frac{1+p}{2}}{fg(1+p)} \end{aligned}$$

Mathematica [C] time = 58.8952, size = 5199, normalized size = 19.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.709, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^3 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

[Out] `int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (fx + e) + a)^3 (g \tan (fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")`

[Out] `integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3 a^3 \cos (fx + e)^2 - 4 a^3 + \left(a^3 \cos (fx + e)^2 - 4 a^3\right) \sin (fx + e)\right)\left(g \tan (fx + e)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")`

[Out] `integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (fx + e) + a)^3 (g \tan (fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")`

[Out] `integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)`

3.124 $\int (a + a \sin(e + fx))^2 (g \tan(e + fx))^p dx$

Optimal. Leaf size=187

$$\frac{a^2(g \tan(e + fx))^{p+3} {}_2F_1\left(2, \frac{p+3}{2}; \frac{p+5}{2}; -\tan^2(e + fx)\right)}{fg^3(p+3)} + \frac{a^2(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{2a^2 \sin(e + fx)}{g}$$

[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rubi [A] time = 0.226524, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2(g \tan(e + fx))^{p+3} {}_2F_1\left(2, \frac{p+3}{2}; \frac{p+5}{2}; -\tan^2(e + fx)\right)}{fg^3(p+3)} + \frac{a^2(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{2a^2 \sin(e + fx)}{g}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]

[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2602

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[e + f*x]^(n+1)*(b*Tan[e + f*x])^(n+1))/(b*(a*Sin[e + f*x])^(n+1)), Int[(a*Sin[e + f*x])^(m+n)/Cos[e + f*x]^n, x]

, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]

Rule 2577

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^ (n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2]))*(a*SIN[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rule 2591

Int[sin[(e_.) + (f_.)*(x_.)]^ (m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^ (n_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (g \tan(e + fx))^p dx &= \int (a^2 (g \tan(e + fx))^p + 2a^2 \sin(e + fx) (g \tan(e + fx))^p + a^2 \sin^2(e + fx) (g \tan(e + fx))^p) dx \\ &= a^2 \int (g \tan(e + fx))^p dx + a^2 \int \sin^2(e + fx) (g \tan(e + fx))^p dx + (2a^2) \int \sin(e + fx) (g \tan(e + fx))^p dx \\ &= \frac{(a^2 g) \text{Subst}\left(\int \frac{x^{2+p}}{(g^2+x^2)^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(a^2 g) \text{Subst}\left(\int \frac{x^p}{g^2+x^2} dx, x, g \tan(e + fx)\right)}{f} \\ &= \frac{a^2 {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{2a^2 \cos^2(e + fx) (g \tan(e + fx))^p}{fg} \end{aligned}$$

Mathematica [C] time = 18.053, size = 2054, normalized size = 10.98

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]

[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(g*Tan[e + f*x])^p*(Cos[(e + f*x)/2]^4*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Tan[e + f*x]^p + 6*Cos[(e + f*x)/2]^2*Sin[(e + f*x)/2]^2*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]*Sin[(e + f*x)/2]^3*Tan[e + f*x]^p + Sin[(e + f*x)/2]^4*Tan[e + f*x]^p)/(f*(1 + p)*(2 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Sec[e + f*x]^2*Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3

+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]*Tan[e + f*x]^p/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*Tan[e + f*x]^p/((1 + p)*(2 + p)) + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(2*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2 + 4*(1 + p)*Tan[(e + f*x)/2]*((-2*(1 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + (2 + p)*(-(((1 + p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 4*(2 + p)*((-3*(1 + p)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p))*Tan[e + f*x]^p/((1 + p)*(2 + p)))

Maple [F] time = 1.421, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)

[Out] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2 \cos(fx + e)\right)^2 - 2a^2 \sin(fx + e) - 2a^2\right) (g \tan(fx + e))^p, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*tan(f*x + e))^p, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int (g \tan(e + fx))^p dx + \int 2(g \tan(e + fx))^p \sin(e + fx) dx + \int (g \tan(e + fx))^p \sin^2(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)

[Out] a**2*(Integral((g*tan(e + f*x))**p, x) + Integral(2*(g*tan(e + f*x))**p*sin(e + f*x), x) + Integral((g*tan(e + f*x))**p*sin(e + f*x)**2, x))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)

3.125 $\int (a + a \sin(e + fx))(g \tan(e + fx))^p dx$

Optimal. Leaf size=129

$$\frac{a(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{a \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2}; \frac{p+4}{2}\right)}{fg(p+2)}$$

[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p))

Rubi [A] time = 0.138781, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2710, 3476, 364, 2602, 2577}

$$\frac{a(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{a \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2}; \frac{p+4}{2}\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]

[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p))

Rule 2710

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2602

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n+1)*(b*Tan[e + f*x])^(n+1))/(b*(a*Sin[e + f*x])^(n+1)), Int[(a*Sin[e + f*x])^(m+n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]

Rule 2577

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2]))*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(g \tan(e + fx))^p dx &= \int (a(g \tan(e + fx))^p + a \sin(e + fx)(g \tan(e + fx))^p) dx \\ &= a \int (g \tan(e + fx))^p dx + a \int \sin(e + fx)(g \tan(e + fx))^p dx \\ &= \frac{(ag) \text{Subst}\left(\int \frac{x^p}{g^2+x^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(a \cos^{1+p}(e + fx) \sin^{-1-p}(e + fx))}{f} \\ &= \frac{a {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{a \cos^2(e + fx)^{\frac{1+p}{2}}}{f} \end{aligned}$$

Mathematica [F] time = 2.02497, size = 0, normalized size = 0.

$$\int (a + a \sin(e + fx))(g \tan(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]

[Out] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]

Maple [F] time = 0.907, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))(g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)

[Out] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)(g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin (f x+e)+a\right)\left(g \tan (f x+e)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int(g \tan (e+f x))^p d x+\int(g \tan (e+f x))^p \sin (e+f x) d x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)

[Out] a*(Integral((g*tan(e + f*x))^p, x) + Integral((g*tan(e + f*x))^p*sin(e + f*x), x))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a \sin (f x+e)+a\right)\left(g \tan (f x+e)\right)^p d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

$$3.126 \quad \int \frac{(g \tan(e+fx))^p}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=108

$$\frac{(g \tan(e+fx))^{p+1}}{afg(p+1)} - \frac{\sec(e+fx) \cos^2(e+fx)^{\frac{p+3}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+3}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{afg^2(p+2)}$$

[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))

Rubi [A] time = 0.12612, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2706, 2607, 32, 2617}

$$\frac{(g \tan(e+fx))^{p+1}}{afg(p+1)} - \frac{\sec(e+fx) \cos^2(e+fx)^{\frac{p+3}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+3}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{afg^2(p+2)}$$

Antiderivative was successfully verified.

[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]

[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))

Rule 2706

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2617

Int[((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e + f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n + 3)/2, Sin[e + f*x]^2]/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]

Rubi steps

$$\int \frac{(g \tan(e + fx))^p}{a + a \sin(e + fx)} dx = \frac{\int \sec^2(e + fx)(g \tan(e + fx))^p dx}{a} - \frac{\int \sec(e + fx)(g \tan(e + fx))^{1+p} dx}{ag}$$

$$= -\frac{\cos^2(e + fx)^{\frac{3+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{3+p}{2}; \frac{4+p}{2}; \sin^2(e + fx)\right) \sec(e + fx)(g \tan(e + fx))^{2+p}}{afg^2(2+p)} + \frac{\text{Subst}\left(\int (g \tan(e + fx))^{1+p} dx, e + fx, \frac{1}{2}(e + fx)\right)}{afg(1+p)}$$

$$= \frac{(g \tan(e + fx))^{1+p}}{afg(1+p)} - \frac{\cos^2(e + fx)^{\frac{3+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{3+p}{2}; \frac{4+p}{2}; \sin^2(e + fx)\right) \sec(e + fx)(g \tan(e + fx))^{2+p}}{afg^2(2+p)}$$

Mathematica [B] time = 4.03293, size = 232, normalized size = 2.15

$$\frac{2 \tan\left(\frac{1}{2}(e + fx)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^2 (g \tan(e + fx))^p \left((p^2 + 5p + 6) {}_2F_1\left(\frac{p+1}{2}, p + 2; \frac{p+3}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right)\right) - (p+1)\right)}{afg(1+p)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]

[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2])^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Tan[(e + f*x)/2]*((6 + 5*p + p^2)*Hypergeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2] - (1 + p)*Tan[(e + f*x)/2]*(2*(3 + p)*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2] - (2 + p)*Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[e + f*x])^p)/(f*(1 + p)*(2 + p)*(3 + p)*(a + a*Sin[e + f*x]))

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{a + a \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)

[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(g \tan(fx + e))^p}{a \sin(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(g \tan(e+fx))^p}{\sin(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e)),x)

[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x) + 1), x)/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)

$$3.127 \quad \int \frac{(g \tan(e+fx))^p}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=138

$$\frac{2 \sec^3(e+fx) \cos^2(e+fx)^{\frac{p+5}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+5}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^2 f g^2 (p+2)} + \frac{2(g \tan(e+fx))^{p+3}}{a^2 f g^3 (p+3)} + \frac{(g \tan(e+fx))^{p+3}}{a^2 f g (p+3)}$$

[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x])^(3 + p))/(a^2*f*g^3*(3 + p))

Rubi [A] time = 0.274754, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2711, 2607, 14, 16, 2617, 32}

$$\frac{2 \sec^3(e+fx) \cos^2(e+fx)^{\frac{p+5}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+5}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^2 f g^2 (p+2)} + \frac{2(g \tan(e+fx))^{p+3}}{a^2 f g^3 (p+3)} + \frac{(g \tan(e+fx))^{p+3}}{a^2 f g (p+3)}$$

Antiderivative was successfully verified.

[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]

[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x])^(3 + p))/(a^2*f*g^3*(3 + p))

Rule 2711

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]

Rule 2607

Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2617


```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e + f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n + 3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 32

```
Int[(a_. + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(g \tan(e + fx))^p}{(a + a \sin(e + fx))^2} dx &= \frac{\int (a^2 \sec^4(e + fx)(g \tan(e + fx))^p - 2a^2 \sec^3(e + fx) \tan(e + fx)(g \tan(e + fx))^p + a^2 \sec^2(e + fx) \tan^2(e + fx)(g \tan(e + fx))^p) dx}{a^4} \\ &= \frac{\int \sec^4(e + fx)(g \tan(e + fx))^p dx}{a^2} + \frac{\int \sec^2(e + fx) \tan^2(e + fx)(g \tan(e + fx))^p dx}{a^2} - \frac{2 \int \sec^3(e + fx) \tan(e + fx)(g \tan(e + fx))^p dx}{a^2} \\ &= \frac{\text{Subst}\left(\int (gx)^p (1 + x^2) dx, x, \tan(e + fx)\right)}{a^2 f} + \frac{\int \sec^2(e + fx)(g \tan(e + fx))^{2+p} dx}{a^2 g^2} - \frac{2 \int \sec^3(e + fx) \tan(e + fx)(g \tan(e + fx))^p dx}{a^2 f g^2} \\ &= -\frac{2 \cos^2(e + fx)^{\frac{5+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{5+p}{2}; \frac{4+p}{2}; \sin^2(e + fx)\right) \sec^3(e + fx)(g \tan(e + fx))^{2+p}}{a^2 f g^2 (2+p)} + \frac{(g \tan(e + fx))^{1+p}}{a^2 f g (1+p)} - \frac{2 \cos^2(e + fx)^{\frac{5+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{5+p}{2}; \frac{4+p}{2}; \sin^2(e + fx)\right) \sec^3(e + fx)(g \tan(e + fx))^p}{a^2 f g^2 (2+p)} \end{aligned}$$

Mathematica [B] time = 14.1776, size = 667, normalized size = 4.83

$$2^{p+1} \tan\left(\frac{1}{2}(e + fx)\right) \left(1 - \tan^2\left(\frac{1}{2}(e + fx)\right)\right)^p \left(-\frac{\tan\left(\frac{1}{2}(e + fx)\right)}{\tan^2\left(\frac{1}{2}(e + fx)\right) - 1}\right)^p \tan^{-p}(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^4 (g \tan(e + fx))^p$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Tan[(e + f*x)/2]*(1 - Tan[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hypergeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) - (2*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p) + (2*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (6*Hypergeometric2F1[(2 + p)/2, 3 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (8*Hypergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (6*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (12*Hypergeometric2F1[(3 + p)/2, 4 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (2*Hypergeometric2F1[3 + p, (4 + p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (8*Hypergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (2*Hypergeometric2F1[4 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p))*(g*Tan[e + f*x])^p/(f*(a + a*Sin[e + f*x])^2*Tan[e + f*x]^p)
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a + a \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)

[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(g \tan(fx + e))^p}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(g*tan(f*x + e))^p/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(e + fx))^p}{\frac{\sin^2(e + fx) + 2 \sin(e + fx) + 1}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**2,x)

[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

$$3.128 \quad \int \frac{(g \tan(e+fx))^p}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=248

$$\frac{3 \sec^5(e+fx) \cos^2(e+fx)^{\frac{p+7}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+7}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^3 f g^2 (p+2)} - \frac{\sec^3(e+fx) \cos^2(e+fx)^{\frac{p+7}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+7}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^3 f g^2 (p+2)}$$

[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^5*(g*Tan[e + f*x])^(2 + p)/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x])^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometric2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Tan[e + f*x])^(4 + p)/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^3*f*g^5*(5 + p))

Rubi [A] time = 0.421813, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2711, 2607, 270, 16, 2617, 14}

$$\frac{3 \sec^5(e+fx) \cos^2(e+fx)^{\frac{p+7}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+7}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^3 f g^2 (p+2)} - \frac{\sec^3(e+fx) \cos^2(e+fx)^{\frac{p+7}{2}} (g \tan(e+fx))^{p+2} {}_2F_1\left(\frac{p+2}{2}, \frac{p+7}{2}; \frac{p+4}{2}; \sin^2(e+fx)\right)}{a^3 f g^2 (p+2)}$$

Antiderivative was successfully verified.

[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]

[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^5*(g*Tan[e + f*x])^(2 + p)/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x])^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometric2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Tan[e + f*x])^(4 + p)/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^3*f*g^5*(5 + p))

Rule 2711

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]

Rule 2607

Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2617

Int[((a_)*sec[(e_)+(f_)*(x_)])^(m_)*((b_)*tan[(e_)+(f_)*(x_)])^(n_), x_Symbol] := Simp[((a*Sec[e+f*x])^m*(b*Tan[e+f*x])^(n+1)*(Cos[e+f*x]^2)^(m+n+1)/2)*Hypergeometric2F1[(n+1)/2, (m+n+1)/2, (n+3)/2, Sin[e+f*x]^2]/(b*f*(n+1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[(n-1)/2] && !IntegerQ[m/2]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rubi steps

$$\begin{aligned} \int \frac{(g \tan(e+fx))^p}{(a+a \sin(e+fx))^3} dx &= \frac{\int (a^3 \sec^6(e+fx)(g \tan(e+fx))^p - 3a^3 \sec^5(e+fx) \tan(e+fx)(g \tan(e+fx))^p + 3a^3 \sec^4(e+fx) \tan^2(e+fx)(g \tan(e+fx))^p - 3a^3 \sec^3(e+fx) \tan^3(e+fx)(g \tan(e+fx))^p) dx}{a^3} \\ &= \frac{\int \sec^6(e+fx)(g \tan(e+fx))^p dx}{a^3} - \frac{\int \sec^3(e+fx) \tan^3(e+fx)(g \tan(e+fx))^p dx}{a^3} - \frac{\int \sec^4(e+fx) \tan^2(e+fx)(g \tan(e+fx))^p dx}{a^3} + \frac{\int \sec^5(e+fx) \tan^3(e+fx)(g \tan(e+fx))^p dx}{a^3} \\ &= \frac{\text{Subst}\left(\int (gx)^p (1+x^2)^2 dx, x, \tan(e+fx)\right)}{a^3 f} - \frac{\int \sec^3(e+fx)(g \tan(e+fx))^{3+p} dx}{a^3 g^3} + \frac{\int \sec^4(e+fx) \tan^2(e+fx)(g \tan(e+fx))^p dx}{a^3} - \frac{\int \sec^5(e+fx) \tan^3(e+fx)(g \tan(e+fx))^p dx}{a^3} \\ &= -\frac{3 \cos^2(e+fx)^{\frac{7+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{7+p}{2}; \frac{4+p}{2}; \sin^2(e+fx)\right) \sec^5(e+fx)(g \tan(e+fx))^{2+p}}{a^3 f g^2 (2+p)} - \frac{3 \cos^2(e+fx)^{\frac{7+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{7+p}{2}; \frac{4+p}{2}; \sin^2(e+fx)\right) \sec^5(e+fx)(g \tan(e+fx))^{2+p}}{a^3 f g^2 (2+p)} \\ &= \frac{(g \tan(e+fx))^{1+p}}{a^3 f g (1+p)} - \frac{3 \cos^2(e+fx)^{\frac{7+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{7+p}{2}; \frac{4+p}{2}; \sin^2(e+fx)\right) \sec^5(e+fx)(g \tan(e+fx))^{2+p}}{a^3 f g^2 (2+p)} \\ &= \frac{(g \tan(e+fx))^{1+p}}{a^3 f g (1+p)} - \frac{3 \cos^2(e+fx)^{\frac{7+p}{2}} {}_2F_1\left(\frac{2+p}{2}, \frac{7+p}{2}; \frac{4+p}{2}; \sin^2(e+fx)\right) \sec^5(e+fx)(g \tan(e+fx))^{2+p}}{a^3 f g^2 (2+p)} \end{aligned}$$

Mathematica [B] time = 31.2325, size = 1276, normalized size = 5.15

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Tan[e+f*x])^p/(a+a*Sin[e+f*x])^3,x]

[Out] (2^(1+p)*(Cos[(e+f*x)/2]+Sin[(e+f*x)/2])^6*Tan[(e+f*x)/2]*(1-Tan[(e+f*x)/2]^2)^p*(-(Tan[(e+f*x)/2]/(-1+Tan[(e+f*x)/2]^2)))^p*(Hypergeometric2F1[(1+p)/2, 2+p, (3+p)/2, Tan[(e+f*x)/2]^2]/(1+p)-(4*Hypergeometric2F1[(1+p)/2, 3+p, (3+p)/2, Tan[(e+f*x)/2]^2]/(1+p)+(8*Hypergeometric2F1[(1+p)/2, 4+p, (3+p)/2, Tan[(e+f*x)/2]^2]/(1+p)-(8*Hypergeometric2F1[(1+p)/2, 5+p, (3+p)/2, Tan[(e+f*x)/2]^2]/(1+p)+(4*Hypergeometric2F1[(1+p)/2, 6+p, (3+p)/2, Tan[(e+f*x)/2]^2]/(1+p)-(2*Hypergeometric2F1[(2+p)/2, 2+p, (4+p)/2, Tan[(e+f*x)/2]^2]*Tan[(e+f*x)/2])/(2+p)+(12*Hypergeometric2F1[(2+p)/2, 3+p, (4+p)/2, Tan[(e+f*x)/2]^2]*Tan[(e+f*x)/2])/(2+p)-(32*Hy

pergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]/(2 + p) + (40*Hypergeometric2F1[(2 + p)/2, 5 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]/(2 + p) - (24*Hypergeometric2F1[(2 + p)/2, 6 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]/(2 + p) + (Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (12*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (48*Hypergeometric2F1[(3 + p)/2, 4 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (80*Hypergeometric2F1[(3 + p)/2, 5 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (60*Hypergeometric2F1[(3 + p)/2, 6 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (4*Hypergeometric2F1[3 + p, (4 + p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (32*Hypergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (80*Hypergeometric2F1[(4 + p)/2, 5 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (80*Hypergeometric2F1[(4 + p)/2, 6 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (8*Hypergeometric2F1[4 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) - (40*Hypergeometric2F1[(5 + p)/2, 5 + p, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (60*Hypergeometric2F1[(5 + p)/2, 6 + p, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (8*Hypergeometric2F1[3 + p/2, 5 + p, 4 + p/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^5)/(6 + p) - (24*Hypergeometric2F1[(6 + p)/2, 6 + p, (8 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^5)/(6 + p) + (4*Hypergeometric2F1[6 + p, (7 + p)/2, (9 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^6)/(7 + p))*(g*Tan[e + f*x])^p/(f*(a + a*Sin[e + f*x])^3*Tan[e + f*x]^p)

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a + a \sin(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)

[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a \sin(fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{(g \tan (fx + e))^p}{3 a^3 \cos (fx + e)^2 - 4 a^3 + (a^3 \cos (fx + e)^2 - 4 a^3) \sin (fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] integral(-(g*tan(f*x + e))^p/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan (fx + e))^p}{(a \sin (fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)

3.129 $\int (a + a \sin(e + fx))^m (g \tan(e + fx))^p dx$

Optimal. Leaf size=111

$$\frac{(1 - \sin(e + fx))^{\frac{p+1}{2}} (a \sin(e + fx) + a)^m (g \tan(e + fx))^{p+1} (\sin(e + fx) + 1)^{\frac{1}{2}(-2m+p+1)} F_1\left(p + 1; \frac{p+1}{2}, \frac{1}{2}(-2m + p + 1); p + 1\right)}{fg(p + 1)}$$

[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e + f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))

Rubi [A] time = 0.12401, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2720, 135, 133}

$$\frac{(1 - \sin(e + fx))^{\frac{p+1}{2}} (a \sin(e + fx) + a)^m (g \tan(e + fx))^{p+1} (\sin(e + fx) + 1)^{\frac{1}{2}(-2m+p+1)} F_1\left(p + 1; \frac{p+1}{2}, \frac{1}{2}(-2m + p + 1); p + 1\right)}{fg(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]

[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e + f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))

Rule 2720

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[((g*Tan[e + f*x])^(p + 1)*(a - b*Sin[e + f*x])^((p + 1)/2)*(a + b*Sin[e + f*x])^((p + 1)/2))/(f*g*(b*Sin[e + f*x])^(p + 1)), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !IntegerQ[p]

Rule 135

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 133

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)]/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m (g \tan(e + fx))^p dx &= \frac{\left((a \sin(e + fx))^{-1-p} (a - a \sin(e + fx))^{\frac{1+p}{2}} (a + a \sin(e + fx))^{\frac{1+p}{2}} (g \tan(e + fx))^p \right)}{fg} \\
&= \frac{\left((1 - \sin(e + fx))^{\frac{1}{2} + \frac{p}{2}} (a \sin(e + fx))^{-1-p} (a - a \sin(e + fx))^{-\frac{1}{2} - \frac{p}{2} + \frac{1+p}{2}} (a + a \sin(e + fx))^{\frac{1+p}{2}} (g \tan(e + fx))^p \right)}{fg} \\
&= \frac{\left((1 - \sin(e + fx))^{\frac{1}{2} + \frac{p}{2}} (a \sin(e + fx))^{-1-p} (1 + \sin(e + fx))^{\frac{1}{2} - m + \frac{p}{2}} (a - a \sin(e + fx))^m (g \tan(e + fx))^p \right)}{fg} \\
&= \frac{F_1\left(1 + p; \frac{1+p}{2}, \frac{1}{2}(1 - 2m + p); 2 + p; \sin(e + fx), -\sin(e + fx)\right) (1 - \sin(e + fx))^{\frac{1}{2} + \frac{p}{2}} (a \sin(e + fx))^{-1-p} (1 + \sin(e + fx))^{\frac{1}{2} - m + \frac{p}{2}} (a - a \sin(e + fx))^m (g \tan(e + fx))^p}{fg}
\end{aligned}$$

Mathematica [B] time = 2.19787, size = 367, normalized size = 3.31

$$\frac{2(p-3) \sin\left(\frac{1}{4}(2e + 2fx - \pi)\right) \cos^3\left(\frac{1}{4}(2e + 2fx - \pi)\right)}{f(p-1) \left((p-3) \cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right) F_1\left(\frac{1-p}{2}; -p, m+1; \frac{3-p}{2}; \cot^2\left(\frac{1}{4}(2e + 2fx - \pi)\right), -\tan^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]

[Out] (-2*(-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^3*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e - Pi + 2*f*x)/4]*(g*Tan[e + f*x])^p)/(f*(-1 + p)*((-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^2 + 2*(p*AppellF1[(3 - p)/2, 1 - p, 1 + m, (5 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m)*AppellF1[(3 - p)/2, -p, 2 + m, (5 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Sin[(2*e - Pi + 2*f*x)/4]^2))

Maple [F] time = 0.847, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)

[Out] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin (f x+e)+a\right)^m\left(g \tan (f x+e)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a \sin (f x+e)+a\right)^m\left(g \tan (f x+e)\right)^p d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

3.130 $\int (a + a \sin(e + fx))^m \tan^3(e + fx) dx$

Optimal. Leaf size=163

$$\frac{a^2 \sin^2(e + fx)(a \sin(e + fx) + a)^{m-1}}{fm(a - a \sin(e + fx))} + \frac{a(m + 4)(a \sin(e + fx) + a)^{m-1} {}_2F_1\left(1, m - 1; m; \frac{1}{2}(\sin(e + fx) + 1)\right)}{4f(1 - m)} + \frac{(2am)}{4f(1 - m)}$$

```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.148753, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2707, 100, 146, 68}

$$\frac{a^2 \sin^2(e + fx)(a \sin(e + fx) + a)^{m-1}}{fm(a - a \sin(e + fx))} + \frac{a(m + 4)(a \sin(e + fx) + a)^{m-1} {}_2F_1\left(1, m - 1; m; \frac{1}{2}(\sin(e + fx) + 1)\right)}{4f(1 - m)} + \frac{(2am)}{4f(1 - m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```

Rule 2707

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 100

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))*((g_) + (h_)*(x_)), x_Symbol] := Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*(m + 1)*x*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
```

, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]

Rule 68

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m \tan^3(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^{-2+m}}{(a-x)^2} dx, x, a \sin(e + fx)\right)}{f} \\ &= -\frac{a^2 \sin^2(e + fx)(a + a \sin(e + fx))^{-1+m}}{fm(a - a \sin(e + fx))} - \frac{\text{Subst}\left(\int \frac{x(a+x)^{-2+m}(-2a^2-ax)}{(a-x)^2} dx, x, a \sin(e + fx)\right)}{fm} \\ &= -\frac{a^2 \sin^2(e + fx)(a + a \sin(e + fx))^{-1+m}}{fm(a - a \sin(e + fx))} + \frac{(a + a \sin(e + fx))^{-1+m} (a(2 - 3m - \sin(e + fx)))}{2f(1 - m)m(1 - \sin(e + fx))} \\ &= \frac{a(4 + m) {}_2F_1\left(1, -1 + m; m; \frac{1}{2}(1 + \sin(e + fx))\right) (a + a \sin(e + fx))^{-1+m}}{4f(1 - m)} - \frac{a^2 \sin^2(e + fx)}{f} \end{aligned}$$

Mathematica [A] time = 0.267321, size = 105, normalized size = 0.64

$$\frac{a(a(\sin(e + fx) + 1))^{m-1} \left(-m(m + 4)(\sin(e + fx) - 1) {}_2F_1\left(1, m - 1; m; \frac{1}{2}(\sin(e + fx) + 1)\right) + 4(m - 1)\sin^2(e + fx) + 4m \right)}{4f(m - 1)m(\sin(e + fx) - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]

[Out] (a*(a*(1 + Sin[e + f*x]))^(-1 + m)*(-2*(-2 + 3*m + m^2) - m*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 4*m*Sin[e + f*x] + 4*(-1 + m)*Sin[e + f*x]^2))/(4*f*(-1 + m)*m*(-1 + Sin[e + f*x]))

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m (\tan(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)

[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \tan(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)

3.131 $\int (a + a \sin(e + fx))^m \tan(e + fx) dx$

Optimal. Leaf size=72

$$\frac{(a \sin(e + fx) + a)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{1}{2}(\sin(e + fx) + 1)\right)}{4af(m + 1)} - \frac{(a \sin(e + fx) + a)^m}{2fm}$$

[Out] $-(a + a*\text{Sin}[e + f*x])^m/(2*f*m) + (\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (1 + \text{Sin}[e + f*x])/2]*(a + a*\text{Sin}[e + f*x])^{(1 + m)})/(4*a*f*(1 + m))$

Rubi [A] time = 0.0504618, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2707, 79, 68}

$$\frac{(a \sin(e + fx) + a)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{1}{2}(\sin(e + fx) + 1)\right)}{4af(m + 1)} - \frac{(a \sin(e + fx) + a)^m}{2fm}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*\text{Tan}[e + f*x], x]$

[Out] $-(a + a*\text{Sin}[e + f*x])^m/(2*f*m) + (\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (1 + \text{Sin}[e + f*x])/2]*(a + a*\text{Sin}[e + f*x])^{(1 + m)})/(4*a*f*(1 + m))$

Rule 2707

$\text{Int}[(a + b*\text{sin}[e + f*x])^m*\text{tan}[e + f*x], x]$
 $\text{Int}[(a + b*\text{sin}[e + f*x])^m*\text{tan}[e + f*x], x] := \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 79

$\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x]$
 $\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] := -\text{Simp}[(b*e - a*f)*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)}/(f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^p, x] /;$ FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSimplerQ[p, 1]

Rule 68

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x]$
 $\text{Int}[(a + b*x)^m*(c + d*x)^n, x] := \text{Simp}[(b*c - a*d)^n*(a + b*x)^{(m + 1)}*\text{Hypergeometric2F1}[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)]/(b^{(n + 1)}*(m + 1)), x] /;$ FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m \tan(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+x)^{-1+m}}}{a-x} dx, x, a \sin(e + fx)\right)}{f} \\
&= -\frac{(a + a \sin(e + fx))^m}{2fm} + \frac{\text{Subst}\left(\int \frac{(a+x)^m}{a-x} dx, x, a \sin(e + fx)\right)}{2f} \\
&= -\frac{(a + a \sin(e + fx))^m}{2fm} + \frac{{}_2F_1\left(1, 1 + m; 2 + m; \frac{1}{2}(1 + \sin(e + fx))\right) (a + a \sin(e + fx))^m}{4af(1 + m)}
\end{aligned}$$

Mathematica [A] time = 0.0706524, size = 63, normalized size = 0.88

$$\frac{(a(\sin(e + fx) + 1))^m \left(m(\sin(e + fx) + 1) {}_2F_1\left(1, m + 1; m + 2; \frac{1}{2}(\sin(e + fx) + 1)\right) - 2(m + 1)\right)}{4fm(m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x], x]

[Out] ((a*(1 + Sin[e + f*x]))^m*(-2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))

Maple [F] time = 0.76, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m \tan(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*tan(f*x+e), x)

[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e), x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a \sin(fx + e) + a)^m \tan(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(e + fx) + 1))^m \tan(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))^m*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```


3.132 $\int \cot(e + fx)(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=43

$$\frac{(a \sin(e + fx) + a)^{m+1} {}_2F_1(1, m + 1; m + 2; \sin(e + fx) + 1)}{af(m + 1)}$$

[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)))

Rubi [A] time = 0.0439523, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2707, 65}

$$\frac{(a \sin(e + fx) + a)^{m+1} {}_2F_1(1, m + 1; m + 2; \sin(e + fx) + 1)}{af(m + 1)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]

[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 65

Int[(b_)*(x_)^(m_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rubi steps

$$\begin{aligned} \int \cot(e + fx)(a + a \sin(e + fx))^m dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x} dx, x, a \sin(e + fx)\right)}{f} \\ &= \frac{{}_2F_1(1, 1 + m; 2 + m; 1 + \sin(e + fx))(a + a \sin(e + fx))^{1+m}}{af(1 + m)} \end{aligned}$$

Mathematica [A] time = 0.0589381, size = 43, normalized size = 1.

$$\frac{(a \sin(e + fx) + a)^{m+1} {}_2F_1(1, m + 1; m + 2; \sin(e + fx) + 1)}{af(m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]

[Out] $-\left(\text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, 1 + \sin[e + f*x]\right] * (a + a*\sin[e + f*x])^{1 + m}\right) / (a*f*(1 + m))$

Maple [F] time = 0.821, size = 0, normalized size = 0.

$$\int \cot(fx + e) (a + a \sin(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)`

[Out] `int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")`

[Out] `integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \cot(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")`

[Out] `integral((a*sin(f*x + e) + a)^m*cot(f*x + e), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a (\sin(e + fx) + 1))^m \cot(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(f*x+e)*(a+a*sin(f*x+e))**m,x)`

[Out] `Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

3.133 $\int \cot^3(e + fx)(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=83

$$\frac{(2-m)(a \sin(e + fx) + a)^{m+2} {}_2F_1(2, m+2; m+3; \sin(e + fx) + 1)}{2a^2 f(m+2)} - \frac{\csc^2(e + fx)(a \sin(e + fx) + a)^{m+2}}{2a^2 f}$$

[Out] $-(\text{Csc}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^{(2 + m)})/(2*a^2*f) - ((2 - m)*\text{Hypergeometric2F1}[2, 2 + m, 3 + m, 1 + \text{Sin}[e + f*x]]*(a + a*\text{Sin}[e + f*x])^{(2 + m)})/(2*a^2*f*(2 + m))$

Rubi [A] time = 0.0685841, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 78, 65}

$$\frac{(2-m)(a \sin(e + fx) + a)^{m+2} {}_2F_1(2, m+2; m+3; \sin(e + fx) + 1)}{2a^2 f(m+2)} - \frac{\csc^2(e + fx)(a \sin(e + fx) + a)^{m+2}}{2a^2 f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^m, x]$

[Out] $-(\text{Csc}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^{(2 + m)})/(2*a^2*f) - ((2 - m)*\text{Hypergeometric2F1}[2, 2 + m, 3 + m, 1 + \text{Sin}[e + f*x]]*(a + a*\text{Sin}[e + f*x])^{(2 + m)})/(2*a^2*f*(2 + m))$

Rule 2707

$\text{Int}[(a + (b*\text{sin}[e + (f*x)])^m)*\text{tan}[e + (f*x)]^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^{(m - (p + 1)/2)})/(a - x)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 78

$\text{Int}[(a + (b*x)*(c + (d*x))^n)*(e + (f*x))^p, x_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/(f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 65

$\text{Int}[(b*x)^m*(c + (d*x))^n, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n + 1)}*\text{Hypergeometric2F1}[-m, n + 1, n + 2, 1 + (d*x)/c]/(d*(n + 1)*(-(d/(b*c)))^m), x] /;$ FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rubi steps

$$\int \cot^3(e + fx)(a + a \sin(e + fx))^m dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^{1+m}}{x^3} dx, x, a \sin(e + fx)\right)}{f}$$

$$= -\frac{\csc^2(e + fx)(a + a \sin(e + fx))^{2+m}}{2a^2 f} - \frac{(2 - m) \text{Subst}\left(\int \frac{(a+x)^{1+m}}{x^2} dx, x, a \sin(e + fx)\right)}{2f}$$

$$= -\frac{\csc^2(e + fx)(a + a \sin(e + fx))^{2+m}}{2a^2 f} - \frac{(2 - m) {}_2F_1(2, 2 + m; 3 + m; 1 + \sin(e + fx))}{2a^2 f(2 + m)}$$

Mathematica [A] time = 0.188158, size = 68, normalized size = 0.82

$$\frac{(\sin(e + fx) + 1)^2(a(\sin(e + fx) + 1))^m \left((m + 2) \csc^2(e + fx) - (m - 2) {}_2F_1(2, m + 2; m + 3; \sin(e + fx) + 1) \right)}{2f(m + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]

[Out] -(((2 + m)*Csc[e + f*x]^2 - (-2 + m)*Hypergeometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^m)/(2*f*(2 + m))

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int (\cot(fx + e))^3 (a + a \sin(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)

[Out] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \cot(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**3*(a+a*sin(f*x+e))**m,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)

3.134 $\int \cot^5(e + fx)(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=123

$$\frac{(m^2 - 9m + 12)(a \sin(e + fx) + a)^{m+3} {}_2F_1(3, m + 3; m + 4; \sin(e + fx) + 1)}{12a^3 f(m + 3)} - \frac{\csc^4(e + fx)(a \sin(e + fx) + a)^{m+3}}{4a^3 f} +$$

[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e + f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f*(3 + m))

Rubi [A] time = 0.0982222, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2707, 89, 78, 65}

$$\frac{(m^2 - 9m + 12)(a \sin(e + fx) + a)^{m+3} {}_2F_1(3, m + 3; m + 4; \sin(e + fx) + 1)}{12a^3 f(m + 3)} - \frac{\csc^4(e + fx)(a \sin(e + fx) + a)^{m+3}}{4a^3 f} +$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]

[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e + f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f*(3 + m))

Rule 2707

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 89

Int[((a_) + (b_)*(x_))^(2*((c_) + (d_)*(x_))^(n_))*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 78

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned} \int \cot^5(e + fx)(a + a \sin(e + fx))^m dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^{2+m}}{x^5} dx, x, a \sin(e + fx)\right)}{f} \\ &= -\frac{\csc^4(e + fx)(a + a \sin(e + fx))^{3+m}}{4a^3 f} + \frac{\text{Subst}\left(\int \frac{(a+x)^{2+m}(-a^2(9-m)+4ax)}{x^4} dx, x, a \sin(e + fx)\right)}{4af} \\ &= \frac{(9 - m) \csc^3(e + fx)(a + a \sin(e + fx))^{3+m}}{12a^3 f} - \frac{\csc^4(e + fx)(a + a \sin(e + fx))^{3+m}}{4a^3 f} \\ &= \frac{(9 - m) \csc^3(e + fx)(a + a \sin(e + fx))^{3+m}}{12a^3 f} - \frac{\csc^4(e + fx)(a + a \sin(e + fx))^{3+m}}{4a^3 f} \end{aligned}$$

Mathematica [A] time = 0.274734, size = 83, normalized size = 0.67

$$\frac{(\sin(e + fx) + 1)^3(a \sin(e + fx) + 1)^m \left((m^2 - 9m + 12) {}_2F_1(3, m + 3; m + 4; \sin(e + fx) + 1) + (m + 3) \csc^3(e + fx)(3 \sin(e + fx) + 1) \right)}{12f(m + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(((3 + m)*Csc[e + f*x]^3*(-9 + m + 3*Csc[e + f*x]) + (12 - 9*m + m^2)*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^m)/(12*f*(3 + m))
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int (\cot(fx + e))^5 (a + a \sin(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```


[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \cot(fx + e)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**5*(a+a*sin(f*x+e))**m,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin(fx + e) + a\right)^m \cot(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)

3.135 $\int (a + a \sin(e + fx))^m \tan^4(e + fx) dx$

Optimal. Leaf size=311

$$-\frac{a^2 \sin^2(e + fx) \tan(e + fx) (a \sin(e + fx) + a)^{m-1}}{fm(a - a \sin(e + fx))} + \frac{a^2 \sin(e + fx) \tan(e + fx) (a \sin(e + fx) + a)^{m-1}}{f(1 - m)(a - a \sin(e + fx))} + \frac{2^{m-\frac{3}{2}} (m^4 + 6m^3 + 12m^2 + 8m + 1)}{f}$$

[Out] $(2^{(-3/2 + m)}(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*\text{Hypergeometric2F1}[1/2, 5/2 - m, 3/2, (1 - \text{Sin}[e + f*x])/2]*\text{Sec}[e + f*x]*(1 - \text{Sin}[e + f*x])*(1 + \text{Sin}[e + f*x])^{(1/2 - m)}*(a + a*\text{Sin}[e + f*x])^m]/(3*f*(1 - m)*m) - (\text{Sec}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 - m^3)*\text{Sin}[e + f*x]))/(3*f*(1 - m)*m*(1 - \text{Sin}[e + f*x])) + (a^2*\text{Sin}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*\text{Tan}[e + f*x])/(f*(1 - m)*(a - a*\text{Sin}[e + f*x])) - (a^2*\text{Sin}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*\text{Tan}[e + f*x])/(f*m*(a - a*\text{Sin}[e + f*x]))$

Rubi [A] time = 0.355058, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2719, 100, 153, 145, 70, 69}

$$-\frac{a^2 \sin^2(e + fx) \tan(e + fx) (a \sin(e + fx) + a)^{m-1}}{fm(a - a \sin(e + fx))} + \frac{a^2 \sin(e + fx) \tan(e + fx) (a \sin(e + fx) + a)^{m-1}}{f(1 - m)(a - a \sin(e + fx))} + \frac{2^{m-\frac{3}{2}} (m^4 + 6m^3 + 12m^2 + 8m + 1)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]

[Out] $(2^{(-3/2 + m)}(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*\text{Hypergeometric2F1}[1/2, 5/2 - m, 3/2, (1 - \text{Sin}[e + f*x])/2]*\text{Sec}[e + f*x]*(1 - \text{Sin}[e + f*x])*(1 + \text{Sin}[e + f*x])^{(1/2 - m)}*(a + a*\text{Sin}[e + f*x])^m]/(3*f*(1 - m)*m) - (\text{Sec}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 - m^3)*\text{Sin}[e + f*x]))/(3*f*(1 - m)*m*(1 - \text{Sin}[e + f*x])) + (a^2*\text{Sin}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*\text{Tan}[e + f*x])/(f*(1 - m)*(a - a*\text{Sin}[e + f*x])) - (a^2*\text{Sin}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^{(-1 + m)}*\text{Tan}[e + f*x])/(f*m*(a - a*\text{Sin}[e + f*x]))$

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 100

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

Rule 153

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegerQ[m]

```

Rule 145

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x] + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m + n + 3, 0] && !LtQ[n, -2]))

```

Rule 70

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

```

Rule 69

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)]/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m \tan^4(e + fx) dx &= \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)}) \operatorname{Subst}\left(\int \frac{x^4(a+x)^{-\frac{5}{2}+m}}{(a-x)^{5/2}} dx, x, \right)}{af} \\
&= -\frac{a^2 \sin^2(e + fx)(a + a \sin(e + fx))^{-1+m} \tan(e + fx)}{fm(a - a \sin(e + fx))} - \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)})}{fm(a - a \sin(e + fx))} \\
&= \frac{a^2 \sin(e + fx)(a + a \sin(e + fx))^{-1+m} \tan(e + fx)}{f(1 - m)(a - a \sin(e + fx))} - \frac{a^2 \sin^2(e + fx)(a + a \sin(e + fx))^{-1+m} \tan(e + fx)}{fm(a - a \sin(e + fx))} \\
&= -\frac{\sec(e + fx)(a + a \sin(e + fx))^{-1+m} (a(6 - m - 7m^2 - m^3) - a(9 - 6m - 8m^2))}{3f(1 - m)m(1 - \sin(e + fx))} \\
&= -\frac{\sec(e + fx)(a + a \sin(e + fx))^{-1+m} (a(6 - m - 7m^2 - m^3) - a(9 - 6m - 8m^2))}{3f(1 - m)m(1 - \sin(e + fx))} \\
&= \frac{2^{-\frac{3}{2}+m} (9 - 12m - 7m^2 + 6m^3 + m^4) {}_2F_1\left(\frac{1}{2}, \frac{5}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) \sec(e + fx)}{3f(1 - m)m}
\end{aligned}$$

Mathematica [F] time = 1.06385, size = 0, normalized size = 0.

$$\int (a + a \sin(e + fx))^m \tan^4(e + fx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]

[Out] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4, x]

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m (\tan(fx + e))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)

[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \tan(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)

3.136 $\int (a + a \sin(e + fx))^m \tan^2(e + fx) dx$

Optimal. Leaf size=157

$$\frac{2^{m-\frac{1}{2}}(-m^2 - m + 1) \sec(e + fx) (\sin(e + fx) + 1)^{\frac{1}{2}-m} (a \sin(e + fx) + a)^m {}_2F_1\left(-\frac{1}{2}, \frac{3}{2} - m; \frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(1 - m)m} + \frac{\sec(e + fx)}{f}$$

[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m - m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)

Rubi [A] time = 0.248016, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2713, 2860, 2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}(-m^2 - m + 1) \sec(e + fx) (\sin(e + fx) + 1)^{\frac{1}{2}-m} (a \sin(e + fx) + a)^m {}_2F_1\left(-\frac{1}{2}, \frac{3}{2} - m; \frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(1 - m)m} + \frac{\sec(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]

[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m - m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)

Rule 2713

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] + Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !LtQ[m, 0]

Rule 2860

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2689

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c
+ d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
a*d))]/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m \tan^2(e + fx) dx &= -\frac{\sec(e + fx)(a + a \sin(e + fx))^{1+m}}{afm} + \frac{\int \sec^2(e + fx)(a + a \sin(e + fx))^m (a + a \sin(e + fx)) dx}{am} \\ &= \frac{\sec(e + fx)(a + a \sin(e + fx))^m}{f(1 - m)m} - \frac{\sec(e + fx)(a + a \sin(e + fx))^{1+m}}{afm} + \frac{(1 - \sec^2(e + fx)) \int \sec^2(e + fx)(a + a \sin(e + fx))^m dx}{afm} \\ &= \frac{\sec(e + fx)(a + a \sin(e + fx))^m}{f(1 - m)m} - \frac{\sec(e + fx)(a + a \sin(e + fx))^{1+m}}{afm} + \frac{(a^2 - \sec^2(e + fx)) \int \sec^2(e + fx)(a + a \sin(e + fx))^m dx}{afm} \\ &= \frac{\sec(e + fx)(a + a \sin(e + fx))^m}{f(1 - m)m} - \frac{\sec(e + fx)(a + a \sin(e + fx))^{1+m}}{afm} + \frac{(a^2 - \sec^2(e + fx)) \int \sec^2(e + fx)(a + a \sin(e + fx))^m dx}{afm} \\ &= \frac{\sec(e + fx)(a + a \sin(e + fx))^m}{f(1 - m)m} + \frac{2^{-\frac{1}{2}+m} (1 - m - m^2) {}_2F_1\left(-\frac{1}{2}, \frac{3}{2} - m; \frac{1}{2}; \frac{1}{2}\right)}{afm} \int \sec^2(e + fx)(a + a \sin(e + fx))^m dx \end{aligned}$$

Mathematica [C] time = 34.458, size = 11184, normalized size = 71.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m (\tan(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m \tan(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \tan(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)

3.137 $\int (a + a \sin(e + fx))^m dx$

Optimal. Leaf size=74

$$\frac{2^{m+\frac{1}{2}} \cos(e + fx) (\sin(e + fx) + 1)^{-m-\frac{1}{2}} (a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f}$$

[Out] $-\left(2^{\frac{1}{2} + m} \cos[e + f*x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, (1 - \sin[e + f*x])/2\right] (1 + \sin[e + f*x])^{-\frac{1}{2} - m} (a + a \sin[e + f*x])^m\right) / f$

Rubi [A] time = 0.0348217, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2652, 2651}

$$\frac{2^{m+\frac{1}{2}} \cos(e + fx) (\sin(e + fx) + 1)^{-m-\frac{1}{2}} (a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \sin[e + f*x])^m, x]$

[Out] $-\left(2^{\frac{1}{2} + m} \cos[e + f*x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, (1 - \sin[e + f*x])/2\right] (1 + \sin[e + f*x])^{-\frac{1}{2} - m} (a + a \sin[e + f*x])^m\right) / f$

Rule 2652

$\text{Int}[(a + b \sin(c + d*x))^n, x_Symbol] \rightarrow \text{Dist}[(a + b \sin(c + d*x))^{\text{IntPart}[n]} (a + b \sin(c + d*x))^{\text{FracPart}[n]}] / (1 + (b \sin(c + d*x))/a)^{\text{FracPart}[n]}, \text{Int}[(1 + (b \sin(c + d*x))/a)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rule 2651

$\text{Int}[(a + b \sin(c + d*x))^n, x_Symbol] \rightarrow -\text{Simp}[(2^{n + \frac{1}{2}} a^{n - \frac{1}{2}} b \cos[c + d*x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - n, \frac{3}{2}, (1 + (b \sin(c + d*x))/a)/2\right]) / (d \sqrt{a + b \sin(c + d*x)})], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m dx &= \left((1 + \sin(e + fx))^{-m} (a + a \sin(e + fx))^m \right) \int (1 + \sin(e + fx))^m dx \\ &= \frac{2^{\frac{1}{2} + m} \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) (1 + \sin(e + fx))^{-\frac{1}{2} - m} (a + a \sin(e + fx))^m}{f} \end{aligned}$$

Mathematica [A] time = 0.181155, size = 90, normalized size = 1.22

$$\frac{\sqrt{2} \cos(e + fx) (a(\sin(e + fx) + 1))^m {}_2F_1\left(\frac{1}{2}, m + \frac{1}{2}; m + \frac{3}{2}; \frac{1}{4} \cos^2(e + fx) \csc^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)\right)}{(2fm + f)\sqrt{1 - \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m,x]

[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (a + a \sin(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m,x)

[Out] int((a+a*sin(f*x+e))^m,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(e + fx) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m,x)

[Out] Integral((a*sin(e + f*x) + a)**m, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m, x)

3.138 $\int \cot^2(e + fx)(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{m+2} F_1\left(m + \frac{3}{2}; -\frac{1}{2}, 2; m + \frac{5}{2}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{a^2 f(2m + 3)}$$

[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(3 + 2*m))

Rubi [A] time = 0.10163, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2719, 137, 136}

$$\frac{2\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{m+2} F_1\left(m + \frac{3}{2}; -\frac{1}{2}, 2; m + \frac{5}{2}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{a^2 f(2m + 3)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]

[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(3 + 2*m))

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x]^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \cot^2(e + fx)(a + a \sin(e + fx))^m dx = \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)}) \operatorname{Subst}\left(\int \frac{\sqrt{a-x}(a+x)^{\frac{1}{2}+m}}{x^2} dx\right)}{af}$$

$$= \frac{(\sqrt{2} \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)}) \operatorname{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m} \sqrt{\frac{1}{2}}}{x^2} dx\right)}{af \sqrt{\frac{a-a \sin(e+fx)}{a}}}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{3}{2} + m; -\frac{1}{2}, 2; \frac{5}{2} + m; \frac{1}{2}(1 + \sin(e + fx)), 1 + \sin(e + fx)\right) \sec(e + fx)}{a^2 f(3 + 2m)}$$

Mathematica [C] time = 99.9376, size = 47487, normalized size = 533.56

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]

[Out] Result too large to show

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int (\cot (fx + e))^2 (a + a \sin (fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)

[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (fx + e) + a)^m \cot (fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a \sin (fx + e) + a\right)^m \cot (fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(e + fx) + 1))^m \cot^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

3.139 $\int \cot^4(e + fx)(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=89

$$\frac{4\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{m+3} F_1\left(m + \frac{5}{2}; -\frac{3}{2}, 4; m + \frac{7}{2}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{a^3 f(2m + 5)}$$

[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(5 + 2*m))

Rubi [A] time = 0.0993711, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2719, 137, 136}

$$\frac{4\sqrt{2}\sqrt{1 - \sin(e + fx)} \sec(e + fx)(a \sin(e + fx) + a)^{m+3} F_1\left(m + \frac{5}{2}; -\frac{3}{2}, 4; m + \frac{7}{2}; \frac{1}{2}(\sin(e + fx) + 1), \sin(e + fx) + 1\right)}{a^3 f(2m + 5)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]

[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(5 + 2*m))

Rule 2719

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && IntegerQ[p/2]

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \cot^4(e + fx)(a + a \sin(e + fx))^m dx = \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{(a-x)^{3/2}(a+x)^{\frac{3}{2}+m}}{x^4} dx, \right)}{af}$$

$$= \frac{(2\sqrt{2} \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)}) \operatorname{Subst} \left(\int \frac{(a+x)^{\frac{3}{2}+m} \left(\frac{1}{2} - \frac{x}{2a}\right)}{x^4} dx, \right)}{f \sqrt{\frac{a - a \sin(e + fx)}{a}}}$$

$$= \frac{4\sqrt{2} F_1 \left(\frac{5}{2} + m; -\frac{3}{2}, 4; \frac{7}{2} + m; \frac{1}{2}(1 + \sin(e + fx)), 1 + \sin(e + fx) \right) \sec(e + fx) \sqrt{1 - \sin(e + fx)}}{a^3 f (5 + 2m)}$$

Mathematica [F] time = 0.740835, size = 0, normalized size = 0.

$$\int \cot^4(e + fx)(a + a \sin(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]

[Out] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m, x]

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int (\cot(fx + e))^4 (a + a \sin(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)

[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left((a \sin(fx + e) + a)^m \cot(fx + e)^4, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**m,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(fx + e) + a)^m \cot(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)

3.140 $\int (a + b \sin(c + dx)) \tan^3(c + dx) dx$

Optimal. Leaf size=88

$$\frac{(2a + 3b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 3b) \log(\sin(c + dx) + 1)}{4d} + \frac{\tan^2(c + dx)(a + b \sin(c + dx))}{2d} + \frac{3b \sin(c + dx)}{2d}$$

[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^2)/(2*d)

Rubi [A] time = 0.0771489, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2721, 819, 774, 633, 31}

$$\frac{(2a + 3b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 3b) \log(\sin(c + dx) + 1)}{4d} + \frac{\tan^2(c + dx)(a + b \sin(c + dx))}{2d} + \frac{3b \sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]

[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^2)/(2*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 819

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !LtQ[m + 2*p + 3, 0])

Rule 774

Int[(((d_) + (e_)*(x_))*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 633

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 31

$\text{Int}[(a + b \sin(x)) \tan^3(x), x] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \sin(x), x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rubi steps

$$\begin{aligned} \int (a + b \sin(c + dx)) \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(b^2-x^2)^2} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{(a + b \sin(c + dx)) \tan^2(c + dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x(2ab^2+3b^2x)}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{3b \sin(c + dx)}{2d} + \frac{(a + b \sin(c + dx)) \tan^2(c + dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-3b^4-2ab^2x}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{3b \sin(c + dx)}{2d} + \frac{(a + b \sin(c + dx)) \tan^2(c + dx)}{2d} - \frac{(2a - 3b) \text{Subst}\left(\int \frac{1}{-b-x} dx, x, b \sin(c + dx)\right)}{4d} \\ &= \frac{(2a + 3b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 3b) \log(1 + \sin(c + dx))}{4d} + \frac{3b \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.119188, size = 77, normalized size = 0.88

$$\frac{a(\tan^2(c + dx) + 2 \log(\cos(c + dx)))}{2d} - \frac{b \sin(c + dx) \tan^2(c + dx)}{d} - \frac{3b(\tanh^{-1}(\sin(c + dx)) - \tan(c + dx) \sec(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]

[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*b*(ArcTanh[Sin[c + d*x]] - Sec[c + d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)

Maple [A] time = 0.032, size = 96, normalized size = 1.1

$$\frac{(\tan(dx + c))^2 a}{2d} + \frac{a \ln(\cos(dx + c))}{d} + \frac{b(\sin(dx + c))^5}{2d(\cos(dx + c))^2} + \frac{b(\sin(dx + c))^3}{2d} + \frac{3b \sin(dx + c)}{2d} - \frac{3b \ln(\sec(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))*tan(d*x+c)^3,x)

[Out] 1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))+1/2/d*b*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*b*sin(d*x+c)^3+3/2*b*sin(d*x+c)/d-3/2/d*b*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.90057, size = 99, normalized size = 1.12

$$\frac{(2a - 3b) \log(\sin(dx + c) + 1) + (2a + 3b) \log(\sin(dx + c) - 1) + 4b \sin(dx + c) - \frac{2(b \sin(dx + c) + a)}{\sin(dx + c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/4*((2*a - 3*b)*log(sin(d*x + c) + 1) + (2*a + 3*b)*log(sin(d*x + c) - 1) + 4*b*sin(d*x + c) - 2*(b*sin(d*x + c) + a)/(sin(d*x + c)^2 - 1))/d

Fricas [A] time = 1.68026, size = 236, normalized size = 2.68

$$\frac{(2a - 3b) \cos(dx + c)^2 \log(\sin(dx + c) + 1) + (2a + 3b) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(2b \cos(dx + c)^2 + b)}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")

[Out] 1/4*((2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a + 3*b)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \tan^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**3,x)

[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**3, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")

[Out] Timed out

3.141 $\int (a + b \sin(c + dx)) \tan(c + dx) dx$

Optimal. Leaf size=55

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d} - \frac{(a-b)\log(\sin(c+dx)+1)}{2d} - \frac{b\sin(c+dx)}{d}$$

[Out] $-\frac{(a+b)\text{Log}[1-\text{Sin}[c+d*x]]}{2*d} - \frac{(a-b)\text{Log}[1+\text{Sin}[c+d*x]]}{2*d} - \frac{b*\text{Sin}[c+d*x]}{d}$

Rubi [A] time = 0.0378478, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2721, 774, 633, 31}

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d} - \frac{(a-b)\log(\sin(c+dx)+1)}{2d} - \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sin}[c + d*x])* \text{Tan}[c + d*x], x]$

[Out] $-\frac{(a+b)\text{Log}[1-\text{Sin}[c+d*x]]}{2*d} - \frac{(a-b)\text{Log}[1+\text{Sin}[c+d*x]]}{2*d} - \frac{b*\text{Sin}[c+d*x]}{d}$

Rule 2721

$\text{Int}[(a + b*\sin(e + f*x))^m * \tan(e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p+1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ $\text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[(p+1)/2]$

Rule 774

$\text{Int}[(d + e*x)*(f + g*x)/(a + c*x^2), x_Symbol] \rightarrow \text{Simp}[e*g*x/c, x] + \text{Dist}[1/c, \text{Int}[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /;$ $\text{FreeQ}\{a, c, d, e, f, g\}, x$

Rule 633

$\text{Int}[(d + e*x)/(a + c*x^2), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x] /;$ $\text{FreeQ}\{a, c, d, e\}, x \&\& \text{NiceSqrtQ}[-(a*c)]$

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ $\text{FreeQ}\{a, b\}, x$

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx)) \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+x)}}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{b \sin(c + dx)}{d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{b \sin(c + dx)}{d} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{-b-x} dx, x, b \sin(c + dx)\right)}{2d} + \frac{(a + b) \text{Subst}\left(\int \frac{1}{b-x} dx, x, b \sin(c + dx)\right)}{2d} \\
&= -\frac{(a + b) \log(1 - \sin(c + dx))}{2d} - \frac{(a - b) \log(1 + \sin(c + dx))}{2d} - \frac{b \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01638, size = 38, normalized size = 0.69

$$-\frac{a \log(\cos(c + dx))}{d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x], x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (b*Sin[c + d*x])/d

Maple [A] time = 0.025, size = 46, normalized size = 0.8

$$-\frac{b \sin(dx + c)}{d} + \frac{b \ln(\sec(dx + c) + \tan(dx + c))}{d} - \frac{a \ln(\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))*tan(d*x+c), x)

[Out] -b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*a*ln(cos(d*x+c))

Maxima [A] time = 1.9426, size = 58, normalized size = 1.05

$$-\frac{(a - b) \log(\sin(dx + c) + 1) + (a + b) \log(\sin(dx + c) - 1) + 2 b \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c), x, algorithm="maxima")

[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(sin(d*x + c) - 1) + 2*b*sin(d*x + c))/d

Fricas [A] time = 1.60971, size = 124, normalized size = 2.25

$$-\frac{(a - b) \log(\sin(dx + c) + 1) + (a + b) \log(-\sin(dx + c) + 1) + 2 b \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(-sin(d*x + c) + 1) + 2*b*
sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x), x)
```

Giac [B] time = 2.82687, size = 1966, normalized size = 35.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*c)^2 + 1)
/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*c)^2
+ 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan
(1/2*c) + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
```

$$\begin{aligned}
&)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) \\
&*\tan(1/2*c)^2 + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3* \\
&\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2 \\
&*c)^2 - 4*b*\tan(1/2*d*x)*\tan(1/2*c)^2 + b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2 \\
&*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1 \\
&/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 \\
&+ 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/ \\
&2*d*x) - 2*\tan(1/2*c) + 1)) - b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*ta \\
&n(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \\
&\tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1 \\
&/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2 \\
&*\tan(1/2*c) + 1)) + a*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x \\
&)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 4 \\
&*b*\tan(1/2*d*x) + 4*b*\tan(1/2*c))/ (d*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(1/ \\
&2*d*x)^2 + d*\tan(1/2*c)^2 + d)
\end{aligned}$$

3.142 $\int \cot(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{a \log(\sin(c + dx))}{d} + \frac{b \sin(c + dx)}{d}$$

[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d

Rubi [A] time = 0.0213609, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2721, 43}

$$\frac{a \log(\sin(c + dx))}{d} + \frac{b \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]

[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + b \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{a}{x}\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{a \log(\sin(c + dx))}{d} + \frac{b \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0331985, size = 43, normalized size = 1.79

$$\frac{a(\log(\tan(c + dx)) + \log(\cos(c + dx)))}{d} + \frac{b \sin(c) \cos(dx)}{d} + \frac{b \cos(c) \sin(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]

[Out] $(a*(\text{Log}[\text{Cos}[c + d*x]] + \text{Log}[\text{Tan}[c + d*x]]))/d + (b*\text{Cos}[d*x]*\text{Sin}[c])/d + (b*\text{Cos}[c]*\text{Sin}[d*x])/d$

Maple [A] time = 0.019, size = 25, normalized size = 1.

$$\frac{a \ln(\sin(dx + c))}{d} + \frac{b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+b*sin(d*x+c)),x)`

[Out] `a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d`

Maxima [A] time = 1.6977, size = 30, normalized size = 1.25

$$\frac{a \log(\sin(dx + c)) + b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")`

[Out] `(a*log(sin(d*x + c)) + b*sin(d*x + c))/d`

Fricas [A] time = 1.50454, size = 62, normalized size = 2.58

$$\frac{a \log\left(\frac{1}{2} \sin(dx + c)\right) + b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")`

[Out] `(a*log(1/2*sin(d*x + c)) + b*sin(d*x + c))/d`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x)`

[Out] `Integral((a + b*sin(c + d*x))*cot(c + d*x), x)`

Giac [A] time = 2.28168, size = 31, normalized size = 1.29

$$\frac{a \log(|\sin(dx + c)|) + b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + b*sin(d*x + c))/d
```

3.143 $\int \cot^3(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=54

$$-\frac{a \csc^2(c + dx)}{2d} - \frac{a \log(\sin(c + dx))}{d} - \frac{b \sin(c + dx)}{d} - \frac{b \csc(c + dx)}{d}$$

[Out] $-\frac{(b \csc[c + d*x])}{d} - \frac{(a \csc[c + d*x]^2)}{(2*d)} - \frac{(a \log[\sin[c + d*x]])}{d} - \frac{(b \sin[c + d*x])}{d}$

Rubi [A] time = 0.0409958, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 766}

$$-\frac{a \csc^2(c + dx)}{2d} - \frac{a \log(\sin(c + dx))}{d} - \frac{b \sin(c + dx)}{d} - \frac{b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]

[Out] $-\frac{(b \csc[c + d*x])}{d} - \frac{(a \csc[c + d*x]^2)}{(2*d)} - \frac{(a \log[\sin[c + d*x]])}{d} - \frac{(b \sin[c + d*x])}{d}$

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 766

Int[((e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + b \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)}{x^3} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-1 + \frac{ab^2}{x^3} + \frac{b^2}{x^2} - \frac{a}{x}\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= -\frac{b \csc(c + dx)}{d} - \frac{a \csc^2(c + dx)}{2d} - \frac{a \log(\sin(c + dx))}{d} - \frac{b \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.210205, size = 60, normalized size = 1.11

$$-\frac{a(\cot^2(c + dx) + 2 \log(\tan(c + dx)) + 2 \log(\cos(c + dx)))}{2d} - \frac{b \sin(c + dx)}{d} - \frac{b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]

[Out] -((b*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]]))/(2*d) - (b*Sin[c + d*x])/d

Maple [A] time = 0.039, size = 83, normalized size = 1.5

$$\frac{(\cot(dx+c))^2 a}{2d} - \frac{a \ln(\sin(dx+c))}{d} - \frac{b(\cos(dx+c))^4}{d \sin(dx+c)} - \frac{(\cos(dx+c))^2 \sin(dx+c) b}{d} - 2 \frac{b \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)),x)

[Out] -1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d-1/d*b/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*b-2*b*sin(d*x+c)/d

Maxima [A] time = 1.44042, size = 61, normalized size = 1.13

$$\frac{2a \log(\sin(dx+c)) + 2b \sin(dx+c) + \frac{2b \sin(dx+c)+a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*b*sin(d*x + c) + (2*b*sin(d*x + c) + a)/sin(d*x + c)^2)/d

Fricas [A] time = 1.60132, size = 167, normalized size = 3.09

$$\frac{2(a \cos(dx+c)^2 - a) \log\left(\frac{1}{2} \sin(dx+c)\right) + 2(b \cos(dx+c)^2 - 2b) \sin(dx+c) - a}{2(d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)),x)

[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**3, x)

Giac [A] time = 2.00709, size = 81, normalized size = 1.5

$$\frac{2a \log(|\sin(dx + c)|) + 2b \sin(dx + c) - \frac{3a \sin(dx+c)^2 - 2b \sin(dx+c) - a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*b*sin(d*x + c) - (3*a*sin(d*x + c)^2 - 2*b*sin(d*x + c) - a)/sin(d*x + c)^2)/d

3.144 $\int \cot^5(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=81

$$-\frac{a \csc^4(c + dx)}{4d} + \frac{a \csc^2(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d} + \frac{b \sin(c + dx)}{d} - \frac{b \csc^3(c + dx)}{3d} + \frac{2b \csc(c + dx)}{d}$$

[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d

Rubi [A] time = 0.0521741, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 766}

$$-\frac{a \csc^4(c + dx)}{4d} + \frac{a \csc^2(c + dx)}{d} + \frac{a \log(\sin(c + dx))}{d} + \frac{b \sin(c + dx)}{d} - \frac{b \csc^3(c + dx)}{3d} + \frac{2b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]

[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 766

Int[((e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + b \sin(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^5} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{ab^4}{x^5} + \frac{b^4}{x^4} - \frac{2ab^2}{x^3} - \frac{2b^2}{x^2} + \frac{a}{x}\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{2b \csc(c + dx)}{d} + \frac{a \csc^2(c + dx)}{d} - \frac{b \csc^3(c + dx)}{3d} - \frac{a \csc^4(c + dx)}{4d} + \frac{a \log(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.238635, size = 87, normalized size = 1.07

$$\frac{a(-\cot^4(c + dx) + 2\cot^2(c + dx) + 4\log(\tan(c + dx)) + 4\log(\cos(c + dx)))}{4d} + \frac{b \sin(c + dx)}{d} - \frac{b \csc^3(c + dx)}{3d} + \frac{2b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]

[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c + d*x])/d

Maple [A] time = 0.04, size = 136, normalized size = 1.7

$$-\frac{a(\cot(dx+c))^4}{4d} + \frac{(\cot(dx+c))^2 a}{2d} + \frac{a \ln(\sin(dx+c))}{d} - \frac{b(\cos(dx+c))^6}{3d(\sin(dx+c))^3} + \frac{b(\cos(dx+c))^6}{d \sin(dx+c)} + \frac{8b \sin(dx+c)}{3d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c)),x)

[Out] -1/4/d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d-1/3/d*b/sin(d*x+c)^3*cos(d*x+c)^6+1/d*b/sin(d*x+c)*cos(d*x+c)^6+8/3*b*sin(d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*b+4/3/d*cos(d*x+c)^2*sin(d*x+c)*b

Maxima [A] time = 1.28052, size = 93, normalized size = 1.15

$$\frac{12 a \log(\sin(dx+c)) + 12 b \sin(dx+c) + \frac{24 b \sin(dx+c)^3 + 12 a \sin(dx+c)^2 - 4 b \sin(dx+c) - 3 a}{\sin(dx+c)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d

Fricas [A] time = 1.53603, size = 292, normalized size = 3.6

$$\frac{12 a \cos(dx+c)^2 - 12 \left(a \cos(dx+c)^4 - 2 a \cos(dx+c)^2 + a \right) \log\left(\frac{1}{2} \sin(dx+c)\right) - 4 \left(3 b \cos(dx+c)^4 - 12 b \cos(dx+c)^2 + 8 b \right)}{12 \left(d \cos(dx+c)^4 - 2 d \cos(dx+c)^2 + d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \cot^5(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c)),x)

[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**5, x)

Giac [A] time = 2.09962, size = 111, normalized size = 1.37

$$\frac{12 a \log(|\sin(dx + c)|) + 12 b \sin(dx + c) - \frac{25 a \sin(dx+c)^4 - 24 b \sin(dx+c)^3 - 12 a \sin(dx+c)^2 + 4 b \sin(dx+c) + 3 a}{\sin(dx+c)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin(d*x + c)^4)/d

3.145 $\int (a + b \sin(c + dx)) \tan^4(c + dx) dx$

Optimal. Leaf size=72

$$\frac{a \tan^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + ax - \frac{b \cos(c + dx)}{d} + \frac{b \sec^3(c + dx)}{3d} - \frac{2b \sec(c + dx)}{d}$$

[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0778072, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2722, 3473, 8, 2590, 270}

$$\frac{a \tan^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + ax - \frac{b \cos(c + dx)}{d} + \frac{b \sec^3(c + dx)}{3d} - \frac{2b \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]

[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2590

Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx)) \tan^4(c + dx) dx &= \int (a \tan^4(c + dx) + b \sin(c + dx) \tan^4(c + dx)) dx \\
&= a \int \tan^4(c + dx) dx + b \int \sin(c + dx) \tan^4(c + dx) dx \\
&= \frac{a \tan^3(c + dx)}{3d} - a \int \tan^2(c + dx) dx - \frac{b \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4} dx, x, \cos(c + dx)\right)}{d} \\
&= -\frac{a \tan(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d} + a \int 1 dx - \frac{b \operatorname{Subst}\left(\int \left(1 + \frac{1}{x^4} - \frac{2}{x^2}\right) dx, x, \cos(c + dx)\right)}{d} \\
&= ax - \frac{b \cos(c + dx)}{d} - \frac{2b \sec(c + dx)}{d} + \frac{b \sec^3(c + dx)}{3d} - \frac{a \tan(c + dx)}{d} + \frac{a \tan^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.037818, size = 81, normalized size = 1.12

$$\frac{a \tan^3(c + dx)}{3d} + \frac{a \tan^{-1}(\tan(c + dx))}{d} - \frac{a \tan(c + dx)}{d} - \frac{b \cos(c + dx)}{d} + \frac{b \sec^3(c + dx)}{3d} - \frac{2b \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]

[Out] (a*ArcTan[Tan[c + d*x]])/d - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)

Maple [A] time = 0.032, size = 98, normalized size = 1.4

$$\frac{1}{d} \left(a \left(\frac{(\tan(dx + c))^3}{3} - \tan(dx + c) + dx + c \right) + b \left(\frac{(\sin(dx + c))^6}{3 (\cos(dx + c))^3} - \frac{(\sin(dx + c))^6}{\cos(dx + c)} - \left(\frac{8}{3} + (\sin(dx + c))^4 + \frac{4 (\sin(dx + c))^2}{\cos(dx + c)} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))*tan(d*x+c)^4,x)

[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2*cos(d*x+c))))

Maxima [A] time = 2.1333, size = 88, normalized size = 1.22

$$\frac{(\tan(dx + c)^3 + 3dx + 3c - 3 \tan(dx + c))a - b \left(\frac{6 \cos(dx + c)^2 - 1}{\cos(dx + c)^3} + 3 \cos(dx + c) \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")

[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d

Fricas [A] time = 1.81522, size = 182, normalized size = 2.53

$$\frac{3 dx \cos(dx + c)^3 - 3 b \cos(dx + c)^4 - 6 b \cos(dx + c)^2 - (4 a \cos(dx + c)^2 - a) \sin(dx + c) + b}{3 d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")

[Out] 1/3*(3*a*d*x*cos(d*x + c)^3 - 3*b*cos(d*x + c)^4 - 6*b*cos(d*x + c)^2 - (4*a*cos(d*x + c)^2 - a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \tan^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**4,x)

[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**4, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

3.146 $\int (a + b \sin(c + dx)) \tan^2(c + dx) dx$

Optimal. Leaf size=38

$$\frac{a \tan(c + dx)}{d} - ax + \frac{b \cos(c + dx)}{d} + \frac{b \sec(c + dx)}{d}$$

[Out] $-(a*x) + (b*\text{Cos}[c + d*x])/d + (b*\text{Sec}[c + d*x])/d + (a*\text{Tan}[c + d*x])/d$

Rubi [A] time = 0.0593762, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2722, 3473, 8, 2590, 14}

$$\frac{a \tan(c + dx)}{d} - ax + \frac{b \cos(c + dx)}{d} + \frac{b \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sin}[c + d*x])*\text{Tan}[c + d*x]^2, x]$

[Out] $-(a*x) + (b*\text{Cos}[c + d*x])/d + (b*\text{Sec}[c + d*x])/d + (a*\text{Tan}[c + d*x])/d$

Rule 2722

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * (g*\text{tan}[e + f*x] + h)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g*\text{Tan}[e + f*x])^p, (a + b*\text{Sin}[e + f*x])^m, x], x] /;$ FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

$\text{Int}[(b*\text{tan}[c + d*x] + e)^n, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{n-1})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

$\text{Int}[a*x, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rule 2590

$\text{Int}[\text{sin}[e + f*x]^m * \text{tan}[e + f*x]^n, x_Symbol] \rightarrow -\text{Dist}[f^{-1}, \text{Subst}[\text{Int}[(1 - x^2)^{(m+n-1)/2}/x^n, x], x, \text{Cos}[e + f*x]], x] /;$ FreeQ[{e, f}, x] && IntegersQ[m, n, (m+n-1)/2]

Rule 14

$\text{Int}[(u + v*(c*x))^m, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a + b*v)] /;

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx)) \tan^2(c + dx) dx &= \int (a \tan^2(c + dx) + b \sin(c + dx) \tan^2(c + dx)) dx \\
&= a \int \tan^2(c + dx) dx + b \int \sin(c + dx) \tan^2(c + dx) dx \\
&= \frac{a \tan(c + dx)}{d} - a \int 1 dx - \frac{b \operatorname{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} \\
&= -ax + \frac{a \tan(c + dx)}{d} - \frac{b \operatorname{Subst}\left(\int \left(-1 + \frac{1}{x^2}\right) dx, x, \cos(c + dx)\right)}{d} \\
&= -ax + \frac{b \cos(c + dx)}{d} + \frac{b \sec(c + dx)}{d} + \frac{a \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0334988, size = 47, normalized size = 1.24

$$-\frac{a \tan^{-1}(\tan(c + dx))}{d} + \frac{a \tan(c + dx)}{d} + \frac{b \cos(c + dx)}{d} + \frac{b \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]

[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d

Maple [A] time = 0.026, size = 59, normalized size = 1.6

$$\frac{1}{d} \left(a (\tan(dx + c) - dx - c) + b \left(\frac{(\sin(dx + c))^4}{\cos(dx + c)} + (2 + (\sin(dx + c))^2) \cos(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))*tan(d*x+c)^2,x)

[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))

Maxima [A] time = 2.72276, size = 53, normalized size = 1.39

$$\frac{(dx + c - \tan(dx + c))a - b \left(\frac{1}{\cos(dx + c)} + \cos(dx + c) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d

Fricas [A] time = 1.70543, size = 108, normalized size = 2.84

$$\frac{adx \cos(dx + c) - b \cos(dx + c)^2 - a \sin(dx + c) - b}{d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")

[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**2,x)

[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**2, x)

Giac [B] time = 8.56166, size = 1361, normalized size = 35.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")

[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)^3 - 8*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*b*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*b*tan(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*b*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*tan(1/2*d*x)^3*tan(1/2*c) + 24*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*b*tan(1/2*d*x)*tan(1/2*c)^3 + 2*b*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)*tan(c) - a*d*x - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(d*x)*tan(c) + a*t

$$\begin{aligned}
& \tan(dx) + a \tan(c) + 2b / (d \tan(dx) \tan(1/2 dx)^4 \tan(1/2 c)^4 \tan(c) - \\
& d \tan(1/2 dx)^4 \tan(1/2 c)^4 - 4d \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c)^3 \tan(c) + 4d \tan(1/2 dx)^3 \tan(1/2 c)^3 - \\
& d \tan(dx) \tan(1/2 dx)^4 \tan(c) - 4d \tan(dx) \tan(1/2 dx)^3 \tan(1/2 c) \tan(c) - 4d \tan(dx) \tan(1/2 dx) \tan(1/2 c)^3 \tan(c) - \\
& d \tan(dx) \tan(1/2 c)^4 \tan(c) + d \tan(1/2 dx)^4 + 4d \tan(1/2 dx)^3 \tan(1/2 c) + 4d \tan(1/2 dx) \tan(1/2 c)^3 + \\
& d \tan(1/2 c)^4 - 4d \tan(dx) \tan(1/2 dx) \tan(1/2 c) \tan(c) + 4d \tan(1/2 dx) \tan(1/2 c) + d \tan(dx) \tan(c) - d)
\end{aligned}$$

3.147 $\int \cot^2(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=41

$$-\frac{a \cot(c + dx)}{d} - ax + \frac{b \cos(c + dx)}{d} - \frac{b \tanh^{-1}(\cos(c + dx))}{d}$$

[Out] $-(a*x) - (b*\text{ArcTanh}[\text{Cos}[c + d*x]])/d + (b*\text{Cos}[c + d*x])/d - (a*\text{Cot}[c + d*x])/d$

Rubi [A] time = 0.0533796, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2722, 2592, 321, 206, 3473, 8}

$$-\frac{a \cot(c + dx)}{d} - ax + \frac{b \cos(c + dx)}{d} - \frac{b \tanh^{-1}(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^2*(a + b*\text{Sin}[c + d*x]), x]$

[Out] $-(a*x) - (b*\text{ArcTanh}[\text{Cos}[c + d*x]])/d + (b*\text{Cos}[c + d*x])/d - (a*\text{Cot}[c + d*x])/d$

Rule 2722

$\text{Int}[(a + (b_*)\sin[(e_*) + (f_*)(x)])^{(m_*)} * ((g_*)\tan[(e_*) + (f_*)(x)])^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g*\text{Tan}[e + f*x])^p, (a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

Rule 2592

$\text{Int}[(a_*)\sin[(e_*) + (f_*)(x)]^{(m_*)} * \tan[(e_*) + (f_*)(x)]^{(n_*)}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(\text{ff}*x)^{(m+n)} / (a^2 - \text{ff}^2*x^2)^{(n+1)/2}, x], x, (a*\text{Sin}[e + f*x])/ff], x] /; \text{FreeQ}\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n+1)/2]$

Rule 321

$\text{Int}[(c_*)(x_*)^{(m_*)} * ((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1)) / (b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m+n*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 206

$\text{Int}[(a_*) + (b_*)(x_*)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 3473

$\text{Int}[(b_*)\tan[(c_*) + (d_*)(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)}) / (d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x],$

$x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + b \sin(c + dx)) dx &= \int (b \cos(c + dx) \cot(c + dx) + a \cot^2(c + dx)) dx \\ &= a \int \cot^2(c + dx) dx + b \int \cos(c + dx) \cot(c + dx) dx \\ &= -\frac{a \cot(c + dx)}{d} - a \int 1 dx - \frac{b \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -ax + \frac{b \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} - \frac{b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -ax - \frac{b \tanh^{-1}(\cos(c + dx))}{d} + \frac{b \cos(c + dx)}{d} - \frac{a \cot(c + dx)}{d} \end{aligned}$$

Mathematica [C] time = 0.0312252, size = 75, normalized size = 1.83

$$-\frac{a \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right)}{d} + \frac{b \cos(c + dx)}{d} + \frac{b \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} - \frac{b \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]

[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d

Maple [A] time = 0.029, size = 57, normalized size = 1.4

$$-ax + \frac{b \cos(dx + c)}{d} - \frac{a \cot(dx + c)}{d} + \frac{b \ln(\csc(dx + c) - \cot(dx + c))}{d} - \frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)),x)

[Out] -a*x+b*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a

Maxima [A] time = 2.20653, size = 73, normalized size = 1.78

$$-\frac{2\left(dx + c + \frac{1}{\tan(dx+c)}\right)a - b(2 \cos(dx + c) - \log(\cos(dx + c) + 1) + \log(\cos(dx + c) - 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/2*(2*(d*x + c + 1/\tan(d*x + c))*a - b*(2*\cos(d*x + c) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)))/d$

Fricas [B] time = 1.83606, size = 236, normalized size = 5.76

$$\frac{b \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - b \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + 2a \cos(dx + c) + 2(adx - b \cos(dx + c))}{2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")`

[Out] $-1/2*(b*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - b*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 2*a*\cos(d*x + c) + 2*(a*d*x - b*\cos(d*x + c))*\sin(d*x + c))/(d*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)),x)`

[Out] `Integral((a + b*sin(c + d*x))*cot(c + d*x)**2, x)`

Giac [B] time = 2.30761, size = 146, normalized size = 3.56

$$\frac{6(dx + c)a - 6b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 3a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{2b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 10b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 3a}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")`

[Out] $-1/6*(6*(d*x + c)*a - 6*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - 3*a*\tan(1/2*d*x + 1/2*c) + (2*b*\tan(1/2*d*x + 1/2*c)^3 + 3*a*\tan(1/2*d*x + 1/2*c)^2 - 10*b*\tan(1/2*d*x + 1/2*c) + 3*a)/(\tan(1/2*d*x + 1/2*c)^3 + \tan(1/2*d*x + 1/2*c)))/d$

3.148 $\int \cot^4(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=82

$$-\frac{a \cot^3(c + dx)}{3d} + \frac{a \cot(c + dx)}{d} + ax - \frac{3b \cos(c + dx)}{2d} - \frac{b \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{3b \tanh^{-1}(\cos(c + dx))}{2d}$$

[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0798351, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a \cot^3(c + dx)}{3d} + \frac{a \cot(c + dx)}{d} + ax - \frac{3b \cos(c + dx)}{2d} - \frac{b \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{3b \tanh^{-1}(\cos(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]

[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\sin(c+dx))dx &= \int (b\cos(c+dx)\cot^3(c+dx) + a\cot^4(c+dx))dx \\
&= a \int \cot^4(c+dx)dx + b \int \cos(c+dx)\cot^3(c+dx)dx \\
&= -\frac{a\cot^3(c+dx)}{3d} - a \int \cot^2(c+dx)dx - \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2}dx, x, \cos(c+dx)\right)}{d} \\
&= \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a \int 1dx + \dots \\
&= ax - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + \dots \\
&= ax + \frac{3b \tanh^{-1}(\cos(c+dx))}{2d} - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)}{2d} + \dots
\end{aligned}$$

Mathematica [C] time = 0.0419753, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)}{3d} {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c+dx)\right) - \frac{b\cos(c+dx)}{d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3b\log(\sin)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]), x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.036, size = 106, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^3}{3d} + \frac{a\cot(dx+c)}{d} + ax + \frac{ca}{d} - \frac{b(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{b(\cos(dx+c))^3}{2d} - \frac{3b\cos(dx+c)}{2d} - \frac{3b\ln(\csc)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)), x)
```

[Out] $-1/3*a*\cot(d*x+c)^3/d+a*\cot(d*x+c)/d+a*x+1/d*c*a-1/2/d*b/\sin(d*x+c)^2*\cos(d*x+c)^5-1/2/d*b*\cos(d*x+c)^3-3/2*b*\cos(d*x+c)/d-3/2/d*b*\ln(\csc(d*x+c)-\cot(d*x+c))$

Maxima [A] time = 1.87341, size = 124, normalized size = 1.51

$$\frac{4\left(3dx + 3c + \frac{3 \tan(dx+c)^2 - 1}{\tan(dx+c)^3}\right)a + 3b\left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2 - 1} - 4 \cos(dx+c) + 3 \log(\cos(dx+c) + 1) - 3 \log(\cos(dx+c) - 1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")`

[Out] $1/12*(4*(3*d*x + 3*c + (3*\tan(d*x + c)^2 - 1)/\tan(d*x + c)^3)*a + 3*b*(2*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) - 4*\cos(d*x + c) + 3*\log(\cos(d*x + c) + 1) - 3*\log(\cos(d*x + c) - 1)))/d$

Fricas [B] time = 1.86448, size = 425, normalized size = 5.18

$$\frac{16a \cos(dx+c)^3 + 9(b \cos(dx+c)^2 - b) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 9(b \cos(dx+c)^2 - b) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c)}{12(d \cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")`

[Out] $1/12*(16*a*\cos(d*x + c)^3 + 9*(b*\cos(d*x + c)^2 - b)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 9*(b*\cos(d*x + c)^2 - b)*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 12*a*\cos(d*x + c) + 6*(2*a*d*x*\cos(d*x + c)^2 - 2*b*\cos(d*x + c)^3 - 2*a*d*x + 3*b*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c)^2 - d)*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx)) \cot^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)),x)`

[Out] `Integral((a + b*sin(c + d*x))*cot(c + d*x)**4, x)`

Giac [A] time = 2.17653, size = 190, normalized size = 2.32

$$a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 24(dx+c)a - 36b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 15a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{4}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}$$

24d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*  
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c)))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/  
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*  
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.149 $\int \cot^6(c + dx)(a + b \sin(c + dx)) dx$

Optimal. Leaf size=122

$$-\frac{a \cot^5(c + dx)}{5d} + \frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - ax + \frac{15b \cos(c + dx)}{8d} - \frac{b \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{5b \cos(c + dx)}{8d}$$

[Out] $-(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.0972308, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a \cot^5(c + dx)}{5d} + \frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - ax + \frac{15b \cos(c + dx)}{8d} - \frac{b \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{5b \cos(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^6*(a + b*\text{Sin}[c + d*x]), x]$

[Out] $-(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d)$

Rule 2722

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x))^m) \cdot ((g \cdot \tan(e + f \cdot x))^p), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g \cdot \tan[e + f \cdot x])^p, (a + b \cdot \sin[e + f \cdot x])^m, x], x] /;$ FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

$\text{Int}[(a \cdot \sin(e + f \cdot x))^m \cdot \tan(e + f \cdot x)^n, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sin}[e + f \cdot x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(\text{ff} \cdot x)^{m+n}/(a^2 - \text{ff}^2 \cdot x^2)^{(n+1)/2}, x], x, (a \cdot \text{Sin}[e + f \cdot x])/ff], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

$\text{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1})/(b \cdot n \cdot (p+1)), x] - \text{Dist}[(c^{n \cdot (m-n+1)})/(b \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^{p+1}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n \cdot (p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

$\text{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1})/(b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^{n \cdot (m-n+1)})/(b \cdot (m + n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n \cdot p

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \cot^6(c + dx)(a + b \sin(c + dx)) dx &= \int (b \cos(c + dx) \cot^5(c + dx) + a \cot^6(c + dx)) dx \\
 &= a \int \cot^6(c + dx) dx + b \int \cos(c + dx) \cot^5(c + dx) dx \\
 &= -\frac{a \cot^5(c + dx)}{5d} - a \int \cot^4(c + dx) dx - \frac{b \operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^3} dx, x, \cos(c + dx)\right)}{d} \\
 &= \frac{a \cot^3(c + dx)}{3d} - \frac{b \cos(c + dx) \cot^4(c + dx)}{4d} - \frac{a \cot^5(c + dx)}{5d} + a \int \cot^2(c + dx) dx \\
 &= -\frac{a \cot(c + dx)}{d} + \frac{5b \cos(c + dx) \cot^2(c + dx)}{8d} + \frac{a \cot^3(c + dx)}{3d} - \frac{b \cos(c + dx)}{4d} \\
 &= -ax + \frac{15b \cos(c + dx)}{8d} - \frac{a \cot(c + dx)}{d} + \frac{5b \cos(c + dx) \cot^2(c + dx)}{8d} + \frac{a \cot^3(c + dx)}{3d} \\
 &= -ax - \frac{15b \tanh^{-1}(\cos(c + dx))}{8d} + \frac{15b \cos(c + dx)}{8d} - \frac{a \cot(c + dx)}{d} + \frac{5b \cos(c + dx)}{4d}
 \end{aligned}$$

Mathematica [C] time = 0.0598431, size = 164, normalized size = 1.34

$$-\frac{a \cot^5(c + dx) {}_2F_1\left(-\frac{5}{2}, 1; -\frac{3}{2}; -\tan^2(c + dx)\right)}{5d} + \frac{b \cos(c + dx)}{d} - \frac{b \csc^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{9b \csc^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{b \sec^4\left(\frac{1}{2}(c + dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]

[Out] (b*Cos[c + d*x])/d + (9*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d) - (15*b*Log[Cos[(c + d*x)/2]])/(8*d) + (15*b*Log[Sin[(c + d*x)/2]])/(8*d) - (9*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d)

Maple [A] time = 0.039, size = 159, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^5}{5d} + \frac{a(\cot(dx+c))^3}{3d} - \frac{a\cot(dx+c)}{d} - ax - \frac{ca}{d} - \frac{b(\cos(dx+c))^7}{4d(\sin(dx+c))^4} + \frac{3b(\cos(dx+c))^7}{8d(\sin(dx+c))^2} + \frac{3b(\cos(dx+c))^7}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c)),x)

[Out] $-1/5*a*\cot(d*x+c)^5/d + 1/3*a*\cot(d*x+c)^3/d - a*\cot(d*x+c)/d - a*x - 1/d*c*a - 1/4/d*b/\sin(d*x+c)^4*\cos(d*x+c)^7 + 3/8/d*b/\sin(d*x+c)^2*\cos(d*x+c)^7 + 3/8/d*b*\cos(d*x+c)^5 + 5/8/d*b*\cos(d*x+c)^3 + 15/8*b*\cos(d*x+c)/d + 15/8/d*b*\ln(\csc(d*x+c) - \cot(d*x+c))$

Maxima [A] time = 2.70003, size = 169, normalized size = 1.39

$$\frac{16\left(15dx + 15c + \frac{15\tan(dx+c)^4 - 5\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a + 15b\left(\frac{2(9\cos(dx+c)^3 - 7\cos(dx+c))}{\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1} - 16\cos(dx+c) + 15\log(\cos(dx+c) + 1) - 15\log(\cos(dx+c) - 1)\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/240*(16*(15*d*x + 15*c + (15*\tan(d*x + c)^4 - 5*\tan(d*x + c)^2 + 3)/\tan(d*x + c)^5)*a + 15*b*(2*(9*\cos(d*x + c)^3 - 7*\cos(d*x + c))/(\cos(d*x + c)^4 - 2*\cos(d*x + c)^2 + 1) - 16*\cos(d*x + c) + 15*\log(\cos(d*x + c) + 1) - 15*\log(\cos(d*x + c) - 1))/d$

Fricas [B] time = 1.7059, size = 620, normalized size = 5.08

$$\frac{368a\cos(dx+c)^5 - 560a\cos(dx+c)^3 + 225(b\cos(dx+c)^4 - 2b\cos(dx+c)^2 + b)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 225(b\cos(dx+c)^4 - 2b\cos(dx+c)^2 + b)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 240a\cos(dx+c) + 30(8a*d*x*\cos(dx+c)^4 - 8*b*\cos(dx+c)^5 - 16*a*d*x*\cos(dx+c)^2 + 25*b*\cos(dx+c)^3 + 8*a*d*x - 15*b*\cos(dx+c))*\sin(dx+c)}{(d*\cos(dx+c)^4 - 2*d*\cos(dx+c)^2 + d)*\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] $-1/240*(368*a*\cos(d*x + c)^5 - 560*a*\cos(d*x + c)^3 + 225*(b*\cos(d*x + c)^4 - 2*b*\cos(d*x + c)^2 + b)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 225*(b*\cos(d*x + c)^4 - 2*b*\cos(d*x + c)^2 + b)*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 240*a*\cos(d*x + c) + 30*(8*a*d*x*\cos(d*x + c)^4 - 8*b*\cos(d*x + c)^5 - 16*a*d*x*\cos(d*x + c)^2 + 25*b*\cos(d*x + c)^3 + 8*a*d*x - 15*b*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c)^4 - 2*d*\cos(d*x + c)^2 + d)*\sin(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.22426, size = 269, normalized size = 2.2

$$6 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 15 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 70 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 240 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 960 (dx + c)a + 1800$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
b*log(abs(tan(1/2*d*x + 1/2*c)))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*b/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.150 $\int (a + b \sin(c + dx))^2 \tan^3(c + dx) dx$

Optimal. Leaf size=111

$$\frac{2ab \sin(c + dx)}{d} + \frac{(a + b)(a + 2b) \log(1 - \sin(c + dx))}{2d} + \frac{(a - 2b)(a - b) \log(\sin(c + dx) + 1)}{2d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))}{2d}$$

[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1 + Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)

Rubi [A] time = 0.172935, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{2ab \sin(c + dx)}{d} + \frac{(a + b)(a + 2b) \log(1 - \sin(c + dx))}{2d} + \frac{(a - 2b)(a - b) \log(\sin(c + dx) + 1)}{2d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]

[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1 + Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 1645

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x)/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && ! (IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 633

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[

-(a*c)]

Rule 31

$\text{Int}[(a + b \cdot x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rubi steps

$$\begin{aligned} \int (a + b \sin(c + dx))^2 \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(b^2-x^2)^2} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\sec^2(c + dx)(a + b \sin(c + dx))^2}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)(-2b^4-2ab^2x-2b^2x^2)}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{\sec^2(c + dx)(a + b \sin(c + dx))^2}{2d} + \frac{\text{Subst}\left(\int \left(4ab^2 + 2b^2x - \frac{2(3ab^4+b^2(a^2+2b^2))}{b^2-x^2}\right) dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))^2}{2d} - \frac{\text{Subst}\left(\int \frac{2(3ab^4+b^2(a^2+2b^2))}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))^2}{2d} - \frac{((a-b)^2 \log(1 - \sin(c + dx)) + (a+b)^2 \log(1 + \sin(c + dx)))}{4d} \\ &= \frac{(a+b)(a+2b) \log(1 - \sin(c + dx))}{2d} + \frac{(a-2b)(a-b) \log(1 + \sin(c + dx))}{2d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))^2}{2d} \end{aligned}$$

Mathematica [A] time = 0.431531, size = 108, normalized size = 0.97

$$\frac{\frac{(a-b)^2}{\sin(c+dx)+1} + 8ab \sin(c + dx) - \frac{(a+b)^2}{\sin(c+dx)-1} + 2(a-2b)(a-b) \log(\sin(c + dx) + 1) + 2(a+b)(a+2b) \log(1 - \sin(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]

[Out] (2*(a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]] + 2*(a - 2*b)*(a - b)*Log[1 + Sin[c + d*x]] - (a + b)^2/(-1 + Sin[c + d*x]) + 8*a*b*Sin[c + d*x] + 2*b^2*Sin[c + d*x]^2 + (a - b)^2/(1 + Sin[c + d*x]))/(4*d)

Maple [A] time = 0.049, size = 172, normalized size = 1.6

$$\frac{a^2 (\tan(dx + c))^2}{2d} + \frac{a^2 \ln(\cos(dx + c))}{d} + \frac{ab (\sin(dx + c))^5}{d (\cos(dx + c))^2} + \frac{ab (\sin(dx + c))^3}{d} + 3 \frac{ab \sin(dx + c)}{d} - 3 \frac{ab \ln(\sec(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x)

[Out] 1/2/d*a^2*tan(d*x+c)^2+1/d*a^2*ln(cos(d*x+c))+1/d*a*b*sin(d*x+c)^5/cos(d*x+c)^2+1/d*a*b*sin(d*x+c)^3+3*a*b*sin(d*x+c)/d-3/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^4+b^2*sin(d*x+c)

$c)^{2/d+2/d} b^2 \ln(\cos(dx+c))$

Maxima [A] time = 1.67617, size = 142, normalized size = 1.28

$$\frac{b^2 \sin(dx+c)^2 + 4ab \sin(dx+c) + (a^2 - 3ab + 2b^2) \log(\sin(dx+c)+1) + (a^2 + 3ab + 2b^2) \log(\sin(dx+c)-1) - \frac{2}{d} (a^2 + b^2) \cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 3*a*b + 2*b^2)*log(sin(d*x + c) + 1) + (a^2 + 3*a*b + 2*b^2)*log(sin(d*x + c) - 1) - (2*a*b*sin(d*x + c) + a^2 + b^2)/(sin(d*x + c)^2 - 1))/d

Fricas [A] time = 1.54829, size = 348, normalized size = 3.14

$$\frac{2b^2 \cos(dx+c)^4 - b^2 \cos(dx+c)^2 - 2(a^2 - 3ab + 2b^2) \cos(dx+c)^2 \log(\sin(dx+c)+1) - 2(a^2 + 3ab + 2b^2) \cos(dx+c)^2 \log(-\sin(dx+c)+1) - 2a^2 - 2b^2 - 4(2ab \cos(dx+c)^2 + ab) \sin(dx+c)}{4d \cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")

[Out] -1/4*(2*b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*(a^2 - 3*a*b + 2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 2*(a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*a^2 - 2*b^2 - 4*(2*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \tan^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**3,x)

[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**3, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")

[Out] Timed out

3.151 $\int (a + b \sin(c + dx))^2 \tan(c + dx) dx$

Optimal. Leaf size=78

$$-\frac{2ab \sin(c + dx)}{d} - \frac{(a - b)^2 \log(\sin(c + dx) + 1)}{2d} - \frac{(a + b)^2 \log(1 - \sin(c + dx))}{2d} - \frac{b^2 \sin^2(c + dx)}{2d}$$

[Out] $-\frac{(a + b)^2 \text{Log}[1 - \text{Sin}[c + d*x]]}{2*d} - \frac{(a - b)^2 \text{Log}[1 + \text{Sin}[c + d*x]]}{2*d} - \frac{2*a*b*\text{Sin}[c + d*x]}{d} - \frac{b^2*\text{Sin}[c + d*x]^2}{2*d}$

Rubi [A] time = 0.0709252, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2721, 801, 633, 31}

$$-\frac{2ab \sin(c + dx)}{d} - \frac{(a - b)^2 \log(\sin(c + dx) + 1)}{2d} - \frac{(a + b)^2 \log(1 - \sin(c + dx))}{2d} - \frac{b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sin}[c + d*x])^2*\text{Tan}[c + d*x], x]$

[Out] $-\frac{(a + b)^2 \text{Log}[1 - \text{Sin}[c + d*x]]}{2*d} - \frac{(a - b)^2 \text{Log}[1 + \text{Sin}[c + d*x]]}{2*d} - \frac{2*a*b*\text{Sin}[c + d*x]}{d} - \frac{b^2*\text{Sin}[c + d*x]^2}{2*d}$

Rule 2721

$\text{Int}[(a + b*\text{sin}[e + f*x])^m*\text{tan}[e + f*x]^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p + 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 801

$\text{Int}[(d + e*x)^m*((f + g*x))/(a + c*x^2), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 633

$\text{Int}[(d + e*x)/(a + c*x^2), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x]] /;$ FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx))^2 \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-2a - x + \frac{2ab^2 + (a^2 + b^2)x}{b^2 - x^2}\right) dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{2ab \sin(c + dx)}{d} - \frac{b^2 \sin^2(c + dx)}{2d} + \frac{\text{Subst}\left(\int \frac{2ab^2 + (a^2 + b^2)x}{b^2 - x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{2ab \sin(c + dx)}{d} - \frac{b^2 \sin^2(c + dx)}{2d} + \frac{(a - b)^2 \text{Subst}\left(\int \frac{1}{-b-x} dx, x, b \sin(c + dx)\right)}{2d} \\
&= -\frac{(a + b)^2 \log(1 - \sin(c + dx))}{2d} - \frac{(a - b)^2 \log(1 + \sin(c + dx))}{2d} - \frac{2ab \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.13574, size = 64, normalized size = 0.82

$$-\frac{4ab \sin(c + dx) + (a - b)^2 \log(\sin(c + dx) + 1) + (a + b)^2 \log(1 - \sin(c + dx)) + b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x], x]

[Out] -((a + b)^2*Log[1 - Sin[c + d*x]] + (a - b)^2*Log[1 + Sin[c + d*x]] + 4*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(2*d)

Maple [A] time = 0.039, size = 82, normalized size = 1.1

$$-\frac{a^2 \ln(\cos(dx + c))}{d} - 2\frac{ab \sin(dx + c)}{d} + 2\frac{ab \ln(\sec(dx + c) + \tan(dx + c))}{d} - \frac{(\sin(dx + c))^2 b^2}{2d} - \frac{b^2 \ln(\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^2*tan(d*x+c), x)

[Out] -1/d*a^2*ln(cos(d*x+c))-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/2*b^2*sin(d*x+c)^2/d-1/d*b^2*ln(cos(d*x+c))

Maxima [A] time = 1.63094, size = 95, normalized size = 1.22

$$-\frac{b^2 \sin(dx + c)^2 + 4ab \sin(dx + c) + (a^2 - 2ab + b^2) \log(\sin(dx + c) + 1) + (a^2 + 2ab + b^2) \log(\sin(dx + c) - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c), x, algorithm="maxima")

[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d

Fricas [A] time = 1.60878, size = 186, normalized size = 2.38

$$\frac{b^2 \cos(dx + c)^2 - 4ab \sin(dx + c) - (a^2 - 2ab + b^2) \log(\sin(dx + c) + 1) - (a^2 + 2ab + b^2) \log(-\sin(dx + c) + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")

[Out] 1/2*(b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) - (a^2 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c),x)

[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x), x)

Giac [B] time = 5.72047, size = 10604, normalized size = 135.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")

[Out] -1/4*(4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(

$$\begin{aligned}
& 1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) \\
& + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/ \\
& (\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 \\
& - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*b^2* \\
& \log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2* \\
& \tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2 \\
& *\tan(1/2*c)^2 + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 \\
& + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d* \\
& x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan \\
& (1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) \\
& + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1) \\
& /(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x) \\
&)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/ \\
& 2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + \\
& 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(c)^2 + 2 \\
& *a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d* \\
& x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/2*d \\
& *x)^2*\tan(c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d* \\
& x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan \\
& (d*x)^2*\tan(1/2*d*x)^2*\tan(c)^2 - 16*a*b*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2 \\
& *c)*\tan(c)^2 + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 \\
& + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x) \\
&)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan \\
& (1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) \\
& + 1))*\tan(d*x)^2*\tan(1/2*c)^2*\tan(c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(t \\
& an(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3 \\
& *\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2* \\
& d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2* \\
& \tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + 2*a^2* \\
& \log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2 \\
& *\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/2*c)^2*\tan \\
& (c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan \\
& (c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2 \\
& *\tan(1/2*c)^2*\tan(c)^2 - 16*a*b*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^2*\tan(c) \\
&)^2 + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/ \\
& 2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan \\
& (1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 \\
& + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan \\
& (1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/ \\
& 2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(\\
& 1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 \\
& - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/ \\
& 2*d*x) + 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + 2*a^2*\log \\
& (4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2 \\
& *\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 \\
& *\tan(c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3* \\
& *\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/ \\
& 2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + b^2*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 \\
& + 4*b^2*\tan(d*x)*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c) - b^2*\tan(d*x)^2*\tan(1/ \\
& 2*d*x)^2*\tan(c)^2 - b^2*\tan(d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + b^2*\tan(1/2*d*x) \\
&)^2*\tan(1/2*c)^2*\tan(c)^2 + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4* \\
& \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 \\
& + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan \\
& (1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - \\
& 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + \\
& 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2* \\
& d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan \\
& (1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 \\
& + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2 + 2*a^2*1
\end{aligned}$$

$$\begin{aligned}
& \log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2 \\
& * \tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/2*d*x)^2 \\
& + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan \\
& (d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(1/ \\
& 2*d*x)^2 - 16*a*b*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c) + 4*a*b*\log(2*(\tan(1 \\
& /2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2 \\
& * \tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c) \\
& ^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan \\
& (1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan(1/2*c)^2 - \\
& 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x) \\
&)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2 \\
& *d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*t \\
& \tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(d*x) \\
& ^2*\tan(1/2*c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d \\
& *x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan \\
& (d*x)^2*\tan(1/2*c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - \\
& 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) \\
& + 1))*\tan(d*x)^2*\tan(1/2*c)^2 - 16*a*b*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^2 \\
& + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2* \\
& d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(\\
& 1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + \\
& 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(1 \\
& /2*d*x)^2*\tan(1/2*c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan \\
& (1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan \\
& (1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/ \\
& 2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2* \\
& \tan(1/2*c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/ \\
& (\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 \\
& - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*b^2*\log(4*(\tan(\\
& c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 \\
& + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 4*a*b* \\
& \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan \\
& (1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^ \\
& 2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2 \\
& *d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(d*x)^2*\tan \\
& (c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan \\
& (1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2 \\
& * \tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c) \\
& ^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))* \\
& \tan(d*x)^2*\tan(c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan \\
& (d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1) \\
&))*\tan(d*x)^2*\tan(c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - \\
& 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) \\
& + 1))*\tan(d*x)^2*\tan(c)^2 + 16*a*b*\tan(d*x)^2*\tan(1/2*d*x)*\tan(c)^2 + 4*a*b \\
& * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan \\
& (1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x) \\
& ^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/ \\
& 2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)^ \\
& 2*\tan(c)^2 - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - \\
& 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^ \\
& 4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1 \\
& /2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + \\
& 1))*\tan(1/2*d*x)^2*\tan(c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c) \\
&)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan \\
& (c) + 1))*\tan(1/2*d*x)^2*\tan(c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4 \\
& *\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan \\
& (d*x)*\tan(c) + 1))*\tan(1/2*d*x)^2*\tan(c)^2 + 16*a*b*\tan(d*x)^2*\tan(1/2*c)*\tan \\
& (c)^2 - 16*a*b*\tan(1/2*d*x)^2*\tan(1/2*c)*\tan(c)^2 + 4*a*b*\log(2*(\tan(1/2*c) \\
&)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*ta
\end{aligned}$$

$$\begin{aligned}
& \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) * \tan(c)^2 + 2*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) * \tan(c)^2 + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) * \tan(c)^2 + 16*a*b*\tan(1/2*d*x)*\tan(c)^2 + 16*a*b*\tan(1/2*c)*\tan(c)^2 + b^2*\tan(d*x)^2 - b^2*\tan(1/2*d*x)^2 - b^2*\tan(1/2*c)^2 + 4*b^2*\tan(d*x)*\tan(c) + b^2*\tan(c)^2 + 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) - 4*a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) + 2*a^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 2*b^2*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 16*a*b*\tan(1/2*d*x) + 16*a*b*\tan(1/2*c) - b^2)/(d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(c)^2 + d*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2*\tan(1/2*d*x)^2 + d*\tan(d*x)^2*\tan(1/2*c)^2 + d*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + d*\tan(1/2*d*x)^2*\tan(c)^2 + d*\tan(1/2*c)^2*\tan(c)^2 + d*\tan(d*x)^2 + d*\tan(1/2*d*x)^2 + d*\tan(1/2*c)^2 + d*\tan(c)^2 + d)
\end{aligned}$$

3.152 $\int \cot(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=46

$$\frac{a^2 \log(\sin(c + dx))}{d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d}$$

[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Rubi [A] time = 0.038808, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 43}

$$\frac{a^2 \log(\sin(c + dx))}{d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]

[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + b \sin(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(2a + \frac{a^2}{x} + x\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{a^2 \log(\sin(c + dx))}{d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0234396, size = 46, normalized size = 1.

$$\frac{a^2 \log(\sin(c + dx))}{d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]

[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Maple [A] time = 0.023, size = 45, normalized size = 1.

$$\frac{a^2 \ln(\sin(dx + c))}{d} + 2 \frac{ab \sin(dx + c)}{d} + \frac{(\sin(dx + c))^2 b^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^2,x)

[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d

Maxima [A] time = 1.89331, size = 54, normalized size = 1.17

$$\frac{b^2 \sin(dx + c)^2 + 2a^2 \log(\sin(dx + c)) + 4ab \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a*b*sin(d*x + c))/d

Fricas [A] time = 1.57997, size = 108, normalized size = 2.35

$$\frac{b^2 \cos(dx + c)^2 - 2a^2 \log\left(\frac{1}{2} \sin(dx + c)\right) - 4ab \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a*b*sin(d*x + c))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))*2*cot(c + d*x), x)

Giac [A] time = 2.32155, size = 55, normalized size = 1.2

$$\frac{b^2 \sin(dx + c)^2 + 2a^2 \log(|\sin(dx + c)|) + 4ab \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a*b*sin(d*x + c))/d

3.153 $\int \cot^3(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=84

$$-\frac{(a^2 - b^2) \log(\sin(c + dx))}{d} - \frac{a^2 \csc^2(c + dx)}{2d} - \frac{2ab \sin(c + dx)}{d} - \frac{2ab \csc(c + dx)}{d} - \frac{b^2 \sin^2(c + dx)}{2d}$$

[Out] $(-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)$

Rubi [A] time = 0.0730727, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$-\frac{(a^2 - b^2) \log(\sin(c + dx))}{d} - \frac{a^2 \csc^2(c + dx)}{2d} - \frac{2ab \sin(c + dx)}{d} - \frac{2ab \csc(c + dx)}{d} - \frac{b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^3*(a + b*\text{Sin}[c + d*x])^2, x]$

[Out] $(-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)$

Rule 2721

$\text{Int}[(a + b*\sin(e + f*x))^m * \tan(e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p+1)/2}], x], x, b*\text{Sin}[e + f*x], x] /;$ FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

$\text{Int}[(d + e*x)^m * (f + g*x)^n * (a + c*x)^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m * (f + g*x)^n * (a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + b \sin(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x^3} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a + \frac{a^2b^2}{x^3} + \frac{2ab^2}{x^2} + \frac{-a^2+b^2}{x} - x\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= -\frac{2ab \csc(c + dx)}{d} - \frac{a^2 \csc^2(c + dx)}{2d} - \frac{(a^2 - b^2) \log(\sin(c + dx))}{d} - \frac{2ab \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.23557, size = 70, normalized size = 0.83

$$-\frac{2(a^2 - b^2) \log(\sin(c + dx)) + a^2 \csc^2(c + dx) + 4ab \sin(c + dx) + 4ab \csc(c + dx) + b^2 \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]

[Out] $-(4*a*b*\text{Csc}[c + d*x] + a^2*\text{Csc}[c + d*x]^2 + 2*(a^2 - b^2)*\text{Log}[\text{Sin}[c + d*x]] + 4*a*b*\text{Sin}[c + d*x] + b^2*\text{Sin}[c + d*x]^2)/(2*d)$

Maple [A] time = 0.053, size = 120, normalized size = 1.4

$$-\frac{a^2(\cot(dx+c))^2}{2d} - \frac{a^2 \ln(\sin(dx+c))}{d} - 2\frac{ab(\cos(dx+c))^4}{d \sin(dx+c)} - 2\frac{ab(\cos(dx+c))^2 \sin(dx+c)}{d} - 4\frac{ab \sin(dx+c)}{d} + \frac{b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x)

[Out] $-1/2/d*a^2*\cot(d*x+c)^2 - a^2*\ln(\sin(d*x+c))/d - 2/d*a*b/\sin(d*x+c)*\cos(d*x+c)^4 - 2/d*a*b*\cos(d*x+c)^2*\sin(d*x+c) - 4*a*b*\sin(d*x+c)/d + 1/2/d*b^2*\cos(d*x+c)^2 + 1/d*b^2*\ln(\sin(d*x+c))$

Maxima [A] time = 1.56796, size = 93, normalized size = 1.11

$$\frac{b^2 \sin(dx+c)^2 + 4ab \sin(dx+c) + 2(a^2 - b^2) \log(\sin(dx+c)) + \frac{4ab \sin(dx+c) + a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/2*(b^2*\sin(d*x + c)^2 + 4*a*b*\sin(d*x + c) + 2*(a^2 - b^2)*\log(\sin(d*x + c)) + (4*a*b*\sin(d*x + c) + a^2)/\sin(d*x + c)^2)/d$

Fricas [A] time = 1.5208, size = 271, normalized size = 3.23

$$\frac{2b^2 \cos(dx+c)^4 - 3b^2 \cos(dx+c)^2 + 2a^2 + b^2 - 4((a^2 - b^2) \cos(dx+c)^2 - a^2 + b^2) \log\left(\frac{1}{2} \sin(dx+c)\right) - 8(ab \cos(dx+c) + a^2)}{4(d \cos(dx+c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $1/4*(2*b^2*\cos(d*x + c)^4 - 3*b^2*\cos(d*x + c)^2 + 2*a^2 + b^2 - 4*((a^2 - b^2)*\cos(d*x + c)^2 - a^2 + b^2)*\log(1/2*\sin(d*x + c)) - 8*(a*b*\cos(d*x + c))^2 - 2*a*b)*\sin(d*x + c))/(d*\cos(d*x + c)^2 - d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**3, x)

Giac [A] time = 2.31265, size = 134, normalized size = 1.6

$$\frac{b^2 \sin(dx + c)^2 + 4ab \sin(dx + c) + 2(a^2 - b^2) \log(|\sin(dx + c)|) - \frac{3a^2 \sin(dx+c)^2 - 3b^2 \sin(dx+c)^2 - 4ab \sin(dx+c) - a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(abs(sin(d*x + c)))) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x + c) - a^2)/sin(d*x + c)^2/d

3.154 $\int \cot^5(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=126

$$\frac{(2a^2 - b^2) \csc^2(c + dx)}{2d} + \frac{(a^2 - 2b^2) \log(\sin(c + dx))}{d} - \frac{a^2 \csc^4(c + dx)}{4d} + \frac{2ab \sin(c + dx)}{d} - \frac{2ab \csc^3(c + dx)}{3d} + \frac{4ab \csc(c + dx)}{d}$$

[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Rubi [A] time = 0.103542, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 948}

$$\frac{(2a^2 - b^2) \csc^2(c + dx)}{2d} + \frac{(a^2 - 2b^2) \log(\sin(c + dx))}{d} - \frac{a^2 \csc^4(c + dx)}{4d} + \frac{2ab \sin(c + dx)}{d} - \frac{2ab \csc^3(c + dx)}{3d} + \frac{4ab \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]

[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 948

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + b \sin(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^5} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(2a + \frac{a^2b^4}{x^5} + \frac{2ab^4}{x^4} + \frac{-2a^2b^2+b^4}{x^3} - \frac{4ab^2}{x^2} + \frac{a^2-2b^2}{x} + x\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{4ab \csc(c + dx)}{d} + \frac{(2a^2 - b^2) \csc^2(c + dx)}{2d} - \frac{2ab \csc^3(c + dx)}{3d} - \frac{a^2 \csc^4(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.756517, size = 107, normalized size = 0.85

$$\frac{6(2a^2 - b^2) \csc^2(c + dx) + 6(2(a^2 - 2b^2) \log(\sin(c + dx)) + 4ab \sin(c + dx) + b^2 \sin^2(c + dx)) - 3a^2 \csc^4(c + dx) - 8ab \csc(c + dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]

[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)

Maple [A] time = 0.047, size = 220, normalized size = 1.8

$$\frac{a^2 (\cot(dx + c))^4}{4d} + \frac{a^2 (\cot(dx + c))^2}{2d} + \frac{a^2 \ln(\sin(dx + c))}{d} - \frac{2ab (\cos(dx + c))^6}{3d (\sin(dx + c))^3} + 2 \frac{ab (\cos(dx + c))^6}{d \sin(dx + c)} + \frac{16ab \sin(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x)

[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/3/d*a*b/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a*b/sin(d*x+c)*cos(d*x+c)^6+16/3*a*b*sin(d*x+c)/d+2/d*a*b*sin(d*x+c)*cos(d*x+c)^4+8/3/d*a*b*cos(d*x+c)^2*sin(d*x+c)-1/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b^2*cos(d*x+c)^4-1/d*b^2*cos(d*x+c)^2-2/d*b^2*ln(sin(d*x+c))

Maxima [A] time = 1.40308, size = 142, normalized size = 1.13

$$\frac{6b^2 \sin(dx + c)^2 + 24ab \sin(dx + c) + 12(a^2 - 2b^2) \log(\sin(dx + c)) + \frac{48ab \sin(dx+c)^3 - 8ab \sin(dx+c) + 6(2a^2 - b^2) \sin(dx+c)^2 - 3a^2}{\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d

Fricas [A] time = 1.62927, size = 437, normalized size = 3.47

$$\frac{6b^2 \cos(dx + c)^6 - 15b^2 \cos(dx + c)^4 + 6(2a^2 + b^2) \cos(dx + c)^2 - 9a^2 + 3b^2 - 12((a^2 - 2b^2) \cos(dx + c)^4 - 2(a^2 - 2b^2) \cos(dx + c)^2 + a^2 - 2b^2) \log(1/2 \sin(dx + c)) - 8(3a*b \cos(dx + c)^4 - 12a*b \cos(dx + c)^2 + 8a*b) \sin(dx + c)}{12(d \cos(dx + c))^4 - 12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x + c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c))^4 - 2

*d*cos(d*x + c)^2 + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 2.22304, size = 186, normalized size = 1.48

$$6b^2 \sin(dx+c)^2 + 24ab \sin(dx+c) + 12(a^2 - 2b^2) \log(|\sin(dx+c)|) - \frac{25a^2 \sin(dx+c)^4 - 50b^2 \sin(dx+c)^4 - 48ab \sin(dx+c)^3 - 12a^2 \sin(dx+c)^2 + 6b^2 \sin(dx+c)^2 + 8ab \sin(dx+c) + 3a^2}{\sin(dx+c)^4} d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*sin(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*x + c) + 3*a^2)/sin(d*x + c)^4)/d

3.155 $\int (a + b \sin(c + dx))^2 \tan^4(c + dx) dx$

Optimal. Leaf size=149

$$\frac{a^2 \tan^3(c + dx)}{3d} - \frac{a^2 \tan(c + dx)}{d} + a^2 x - \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec^3(c + dx)}{3d} - \frac{4ab \sec(c + dx)}{d} + \frac{5b^2 \tan^3(c + dx)}{6d}$$

[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)

Rubi [A] time = 0.161982, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{a^2 \tan^3(c + dx)}{3d} - \frac{a^2 \tan(c + dx)}{d} + a^2 x - \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec^3(c + dx)}{3d} - \frac{4ab \sec(c + dx)}{d} + \frac{5b^2 \tan^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]

[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2590

Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx))^2 \tan^4(c + dx) dx &= \int (a^2 \tan^4(c + dx) + 2ab \sin(c + dx) \tan^4(c + dx) + b^2 \sin^2(c + dx) \tan^4(c + dx)) dx \\
&= a^2 \int \tan^4(c + dx) dx + (2ab) \int \sin(c + dx) \tan^4(c + dx) dx + b^2 \int \sin^2(c + dx) \tan^4(c + dx) dx \\
&= \frac{a^2 \tan^3(c + dx)}{3d} - a^2 \int \tan^2(c + dx) dx - \frac{(2ab) \text{Subst}\left(\int \frac{(1-x^2)^2}{x^4} dx, x, \cos(c + dx)\right)}{d} \\
&= -\frac{a^2 \tan(c + dx)}{d} + \frac{a^2 \tan^3(c + dx)}{3d} - \frac{b^2 \sin^2(c + dx) \tan^3(c + dx)}{2d} + a^2 \int 1 dx - \dots \\
&= a^2 x - \frac{2ab \cos(c + dx)}{d} - \frac{4ab \sec(c + dx)}{d} + \frac{2ab \sec^3(c + dx)}{3d} - \frac{a^2 \tan(c + dx)}{d} + \dots \\
&= a^2 x - \frac{2ab \cos(c + dx)}{d} - \frac{4ab \sec(c + dx)}{d} + \frac{2ab \sec^3(c + dx)}{3d} - \frac{a^2 \tan(c + dx)}{d} - \dots \\
&= a^2 x + \frac{5b^2 x}{2} - \frac{2ab \cos(c + dx)}{d} - \frac{4ab \sec(c + dx)}{d} + \frac{2ab \sec^3(c + dx)}{3d} - \frac{a^2 \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.686576, size = 176, normalized size = 1.18

$$\sec^3(c + dx) \left(-36(2a^2 + 5b^2)(c + dx) \cos(c + dx) + 32a^2 \sin(3(c + dx)) - 24a^2 c \cos(3(c + dx)) - 24a^2 dx \cos(3(c + dx)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```



```
[Out] -(Sec[c + d*x]^3*(200*a*b - 36*(2*a^2 + 5*b^2)*(c + d*x)*Cos[c + d*x] + 288
*a*b*Cos[2*(c + d*x)] - 24*a^2*c*Cos[3*(c + d*x)] - 60*b^2*c*Cos[3*(c + d*x
)] - 24*a^2*d*x*Cos[3*(c + d*x)] - 60*b^2*d*x*Cos[3*(c + d*x)] + 24*a*b*Cos
[4*(c + d*x)] + 30*b^2*Sin[c + d*x] + 32*a^2*Sin[3*(c + d*x)] + 65*b^2*Sin[
3*(c + d*x)] + 3*b^2*Sin[5*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.044, size = 185, normalized size = 1.2

$$\frac{1}{d} \left(a^2 \left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c) + dx+c \right) + 2ab \left(\frac{1}{3} \frac{(\sin(dx+c))^6}{(\cos(dx+c))^3} - \frac{(\sin(dx+c))^6}{\cos(dx+c)} - (8/3 + (\sin(dx+c))^4) + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin
(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A] time = 2.50033, size = 161, normalized size = 1.08

$$\frac{2 \left(\tan(dx+c)^3 + 3dx + 3c - 3 \tan(dx+c) \right) a^2 + \left(2 \tan(dx+c)^3 + 15dx + 15c - \frac{3 \tan(dx+c)}{\tan(dx+c)^2 + 1} - 12 \tan(dx+c) \right) b^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + (2*tan(d*x + c
)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c)
)*b^2 - 4*a*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.46423, size = 281, normalized size = 1.89

$$\frac{3 \left(2a^2 + 5b^2 \right) dx \cos(dx+c)^3 - 12ab \cos(dx+c)^4 - 24ab \cos(dx+c)^2 + 4ab - \left(3b^2 \cos(dx+c)^4 + 2 \left(4a^2 + 7b^2 \right) \dots \right)}{6d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^2 + 5*b^2)*d*x*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)^4 - 24*a*b*
cos(d*x + c)^2 + 4*a*b - (3*b^2*cos(d*x + c)^4 + 2*(4*a^2 + 7*b^2)*cos(d*x
+ c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int (a + b \sin(c + dx))^2 \tan^2(c + dx) dx$

Optimal. Leaf size=94

$$\frac{a^2 \tan(c + dx)}{d} + a^2(-x) + \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec(c + dx)}{d} + \frac{3b^2 \tan(c + dx)}{2d} - \frac{b^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{3b^2 x}{2}$$

[Out] $-(a^2 x) - (3b^2 x)/2 + (2ab \cos[c + dx])/d + (2ab \sec[c + dx])/d + (a^2 \tan[c + dx])/d + (3b^2 \tan[c + dx])/(2d) - (b^2 \sin[c + dx]^2 \tan[c + dx])/(2d)$

Rubi [A] time = 0.122315, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203}

$$\frac{a^2 \tan(c + dx)}{d} + a^2(-x) + \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec(c + dx)}{d} + \frac{3b^2 \tan(c + dx)}{2d} - \frac{b^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{3b^2 x}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b \sin[c + dx])^2 \tan^2[c + dx], x]$

[Out] $-(a^2 x) - (3b^2 x)/2 + (2ab \cos[c + dx])/d + (2ab \sec[c + dx])/d + (a^2 \tan[c + dx])/d + (3b^2 \tan[c + dx])/(2d) - (b^2 \sin[c + dx]^2 \tan[c + dx])/(2d)$

Rule 2722

$\text{Int}[(a + b \sin(e + f x))^m (g \tan(e + f x))^p, x] \text{Symbol} \rightarrow \text{Int}[\text{ExpandIntegrand}[(g \tan[e + f x])^p, (a + b \sin[e + f x])^m, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x \} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

Rule 3473

$\text{Int}[(b \tan(c + dx))^n, x] \text{Symbol} \rightarrow \text{Simp}[(b \tan[c + dx])^{n-1}]/(d(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \tan[c + dx])^{n-2}], x], x] /; \text{FreeQ}\{b, c, d\}, x \} \&\& \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a x, x] \text{Symbol} \rightarrow \text{Simp}[a x], x] /; \text{FreeQ}[a, x]$

Rule 2590

$\text{Int}[\sin(e + f x)^m \tan(e + f x)^n, x] \text{Symbol} \rightarrow -\text{Dist}[f^{-1}, \text{Subst}[\text{Int}[(1 - x^2)^{(m+n-1)/2}/x^n], x], \text{Cos}[e + f x], x] /; \text{FreeQ}\{e, f\}, x \} \&\& \text{IntegersQ}[m, n, (m+n-1)/2]$

Rule 14

$\text{Int}[u (c + dx)^m, x] \text{Symbol} \rightarrow \text{Int}[\text{ExpandIntegrand}[(c x)^m u], x] /; \text{FreeQ}\{c, m\}, x \} \&\& \text{SumQ}[u] \&\& \text{!LinearQ}[u, x] \&\& \text{!MatchQ}[u, (a + b v)] /; \text{FreeQ}\{a, b\}, x \} \&\& \text{InverseFunctionQ}[v]$

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int (a + b \sin(c + dx))^2 \tan^2(c + dx) dx &= \int (a^2 \tan^2(c + dx) + 2ab \sin(c + dx) \tan^2(c + dx) + b^2 \sin^2(c + dx) \tan^2(c + dx)) dx \\ &= a^2 \int \tan^2(c + dx) dx + (2ab) \int \sin(c + dx) \tan^2(c + dx) dx + b^2 \int \sin^2(c + dx) \tan^2(c + dx) dx \\ &= \frac{a^2 \tan(c + dx)}{d} - a^2 \int 1 dx - \frac{(2ab) \text{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} + \frac{b^2 \text{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -a^2 x + \frac{a^2 \tan(c + dx)}{d} - \frac{b^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{(2ab) \text{Subst}\left(\int \left(-1 + \frac{1}{x^2}\right) dx, x, \cos(c + dx)\right)}{d} \\ &= -a^2 x + \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec(c + dx)}{d} + \frac{a^2 \tan(c + dx)}{d} + \frac{3b^2 \tan(c + dx)}{2d} \\ &= -a^2 x - \frac{3b^2 x}{2} + \frac{2ab \cos(c + dx)}{d} + \frac{2ab \sec(c + dx)}{d} + \frac{a^2 \tan(c + dx)}{d} + \frac{3b^2 \tan(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.457161, size = 77, normalized size = 0.82

$$\frac{-4(2a^2 + 3b^2)(c + dx) + (8a^2 + 9b^2) \tan(c + dx) + b \sec(c + dx)(8a \cos(2(c + dx)) + 24a + b \sin(3(c + dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)] + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

Maple [A] time = 0.039, size = 116, normalized size = 1.2

$$\frac{1}{d} \left(a^2 (\tan(dx+c) - dx - c) + 2ab \left(\frac{(\sin(dx+c))^4}{\cos(dx+c)} + (2 + (\sin(dx+c))^2) \cos(dx+c) \right) + b^2 \left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + (\sin(dx+c))^3 + \frac{3}{2} \sin(dx+c) \right) \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x)

[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))

Maxima [A] time = 2.52019, size = 112, normalized size = 1.19

$$\frac{2(dx+c - \tan(dx+c))a^2 + \left(3dx + 3c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2 \tan(dx+c)\right)b^2 - 4ab\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))/d

Fricas [A] time = 1.3702, size = 190, normalized size = 2.02

$$\frac{(2a^2 + 3b^2)dx \cos(dx+c) - 4ab \cos(dx+c)^2 - 4ab - (b^2 \cos(dx+c)^2 + 2a^2 + 2b^2) \sin(dx+c)}{2d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")

[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**2, x)

Giac [B] time = 30.6123, size = 10355, normalized size = 110.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")

[Out]
$$-1/2*(2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 12*b^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2$$

$$\begin{aligned}
& 2*c)*\tan(c)^2 + 8*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 1 \\
& 2*b^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 8*a^2*d*x*\tan(1/2 \\
& *d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 12*b^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) \\
& ^2 - 32*a*b*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 2*a^2*d*x*t \\
& \tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^2 + 3*b^2*d*x*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^ \\
& 2 + 8*a*b*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 2*a^2*d*x*\tan(d*x)*\tan(1/2 \\
& *d*x)^4*\tan(c)^3 - 3*b^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c)^3 - 8*a*b*\tan(d \\
& *x)^3*\tan(1/2*d*x)^4*\tan(c)^3 - 8*a^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c \\
&)*\tan(c)^3 - 12*b^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 8*a^2 \\
& *d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 12*b^2*d*x*\tan(d*x)*\tan(\\
& 1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 32*a*b*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c \\
&)*\tan(c)^3 - 96*a*b*\tan(d*x)^3*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c)^3 - 8*a^2 \\
& *d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 12*b^2*d*x*\tan(d*x)*\tan(\\
& 1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 32*a*b*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^ \\
& 3*\tan(c)^3 + 32*a*b*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - 2*a^2*d \\
& *x*\tan(d*x)*\tan(1/2*c)^4*\tan(c)^3 - 3*b^2*d*x*\tan(d*x)*\tan(1/2*c)^4*\tan(c)^ \\
& 3 - 8*a*b*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^3 - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^ \\
& 3*\tan(1/2*c)^3 - 8*b^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3 + 2*a^2*\tan(d \\
& *x)*\tan(1/2*d*x)^4*\tan(1/2*c)^4 + 3*b^2*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c)^ \\
& 4 - 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) + 2*a^2*\tan(1/2*d*x \\
&)^4*\tan(1/2*c)^4*\tan(c) + 3*b^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c) - 2*a^2* \\
& \tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c)^2 - 3*b^2*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c) \\
& ^2 - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 - 12*b^2*\tan(d*x)^ \\
& 3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 - 8*a^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/ \\
& 2*c)^3*\tan(c)^2 - 12*b^2*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 - 8* \\
& a^2*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 - 2*a^2*\tan(d*x)^3*\tan(1/ \\
& 2*c)^4*\tan(c)^2 - 3*b^2*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^2 - 2*a^2*\tan(d*x)^2 \\
& *\tan(1/2*d*x)^4*\tan(c)^3 - 3*b^2*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^3 - 8*a^2 \\
& *\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 12*b^2*\tan(d*x)^2*\tan(1/2* \\
& d*x)^3*\tan(1/2*c)*\tan(c)^3 - 8*a^2*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan \\
& (c)^3 - 12*b^2*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - 8*a^2*\tan(1/ \\
& 2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - 8*b^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 \\
& - 2*a^2*\tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^3 - 3*b^2*\tan(d*x)^2*\tan(1/2*c)^4*t \\
& \tan(c)^3 + 2*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^4 + 3*b^2*d*x*\tan(d*x)^2*\tan(1/ \\
& 2*d*x)^4 + 8*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c) + 12*b^2*d*x*\tan(\\
& d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c) + 8*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/ \\
& 2*c)^3 + 12*b^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3 + 8*a^2*d*x*\tan(1/ \\
& 2*d*x)^3*\tan(1/2*c)^3 + 12*b^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)^3 - 32*a*b*\tan \\
& (d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3 + 2*a^2*d*x*\tan(d*x)^2*\tan(1/2*c)^4 + 3 \\
& *b^2*d*x*\tan(d*x)^2*\tan(1/2*c)^4 + 8*a*b*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 2*a^ \\
& 2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c) - 3*b^2*d*x*\tan(d*x)*\tan(1/2*d*x)^4*t \\
& \tan(c) - 8*a*b*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c) - 8*a^2*d*x*\tan(d*x)^3*\tan(1/ \\
& 2*d*x)*\tan(1/2*c)*\tan(c) - 12*b^2*d*x*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)* \\
& \tan(c) - 8*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 12*b^2*d*x*\tan \\
& (d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 32*a*b*\tan(d*x)^3*\tan(1/2*d*x)^3*t \\
& \tan(1/2*c)*\tan(c) - 96*a*b*\tan(d*x)^3*\tan(1/2*d*x)^2*\tan(1/2*c)^2*\tan(c) - 8 \\
& *a^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - 12*b^2*d*x*\tan(d*x)* \\
& \tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c) - 32*a*b*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)^ \\
& 3*\tan(c) + 32*a*b*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) - 2*a^2*d*x*t \\
& \tan(d*x)*\tan(1/2*c)^4*\tan(c) - 3*b^2*d*x*\tan(d*x)*\tan(1/2*c)^4*\tan(c) - 8*a* \\
& b*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c) + 2*a^2*d*x*\tan(1/2*d*x)^4*\tan(c)^2 + 3*b^ \\
& 2*d*x*\tan(1/2*d*x)^4*\tan(c)^2 + 8*a*b*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^2 + \\
& 8*a^2*d*x*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 12*b^2*d*x*\tan(d*x) \\
& ^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 8*a^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)* \\
& \tan(c)^2 + 12*b^2*d*x*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 32*a*b*\tan(d*x)^2 \\
& *\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 96*a*b*\tan(d*x)^2*\tan(1/2*d*x)^2*\tan(\\
& 1/2*c)^2*\tan(c)^2 + 8*a^2*d*x*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 12*b^2*d \\
& *x*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + 32*a*b*\tan(d*x)^2*\tan(1/2*d*x)*\tan(\\
& 1/2*c)^3*\tan(c)^2 - 32*a*b*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + 2*a^2*d*x
\end{aligned}$$

$$\begin{aligned}
& * \tan(1/2*c)^4 * \tan(c)^2 + 3*b^2*d*x*\tan(1/2*c)^4 * \tan(c)^2 + 8*a*b*\tan(d*x)^2 \\
& * \tan(1/2*c)^4 * \tan(c)^2 + 2*a^2*d*x*\tan(d*x)^3 * \tan(c)^3 + 3*b^2*d*x*\tan(d*x) \\
& ^3 * \tan(c)^3 - 8*a*b*\tan(d*x)*\tan(1/2*d*x)^4 * \tan(c)^3 - 8*a^2*d*x*\tan(d*x)* \\
& \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 12*b^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2* \\
& c)*\tan(c)^3 + 32*a*b*\tan(d*x)^3 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 32*a*b*t \\
& \tan(d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c)*\tan(c)^3 - 96*a*b*\tan(d*x)*\tan(1/2*d*x)^2 \\
& * \tan(1/2*c)^2 * \tan(c)^3 - 32*a*b*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c)^3 \\
& - 8*a*b*\tan(d*x)*\tan(1/2*c)^4 * \tan(c)^3 - 2*a^2*\tan(d*x)^3 * \tan(1/2*d*x)^4 - \\
& 2*b^2*\tan(d*x)^3 * \tan(1/2*d*x)^4 - 8*a^2*\tan(d*x)^3 * \tan(1/2*d*x)^3 * \tan(1/2* \\
& c) - 8*b^2*\tan(d*x)^3 * \tan(1/2*d*x)^3 * \tan(1/2*c) - 8*a^2*\tan(d*x)^3 * \tan(1/2* \\
& d*x)*\tan(1/2*c)^3 - 8*b^2*\tan(d*x)^3 * \tan(1/2*d*x)*\tan(1/2*c)^3 - 8*a^2*\tan \\
& (d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c)^3 - 12*b^2*\tan(d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c \\
&)^3 - 2*a^2*\tan(d*x)^3 * \tan(1/2*c)^4 - 2*b^2*\tan(d*x)^3 * \tan(1/2*c)^4 - 2*a^2 \\
& * \tan(d*x)^2 * \tan(1/2*d*x)^4 * \tan(c) - 8*a^2*\tan(d*x)^2 * \tan(1/2*d*x)^3 * \tan(1/2 \\
& *c)*\tan(c) - 8*a^2*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c) - 8*a^2*\tan \\
& (1/2*d*x)^3 * \tan(1/2*c)^3 * \tan(c) - 12*b^2*\tan(1/2*d*x)^3 * \tan(1/2*c)^3 * \tan(c) \\
& - 2*a^2*\tan(d*x)^2 * \tan(1/2*c)^4 * \tan(c) - 2*a^2*\tan(d*x)*\tan(1/2*d*x)^4 * \tan \\
& (c)^2 - 8*a^2*\tan(d*x)^3 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 - 12*b^2*\tan(d*x)^ \\
& 3 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)^3 * \tan(1/2* \\
& c)*\tan(c)^2 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c)^2 - 2*a^2*\tan \\
& (d*x)*\tan(1/2*c)^4 * \tan(c)^2 - 2*a^2*\tan(1/2*d*x)^4 * \tan(c)^3 - 2*b^2*\tan(1/2 \\
& *d*x)^4 * \tan(c)^3 - 8*a^2*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 12*b \\
& ^2*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 8*a^2*\tan(1/2*d*x)^3 * \tan(1 \\
& /2*c)*\tan(c)^3 - 8*b^2*\tan(1/2*d*x)^3 * \tan(1/2*c)*\tan(c)^3 - 8*a^2*\tan(1/2*d \\
& *x)*\tan(1/2*c)^3 * \tan(c)^3 - 8*b^2*\tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c)^3 - 2*a^ \\
& 2*\tan(1/2*c)^4 * \tan(c)^3 - 2*b^2*\tan(1/2*c)^4 * \tan(c)^3 + 2*a^2*d*x*\tan(1/2*d \\
& *x)^4 + 3*b^2*d*x*\tan(1/2*d*x)^4 + 8*a*b*\tan(d*x)^2 * \tan(1/2*d*x)^4 + 8*a^2* \\
& d*x*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2*c) + 12*b^2*d*x*\tan(d*x)^2 * \tan(1/2*d*x) \\
& * \tan(1/2*c) + 8*a^2*d*x*\tan(1/2*d*x)^3 * \tan(1/2*c) + 12*b^2*d*x*\tan(1/2*d*x) \\
& ^3 * \tan(1/2*c) + 32*a*b*\tan(d*x)^2 * \tan(1/2*d*x)^3 * \tan(1/2*c) + 96*a*b*\tan(d* \\
& x)^2 * \tan(1/2*d*x)^2 * \tan(1/2*c)^2 + 8*a^2*d*x*\tan(1/2*d*x)*\tan(1/2*c)^3 + 12 \\
& *b^2*d*x*\tan(1/2*d*x)*\tan(1/2*c)^3 + 32*a*b*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2 \\
& *c)^3 - 32*a*b*\tan(1/2*d*x)^3 * \tan(1/2*c)^3 + 2*a^2*d*x*\tan(1/2*c)^4 + 3*b^2 \\
& *d*x*\tan(1/2*c)^4 + 8*a*b*\tan(d*x)^2 * \tan(1/2*c)^4 + 2*a^2*d*x*\tan(d*x)^3 * \tan \\
& (c) + 3*b^2*d*x*\tan(d*x)^3 * \tan(c) - 8*a*b*\tan(d*x)*\tan(1/2*d*x)^4 * \tan(c) - \\
& 8*a^2*d*x*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 12*b^2*d*x*\tan(d*x)*\tan \\
& (1/2*d*x)*\tan(1/2*c)*\tan(c) + 32*a*b*\tan(d*x)^3 * \tan(1/2*d*x)*\tan(1/2*c)*\tan \\
& (c) - 32*a*b*\tan(d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c)*\tan(c) - 96*a*b*\tan(d*x)*\tan \\
& (1/2*d*x)^2 * \tan(1/2*c)^2 * \tan(c) - 32*a*b*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c) \\
& ^3 * \tan(c) - 8*a*b*\tan(d*x)*\tan(1/2*c)^4 * \tan(c) - 2*a^2*d*x*\tan(d*x)^2 * \tan(c \\
&)^2 - 3*b^2*d*x*\tan(d*x)^2 * \tan(c)^2 + 8*a*b*\tan(1/2*d*x)^4 * \tan(c)^2 + 8*a^2 \\
& *d*x*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 12*b^2*d*x*\tan(1/2*d*x)*\tan(1/2*c)* \\
& \tan(c)^2 - 32*a*b*\tan(d*x)^2 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 32*a*b*\tan \\
& (1/2*d*x)^3 * \tan(1/2*c)*\tan(c)^2 + 96*a*b*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 * \tan(c)^ \\
& 2 + 32*a*b*\tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c)^2 + 8*a*b*\tan(1/2*c)^4 * \tan(c)^2 \\
& + 2*a^2*d*x*\tan(d*x)*\tan(c)^3 + 3*b^2*d*x*\tan(d*x)*\tan(c)^3 - 8*a*b*\tan(d* \\
& x)^3 * \tan(c)^3 + 32*a*b*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 2*a^2*\tan \\
& (d*x)*\tan(1/2*d*x)^4 - 3*b^2*\tan(d*x)*\tan(1/2*d*x)^4 - 8*a^2*\tan(d*x)^3 * \tan \\
& (1/2*d*x)*\tan(1/2*c) - 8*b^2*\tan(d*x)^3 * \tan(1/2*d*x)*\tan(1/2*c) - 8*a^2*\tan \\
& (d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c) - 12*b^2*\tan(d*x)*\tan(1/2*d*x)^3 * \tan(1/2*c \\
&) - 8*a^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3 - 12*b^2*\tan(d*x)*\tan(1/2*d*x) \\
& * \tan(1/2*c)^3 - 2*a^2*\tan(d*x)*\tan(1/2*c)^4 - 3*b^2*\tan(d*x)*\tan(1/2*c)^4 - \\
& 2*a^2*\tan(1/2*d*x)^4 * \tan(c) - 3*b^2*\tan(1/2*d*x)^4 * \tan(c) - 8*a^2*\tan(d*x) \\
& ^2 * \tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 8*a^2*\tan(1/2*d*x)^3 * \tan(1/2*c)*\tan(c) \\
& - 12*b^2*\tan(1/2*d*x)^3 * \tan(1/2*c)*\tan(c) - 8*a^2*\tan(1/2*d*x)*\tan(1/2*c)^3 \\
& * \tan(c) - 12*b^2*\tan(1/2*d*x)*\tan(1/2*c)^3 * \tan(c) - 2*a^2*\tan(1/2*c)^4 * \tan \\
& (c) - 3*b^2*\tan(1/2*c)^4 * \tan(c) + 2*a^2*\tan(d*x)^3 * \tan(c)^2 + 3*b^2*\tan(d*x) \\
& ^3 * \tan(c)^2 - 8*a^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 2*a^2*\tan(d \\
& *x)^2 * \tan(c)^3 + 3*b^2*\tan(d*x)^2 * \tan(c)^3 - 8*a^2*\tan(1/2*d*x)*\tan(1/2*c)*
\end{aligned}$$

$$\begin{aligned}
& \tan(c)^3 - 8*b^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 2*a^2*d*x*\tan(d*x)^2 - \\
& 3*b^2*d*x*\tan(d*x)^2 + 8*a*b*\tan(1/2*d*x)^4 + 8*a^2*d*x*\tan(1/2*d*x)*\tan(1/ \\
& 2*c) + 12*b^2*d*x*\tan(1/2*d*x)*\tan(1/2*c) - 32*a*b*\tan(d*x)^2*\tan(1/2*d*x)* \\
& \tan(1/2*c) + 32*a*b*\tan(1/2*d*x)^3*\tan(1/2*c) + 96*a*b*\tan(1/2*d*x)^2*\tan(1 \\
& /2*c)^2 + 32*a*b*\tan(1/2*d*x)*\tan(1/2*c)^3 + 8*a*b*\tan(1/2*c)^4 + 2*a^2*d*x \\
& *\tan(d*x)*\tan(c) + 3*b^2*d*x*\tan(d*x)*\tan(c) - 8*a*b*\tan(d*x)^3*\tan(c) + 32 \\
& *a*b*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 2*a^2*d*x*\tan(c)^2 - 3*b^2*d \\
& *x*\tan(c)^2 + 8*a*b*\tan(d*x)^2*\tan(c)^2 - 32*a*b*\tan(1/2*d*x)*\tan(1/2*c)*\tan \\
& (c)^2 - 8*a*b*\tan(d*x)*\tan(c)^3 + 2*a^2*\tan(d*x)^3 + 2*b^2*\tan(d*x)^3 - 8* \\
& a^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c) - 12*b^2*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2 \\
& *c) + 2*a^2*\tan(d*x)^2*\tan(c) - 8*a^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 12*b \\
& ^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) + 2*a^2*\tan(d*x)*\tan(c)^2 + 2*a^2*\tan(c)^ \\
& 3 + 2*b^2*\tan(c)^3 - 2*a^2*d*x - 3*b^2*d*x + 8*a*b*\tan(d*x)^2 - 32*a*b*\tan(\\
& 1/2*d*x)*\tan(1/2*c) - 8*a*b*\tan(d*x)*\tan(c) + 8*a*b*\tan(c)^2 + 2*a^2*\tan(d* \\
& x) + 3*b^2*\tan(d*x) + 2*a^2*\tan(c) + 3*b^2*\tan(c) + 8*a*b)/(d*\tan(d*x)^3*\tan \\
& (1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^3 + d*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(1/2*c) \\
& ^4*\tan(c) - d*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - 4*d*\tan(d*x) \\
& ^3*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 + d*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/ \\
& 2*c)^4*\tan(c)^3 - d*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 4*d*\tan(d*x)^3 \\
& *\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) + d*\tan(d*x)*\tan(1/2*d*x)^4*\tan(1/2*c) \\
& ^4*\tan(c) + 4*d*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 - d*\tan(1/2* \\
& d*x)^4*\tan(1/2*c)^4*\tan(c)^2 - d*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c)^3 - 4*d*t \\
& \tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^3 - 4*d*\tan(d*x)^3*\tan(1/2*d*x)* \\
& \tan(1/2*c)^3*\tan(c)^3 - 4*d*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^3 - \\
& d*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c)^3 + 4*d*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2 \\
& *c)^3 - d*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - d*\tan(d*x)^3*\tan(1/2*d*x)^4*\tan(c) \\
& - 4*d*\tan(d*x)^3*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 4*d*\tan(d*x)^3*\tan(1/2* \\
& d*x)*\tan(1/2*c)^3*\tan(c) - 4*d*\tan(d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c) \\
& - d*\tan(d*x)^3*\tan(1/2*c)^4*\tan(c) + d*\tan(d*x)^2*\tan(1/2*d*x)^4*\tan(c)^2 + \\
& 4*d*\tan(d*x)^2*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c)^2 + 4*d*\tan(d*x)^2*\tan(1/2 \\
& *d*x)*\tan(1/2*c)^3*\tan(c)^2 + 4*d*\tan(1/2*d*x)^3*\tan(1/2*c)^3*\tan(c)^2 + d* \\
& \tan(d*x)^2*\tan(1/2*c)^4*\tan(c)^2 - d*\tan(d*x)*\tan(1/2*d*x)^4*\tan(c)^3 - 4*d \\
& *\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^3 - 4*d*\tan(d*x)*\tan(1/2*d*x)^3* \\
& \tan(1/2*c)*\tan(c)^3 - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^3 - d*t \\
& \tan(d*x)*\tan(1/2*c)^4*\tan(c)^3 + d*\tan(d*x)^2*\tan(1/2*d*x)^4 + 4*d*\tan(d*x)^ \\
& 2*\tan(1/2*d*x)^3*\tan(1/2*c) + 4*d*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)^3 + 4* \\
& d*\tan(1/2*d*x)^3*\tan(1/2*c)^3 + d*\tan(d*x)^2*\tan(1/2*c)^4 - d*\tan(d*x)*\tan(\\
& 1/2*d*x)^4*\tan(c) - 4*d*\tan(d*x)^3*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - 4*d*\tan \\
& (d*x)*\tan(1/2*d*x)^3*\tan(1/2*c)*\tan(c) - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2* \\
& c)^3*\tan(c) - d*\tan(d*x)*\tan(1/2*c)^4*\tan(c) + d*\tan(1/2*d*x)^4*\tan(c)^2 + \\
& 4*d*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + 4*d*\tan(1/2*d*x)^3*\tan(1/ \\
& 2*c)*\tan(c)^2 + 4*d*\tan(1/2*d*x)*\tan(1/2*c)^3*\tan(c)^2 + d*\tan(1/2*c)^4*\tan \\
& (c)^2 + d*\tan(d*x)^3*\tan(c)^3 - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) \\
& ^3 + d*\tan(1/2*d*x)^4 + 4*d*\tan(d*x)^2*\tan(1/2*d*x)*\tan(1/2*c) + 4*d*\tan(1/ \\
& 2*d*x)^3*\tan(1/2*c) + 4*d*\tan(1/2*d*x)*\tan(1/2*c)^3 + d*\tan(1/2*c)^4 + d*\tan \\
& (d*x)^3*\tan(c) - 4*d*\tan(d*x)*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c) - d*\tan(d*x)^ \\
& 2*\tan(c)^2 + 4*d*\tan(1/2*d*x)*\tan(1/2*c)*\tan(c)^2 + d*\tan(d*x)*\tan(c)^3 - d \\
& *\tan(d*x)^2 + 4*d*\tan(1/2*d*x)*\tan(1/2*c) + d*\tan(d*x)*\tan(c) - d*\tan(c)^2 \\
& - d)
\end{aligned}$$

3.157 $\int \cot^2(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=78

$$-\frac{a^2 \cot(c + dx)}{d} + a^2(-x) + \frac{2ab \cos(c + dx)}{d} - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} + \frac{b^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{b^2 x}{2}$$

[Out] $-(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)$

Rubi [A] time = 0.0855624, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2 \cot(c + dx)}{d} + a^2(-x) + \frac{2ab \cos(c + dx)}{d} - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} + \frac{b^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{b^2 x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]

[Out] $-(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)$

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^(n + 1)/2], x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + b \sin(c + dx))^2 dx &= \int (b^2 \cos^2(c + dx) + 2ab \cos(c + dx) \cot(c + dx) + a^2 \cot^2(c + dx)) dx \\ &= a^2 \int \cot^2(c + dx) dx + (2ab) \int \cos(c + dx) \cot(c + dx) dx + b^2 \int \cos^2(c + dx) dx \\ &= -\frac{a^2 \cot(c + dx)}{d} + \frac{b^2 \cos(c + dx) \sin(c + dx)}{2d} - a^2 \int 1 dx + \frac{1}{2} b^2 \int 1 dx - \frac{b^2}{2d} \int \cos(2(c + dx)) dx \\ &= -a^2 x + \frac{b^2 x}{2} + \frac{2ab \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx)}{d} + \frac{b^2 \cos(c + dx) \sin(c + dx)}{2d} \\ &= -a^2 x + \frac{b^2 x}{2} - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \cos(c + dx)}{d} - \frac{a^2 \cot(c + dx)}{d} + \frac{b^2 \sin^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.416328, size = 116, normalized size = 1.49

$$\frac{2a^2 \tan\left(\frac{1}{2}(c + dx)\right) - 2a^2 \cot\left(\frac{1}{2}(c + dx)\right) - 4a^2 c - 4a^2 dx + 8ab \cos(c + dx) + 8ab \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 8ab \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]

[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Cot[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]] + b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)

Maple [A] time = 0.039, size = 102, normalized size = 1.3

$$-a^2 x - \frac{a^2 \cot(dx + c)}{d} - \frac{a^2 c}{d} + 2 \frac{ab \cos(dx + c)}{d} + 2 \frac{ab \ln(\csc(dx + c) - \cot(dx + c))}{d} + \frac{b^2 \cos(dx + c) \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x)

[Out] -a^2*x - a^2*cot(d*x+c)/d - 1/d*a^2*c + 2*a*b*cos(d*x+c)/d + 2/d*a*b*ln(csc(d*x+c) - cot(d*x+c)) + 1/2*b^2*cos(d*x+c)*sin(d*x+c)/d + 1/2*b^2*x + 1/2/d*b^2*c

Maxima [A] time = 2.59493, size = 107, normalized size = 1.37

$$\frac{4 \left(dx + c + \frac{1}{\tan(dx+c)} \right) a^2 - (2 dx + 2 c + \sin(2 dx + 2 c)) b^2 - 4 ab(2 \cos(dx + c) - \log(\cos(dx + c) + 1) + \log(\cos(dx + c) - 1))}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d

Fricas [A] time = 1.64996, size = 308, normalized size = 3.95

$$\frac{b^2 \cos(dx + c)^3 + 2 ab \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 2 ab \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + (2 a^2 - b^2) \cos(dx + c)}{2 d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x + c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**2, x)

Giac [A] time = 2.23738, size = 200, normalized size = 2.56

$$\frac{4 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - (2 a^2 - b^2)(dx + c) - \frac{4 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a^2}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)} - \frac{2 \left(b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^3 - 4 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 + 1}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) - 2*(b^2*tan(1/2*d*x + 1/2*c))^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2/d

3.158 $\int \cot^4(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=133

$$-\frac{a^2 \cot^3(c + dx)}{3d} + \frac{a^2 \cot(c + dx)}{d} + a^2 x - \frac{3ab \cos(c + dx)}{d} - \frac{ab \cos(c + dx) \cot^2(c + dx)}{d} + \frac{3ab \tanh^{-1}(\cos(c + dx))}{d}$$

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
d*x]^3)/(3*d)
```

Rubi [A] time = 0.149396, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 2591, 288, 321, 203, 2592, 206, 3473, 8}

$$-\frac{a^2 \cot^3(c + dx)}{3d} + \frac{a^2 \cot(c + dx)}{d} + a^2 x - \frac{3ab \cos(c + dx)}{d} - \frac{ab \cos(c + dx) \cot^2(c + dx)}{d} + \frac{3ab \tanh^{-1}(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
d*x]^3)/(3*d)
```

Rule 2722

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(
x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_S
ymbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \cot^4(c + dx)(a + b \sin(c + dx))^2 dx &= \int (b^2 \cos^2(c + dx) \cot^2(c + dx) + 2ab \cos(c + dx) \cot^3(c + dx) + a^2 \cot^4(c + dx)) dx \\
 &= a^2 \int \cot^4(c + dx) dx + (2ab) \int \cos(c + dx) \cot^3(c + dx) dx + b^2 \int \cos^2(c + dx) dx \\
 &= -\frac{a^2 \cot^3(c + dx)}{3d} - a^2 \int \cot^2(c + dx) dx - \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2} dx, x, \cos(c + dx)\right)}{d} \\
 &= \frac{a^2 \cot(c + dx)}{d} + \frac{b^2 \cos^2(c + dx) \cot(c + dx)}{2d} - \frac{ab \cos(c + dx) \cot^2(c + dx)}{d} - \frac{a^2}{d} \\
 &= a^2 x - \frac{3ab \cos(c + dx)}{d} + \frac{a^2 \cot(c + dx)}{d} - \frac{3b^2 \cot(c + dx)}{2d} + \frac{b^2 \cos^2(c + dx) \cot(c + dx)}{2d} \\
 &= a^2 x - \frac{3b^2 x}{2} + \frac{3ab \tanh^{-1}(\cos(c + dx))}{d} - \frac{3ab \cos(c + dx)}{d} + \frac{a^2 \cot(c + dx)}{d} - \frac{3b^2}{2d}
 \end{aligned}$$

Mathematica [B] time = 6.23321, size = 293, normalized size = 2.2

$$\frac{(2a^2 - 3b^2)(c + dx)}{2d} + \frac{\csc\left(\frac{1}{2}(c + dx)\right)\left(4a^2 \cos\left(\frac{1}{2}(c + dx)\right) - 3b^2 \cos\left(\frac{1}{2}(c + dx)\right)\right)}{6d} + \frac{\sec\left(\frac{1}{2}(c + dx)\right)\left(3b^2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{6d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*cos[c + d*x])/d + ((4*a^2*cos[(c + d*x)/2] - 3*b^2*cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c + d*x)/2]^2)/(4*d) - (a^2*cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*sin[(c + d*x)/2] + 3*b^2*sin[(c + d*x)/2]))/(6*d) - (b^2*sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.049, size = 199, normalized size = 1.5

$$-\frac{a^2(\cot(dx+c))^3}{3d} + \frac{a^2 \cot(dx+c)}{d} + a^2x + \frac{a^2c}{d} - \frac{ab(\cos(dx+c))^5}{d(\sin(dx+c))^2} - \frac{ab(\cos(dx+c))^3}{d} - 3\frac{ab\cos(dx+c)}{d} - 3\frac{ab}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*a^2*c-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5-1/d*a*b*cos(d*x+c)^3-3*a*b*cos(d*x+c)/d-3/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^5-1/d*b^2*sin(d*x+c)*cos(d*x+c)^3-3/2*b^2*cos(d*x+c)*sin(d*x+c)/d-3/2*b^2*x-3/2/d*b^2*c
```

Maxima [A] time = 2.45315, size = 186, normalized size = 1.4

$$\frac{2\left(3dx + 3c + \frac{3 \tan(dx+c)^2 - 1}{\tan(dx+c)^3}\right)a^2 - 3\left(3dx + 3c + \frac{3 \tan(dx+c)^2 + 2}{\tan(dx+c)^3 + \tan(dx+c)}\right)b^2 + 3ab\left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2 - 1} - 4 \cos(dx+c) + 3 \log(\cos(dx+c) - 1)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.82669, size = 539, normalized size = 4.05

$$\frac{3b^2 \cos(dx+c)^5 + 4(2a^2 - 3b^2) \cos(dx+c)^3 + 9(ab \cos(dx+c)^2 - ab) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 9(a^2 \cos(dx+c)^2 - a^2 \sin(dx+c)^2)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)
```

)² - d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^2 \cot^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**4, x)

Giac [A] time = 2.05877, size = 325, normalized size = 2.44

$$a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 6 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 72 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 15 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 12 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/24*(a²*tan(1/2*d*x + 1/2*c)³ + 6*a*b*tan(1/2*d*x + 1/2*c)² - 72*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a²*tan(1/2*d*x + 1/2*c) + 12*b²*tan(1/2*d*x + 1/2*c) + 12*(2*a² - 3*b²)*(d*x + c) + 24*(b²*tan(1/2*d*x + 1/2*c)³ - 4*a*b*tan(1/2*d*x + 1/2*c)² - b²*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)² + 1)² + (132*a*b*tan(1/2*d*x + 1/2*c)³ + 15*a²*tan(1/2*d*x + 1/2*c)² - 12*b²*tan(1/2*d*x + 1/2*c)² - 6*a*b*tan(1/2*d*x + 1/2*c) - a²)/tan(1/2*d*x + 1/2*c)³/d

3.159 $\int \cot^6(c + dx)(a + b \sin(c + dx))^2 dx$

Optimal. Leaf size=202

$$\frac{a^2 \cot^5(c + dx)}{5d} + \frac{a^2 \cot^3(c + dx)}{3d} - \frac{a^2 \cot(c + dx)}{d} - a^2 x + \frac{15ab \cos(c + dx)}{4d} - \frac{ab \cos(c + dx) \cot^4(c + dx)}{2d} + \frac{5ab \cos^2(c + dx)}{2d}$$

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.169485, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 2591, 288, 302, 203, 2592, 321, 206, 3473, 8}

$$\frac{a^2 \cot^5(c + dx)}{5d} + \frac{a^2 \cot^3(c + dx)}{3d} - \frac{a^2 \cot(c + dx)}{d} - a^2 x + \frac{15ab \cos(c + dx)}{4d} - \frac{ab \cos(c + dx) \cot^4(c + dx)}{2d} + \frac{5ab \cos^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 2722

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, n]
```

$Q[m, 2*n - 1]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 2592

$\text{Int}[(a_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*\tan[(e_ + (f_)*(x_))]^{(n_)}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(\text{ff}*x)^{(m + n)}/(a^2 - \text{ff}^2*x^2)^{(n + 1)/2}, x], x, (a*\text{Sin}[e + f*x])/ff], x] \text{ ; FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n + 1)/2]$

Rule 321

$\text{Int}[(c_)*(x_)]^{(m_)}*((a_ + (b_)*(x_)]^{(n_)}])^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)]^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 3473

$\text{Int}[(b_)*\tan[(c_ + (d_)*(x_))]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n - 2)}, x], x] \text{ ; FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\sin(c+dx))^2 dx &= \int (b^2 \cos^2(c+dx) \cot^4(c+dx) + 2ab \cos(c+dx) \cot^5(c+dx) + a^2 \cot^6(c+dx)) dx \\
&= a^2 \int \cot^6(c+dx) dx + (2ab) \int \cos(c+dx) \cot^5(c+dx) dx + b^2 \int \cos^2(c+dx) dx \\
&= -\frac{a^2 \cot^5(c+dx)}{5d} - a^2 \int \cot^4(c+dx) dx - \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^3} dx, x, \cos(c+dx)\right)}{d} \\
&= \frac{a^2 \cot^3(c+dx)}{3d} + \frac{b^2 \cos^2(c+dx) \cot^3(c+dx)}{2d} - \frac{ab \cos(c+dx) \cot^4(c+dx)}{2d} \\
&= -\frac{a^2 \cot(c+dx)}{d} + \frac{5ab \cos(c+dx) \cot^2(c+dx)}{4d} + \frac{a^2 \cot^3(c+dx)}{3d} + \frac{b^2 \cos^2(c+dx)}{2d} \\
&= -a^2 x + \frac{15ab \cos(c+dx)}{4d} - \frac{a^2 \cot(c+dx)}{d} + \frac{5b^2 \cot(c+dx)}{2d} + \frac{5ab \cos(c+dx)}{4d} \\
&= -a^2 x + \frac{5b^2 x}{2} - \frac{15ab \tanh^{-1}(\cos(c+dx))}{4d} + \frac{15ab \cos(c+dx)}{4d} - \frac{a^2 \cot(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 1.09302, size = 351, normalized size = 1.74

$$(560b^2 - 368a^2) \cot\left(\frac{1}{2}(c+dx)\right) + 368a^2 \tan\left(\frac{1}{2}(c+dx)\right) + 96a^2 \sin^6\left(\frac{1}{2}(c+dx)\right) \csc^5(c+dx) - 328a^2 \sin^4\left(\frac{1}{2}(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]

[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 328*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Csc[c + d*x]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)

Maple [A] time = 0.052, size = 302, normalized size = 1.5

$$-\frac{a^2 (\cot(dx+c))^5}{5d} + \frac{a^2 (\cot(dx+c))^3}{3d} - \frac{a^2 \cot(dx+c)}{d} - a^2 x - \frac{a^2 c}{d} - \frac{ab (\cos(dx+c))^7}{2d (\sin(dx+c))^4} + \frac{3ab (\cos(dx+c))^7}{4d (\sin(dx+c))^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x)

[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*a^2*c-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7+3/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7+3/4/d*a*b*cos(d*x+c)^5+5/4/d*a*b*cos(d*x+c)^3+15/4*a*b*cos(d*x+c)/d+15/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*b^2/sin(d*x+c)*cos(d*x+c)^7+4/3/d*b^2*sin(d*x+c)*cos(d*x+c)^5+5/3/d*b^2*sin(d*x+c)*cos(d*x+c)^3+5/2*b^2*cos(d*x+c)*sin(d*x+c)/d+5/2*b^2*x+5/2/d*b^2*c

Maxima [A] time = 1.67638, size = 247, normalized size = 1.22

$$\frac{8 \left(15 dx + 15 c + \frac{15 \tan(dx+c)^4 - 5 \tan(dx+c)^2 + 3}{\tan(dx+c)^5} \right) a^2 - 20 \left(15 dx + 15 c + \frac{15 \tan(dx+c)^4 + 10 \tan(dx+c)^2 - 2}{\tan(dx+c)^5 + \tan(dx+c)^3} \right) b^2 + 15 ab \left(\frac{2(9 \cos(dx+c)^3 - 7 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 16 \cos(dx+c) + 15 \log(\cos(dx+c) + 1) - 15 \log(\cos(dx+c) - 1) \right)}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1))/d

Fricas [A] time = 1.63935, size = 786, normalized size = 3.89

$$60 b^2 \cos(dx + c)^7 + 92 (2 a^2 - 5 b^2) \cos(dx + c)^5 - 140 (2 a^2 - 5 b^2) \cos(dx + c)^3 + 225 (ab \cos(dx + c)^4 - 2 ab \cos(dx + c)^2 + a^2 b) \log(1/2 \cos(dx + c) + 1/2) \sin(dx + c) - 225 (a^2 b \cos(dx + c)^4 - 2 a^2 b \cos(dx + c)^2 + a^2 b) \log(-1/2 \cos(dx + c) + 1/2) \sin(dx + c) + 60 (2 a^2 - 5 b^2) \cos(dx + c) + 30 (2 (2 a^2 - 5 b^2) d x \cos(dx + c)^4 - 8 a^2 b \cos(dx + c)^5 - 4 (2 a^2 - 5 b^2) d x \cos(dx + c)^2 + 25 a^2 b \cos(dx + c)^3 + 2 (2 a^2 - 5 b^2) d x - 15 a^2 b \cos(dx + c)) \sin(dx + c) / ((d \cos(dx + c)^4 - 2 d \cos(dx + c)^2 + d) \sin(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4 - 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 2.77852, size = 455, normalized size = 2.25

$$3 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 15 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 35 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 20 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 240 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 15 a^2 \log\left(\frac{\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}\right) - 15 b^2 \log\left(\frac{\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{480}(3a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 + 15ab \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 35a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 20b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 240ab \tan(\frac{1}{2}dx + \frac{1}{2}c) + 1800ab \log(\tan(\frac{1}{2}dx + \frac{1}{2}c))) + 330a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 540b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 240(2a^2 - 5b^2)(dx + c) - 480(b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 4ab \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 4ab)/(\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 - (4110ab \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 + 330a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 540b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 240ab \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 35a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 20b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 15ab \tan(\frac{1}{2}dx + \frac{1}{2}c) + 3a^2)/\tan(\frac{1}{2}dx + \frac{1}{2}c)^5)/d$

3.160 $\int (a + b \sin(c + dx))^3 \tan^3(c + dx) dx$

Optimal. Leaf size=150

$$\frac{b(6a^2 + 5b^2) \sin(c + dx)}{2d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{(a + b)^2(2a + 5b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 5b)(a - b)^2 \log(\sin(c + dx))}{4d}$$

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]]/(4*d) + ((2*a - 5*b)*(a - b)^2*Log[1 + Sin[c + d*x]]/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (3*a*b^2*SIN[c + d*x]^2)/(2*d) + (b^3*SIN[c + d*x]^3)/(3*d) + (Sec[c + d*x]^2*(a + b*SIN[c + d*x])^3)/(2*d)
```

Rubi [A] time = 0.241513, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{b(6a^2 + 5b^2) \sin(c + dx)}{2d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{(a + b)^2(2a + 5b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 5b)(a - b)^2 \log(\sin(c + dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*SIN[c + d*x])^3*TAN[c + d*x]^3,x]
```

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]]/(4*d) + ((2*a - 5*b)*(a - b)^2*Log[1 + Sin[c + d*x]]/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (3*a*b^2*SIN[c + d*x]^2)/(2*d) + (b^3*SIN[c + d*x]^3)/(3*d) + (Sec[c + d*x]^2*(a + b*SIN[c + d*x])^3)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*SIN[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
```

*d)/(2*q), Int[1/(q + c*x), x], x] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int (a + b \sin(c + dx))^3 \tan^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{(b^2-x^2)^2} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\sec^2(c + dx)(a + b \sin(c + dx))^3}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2(-3b^4-2ab^2x-2b^2x^2)}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{\sec^2(c + dx)(a + b \sin(c + dx))^3}{2d} + \frac{\text{Subst}\left(\int (6a^2b^2 + 5b^4 + 6ab^2x + 2b^2x^2 - 2b^2x) dx, x, b \sin(c + dx)\right)}{2b^2d} \\ &= \frac{b(6a^2 + 5b^2) \sin(c + dx)}{2d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))^3}{2d} \\ &= \frac{b(6a^2 + 5b^2) \sin(c + dx)}{2d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d} + \frac{\sec^2(c + dx)(a + b \sin(c + dx))^3}{2d} \\ &= \frac{(a + b)^2(2a + 5b) \log(1 - \sin(c + dx))}{4d} + \frac{(2a - 5b)(a - b)^2 \log(1 + \sin(c + dx))}{4d} \end{aligned}$$

Mathematica [A] time = 0.263131, size = 141, normalized size = 0.94

$$\frac{12b(3a^2 + 2b^2) \sin(c + dx) + 18ab^2 \sin^2(c + dx) + \frac{3(a-b)^3}{\sin(c+dx)+1} - \frac{3(a+b)^3}{\sin(c+dx)-1} + 3(2a - 5b)(a - b)^2 \log(\sin(c + dx) + 1) + \frac{3(2a - 5b)(a - b)^2 \log(\sin(c + dx) - 1)}{12d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]

[Out] (3*(a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(2*a - 5*b)*(a - b)^2*Log[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c + d*x]) + 12*b*(3*a^2 + 2*b^2)*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x]^2 + 4*b^3*Sin[c + d*x]^3 + (3*(a - b)^3)/(1 + Sin[c + d*x]))/(12*d)

Maple [B] time = 0.056, size = 279, normalized size = 1.9

$$\frac{a^3 (\tan(dx + c))^2}{2d} + \frac{a^3 \ln(\cos(dx + c))}{d} + \frac{3a^2b (\sin(dx + c))^5}{2d (\cos(dx + c))^2} + \frac{3a^2b (\sin(dx + c))^3}{2d} + \frac{9a^2b \sin(dx + c)}{2d} - \frac{9a^2b \ln(\sec(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x)

[Out] 1/2/d*a^3*tan(d*x+c)^2+1/d*a^3*ln(cos(d*x+c))+3/2/d*a^2*b*sin(d*x+c)^5/cos(d*x+c)^2+3/2/d*a^2*b*sin(d*x+c)^3+9/2*a^2*b*sin(d*x+c)/d-9/2/d*a^2*b*ln(sec(dx + c))

$$(d*x+c)+\tan(d*x+c))+3/2/d*a*b^2*\sin(d*x+c)^6/\cos(d*x+c)^2+3/2/d*a*b^2*\sin(d*x+c)^4+3*a*b^2*\sin(d*x+c)^2/d+6/d*a*b^2*\ln(\cos(d*x+c))+1/2/d*b^3*\sin(d*x+c)^7/\cos(d*x+c)^2+1/2/d*b^3*\sin(d*x+c)^5+5/6*b^3*\sin(d*x+c)^3/d+5/2/d*b^3*\sin(d*x+c)-5/2/d*b^3*\ln(\sec(d*x+c)+\tan(d*x+c))$$

Maxima [A] time = 1.54646, size = 219, normalized size = 1.46

$$\frac{4b^3 \sin(dx + c)^3 + 18ab^2 \sin(dx + c)^2 + 3(2a^3 - 9a^2b + 12ab^2 - 5b^3) \log(\sin(dx + c) + 1) + 3(2a^3 + 9a^2b + 12ab^2 - 5b^3) \log(\sin(dx + c) - 1) + 12(3a^2b + 2b^3) \sin(dx + c) - 6(a^3 + 3ab^2 + (3a^2b + b^3) \sin(dx + c)) / (\sin(dx + c)^2 - 1)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 3*(2*a^3 - 9*a^2*b + 12*a*b^2 - 5*b^3)*log(sin(d*x + c) + 1) + 3*(2*a^3 + 9*a^2*b + 12*a*b^2 + 5*b^3)*log(sin(d*x + c) - 1) + 12*(3*a^2*b + 2*b^3)*sin(d*x + c) - 6*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^2 - 1)/d

Fricas [A] time = 1.72138, size = 470, normalized size = 3.13

$$\frac{18ab^2 \cos(dx + c)^4 - 9ab^2 \cos(dx + c)^2 - 3(2a^3 - 9a^2b + 12ab^2 - 5b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 3(2a^3 + 9a^2b + 12ab^2 - 5b^3) \cos(dx + c)^2 \log(\sin(dx + c) - 1) + 12(3a^2b + 2b^3) \sin(dx + c) - 6(a^3 + 3ab^2 + (3a^2b + b^3) \sin(dx + c)) / (d \cos(dx + c)^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")

[Out] -1/12*(18*a*b^2*cos(d*x + c)^4 - 9*a*b^2*cos(d*x + c)^2 - 3*(2*a^3 - 9*a^2*b + 12*a*b^2 - 5*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^3 + 9*a^2*b + 12*a*b^2 + 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 6*a^3 - 18*a*b^2 + 2*(2*b^3*cos(d*x + c)^4 - 9*a^2*b - 3*b^3 - 2*(9*a^2*b + 7*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**3,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.161 $\int (a + b \sin(c + dx))^3 \tan(c + dx) dx$

Optimal. Leaf size=105

$$\frac{b(3a^2 + b^2) \sin(c + dx)}{d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{(a - b)^3 \log(\sin(c + dx) + 1)}{2d} - \frac{(a + b)^3 \log(1 - \sin(c + dx))}{2d} - \frac{b^3 \sin^3(c + dx)}{3d}$$

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*SIN[c + d*x]^2)/(2*d)
- (b^3*SIN[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.113594, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2721, 801, 633, 31}

$$\frac{b(3a^2 + b^2) \sin(c + dx)}{d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{(a - b)^3 \log(\sin(c + dx) + 1)}{2d} - \frac{(a + b)^3 \log(1 - \sin(c + dx))}{2d} - \frac{b^3 \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*SIN[c + d*x])^3*TAN[c + d*x],x]
```

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*SIN[c + d*x]^2)/(2*d)
- (b^3*SIN[c + d*x]^3)/(3*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p
_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*SIN[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[(((d_) + (e_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[(((a_) + (b_)*(x_))^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx))^3 \tan(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{b^2-x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-3a^2 - b^2 - 3ax - x^2 + \frac{3a^2b^2 + b^4 + a(a^2 + 3b^2)x}{b^2 - x^2}\right) dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{b(3a^2 + b^2) \sin(c + dx)}{d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d} + \frac{\text{Subst}\left(\int \frac{x^5}{b^2 - x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{b(3a^2 + b^2) \sin(c + dx)}{d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d} + \frac{(a - b)^3 \text{Subst}\left(\int \frac{x^5}{b^2 - x^2} dx, x, b \sin(c + dx)\right)}{d} \\
&= -\frac{(a + b)^3 \log(1 - \sin(c + dx))}{2d} - \frac{(a - b)^3 \log(1 + \sin(c + dx))}{2d} - \frac{b(3a^2 + b^2) \sin^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.189961, size = 90, normalized size = 0.86

$$\frac{6b(3a^2 + b^2) \sin(c + dx) + 9ab^2 \sin^2(c + dx) + 3((a - b)^3 \log(\sin(c + dx) + 1) + (a + b)^3 \log(1 - \sin(c + dx))) + 2b^3 \sin^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x], x]

[Out] -(3*((a + b)^3*Log[1 - Sin[c + d*x]] + (a - b)^3*Log[1 + Sin[c + d*x]])) + 6*b*(3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c + d*x]^3)/(6*d)

Maple [A] time = 0.046, size = 139, normalized size = 1.3

$$-\frac{a^3 \ln(\cos(dx + c))}{d} - 3 \frac{a^2 b \sin(dx + c)}{d} + 3 \frac{a^2 b \ln(\sec(dx + c) + \tan(dx + c))}{d} - \frac{3(\sin(dx + c))^2 ab^2}{2d} - 3 \frac{ab^2 \ln(\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^3*tan(d*x+c), x)

[Out] -1/d*a^3*ln(cos(d*x+c))-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3/2*a*b^2*sin(d*x+c)^2/d-3/d*a*b^2*ln(cos(d*x+c))-1/3*b^3*sin(d*x+c)^3/d-1/d*b^3*sin(d*x+c)+1/d*b^3*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.70671, size = 153, normalized size = 1.46

$$\frac{2b^3 \sin(dx + c)^3 + 9ab^2 \sin(dx + c)^2 + 3(a^3 - 3a^2b + 3ab^2 - b^3) \log(\sin(dx + c) + 1) + 3(a^3 + 3a^2b + 3ab^2 + b^3) \log(\sin(dx + c) - 1)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c), x, algorithm="maxima")

[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x + c) - 1))

$$\sin(dx + c) - 1) + 6*(3*a^2*b + b^3)*\sin(dx + c))/d$$

Fricas [A] time = 1.60646, size = 277, normalized size = 2.64

$$\frac{9ab^2 \cos(dx + c)^2 - 3(a^3 - 3a^2b + 3ab^2 - b^3) \log(\sin(dx + c) + 1) - 3(a^3 + 3a^2b + 3ab^2 + b^3) \log(-\sin(dx + c) + 1)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a*b^2*cos(d*x + c)^2 - 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*x + c) + 1) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x + c) + 1) + 2*(b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^3 \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))^3*tan(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.162 $\int \cot(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=67

$$\frac{3a^2b \sin(c + dx)}{d} + \frac{a^3 \log(\sin(c + dx))}{d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d}$$

[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0449595, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 43}

$$\frac{3a^2b \sin(c + dx)}{d} + \frac{a^3 \log(\sin(c + dx))}{d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]

[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + b \sin(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(3a^2 + \frac{a^3}{x} + 3ax + x^2\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{a^3 \log(\sin(c + dx))}{d} + \frac{3a^2b \sin(c + dx)}{d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0252867, size = 67, normalized size = 1.

$$\frac{3a^2b \sin(c + dx)}{d} + \frac{a^3 \log(\sin(c + dx))}{d} + \frac{3ab^2 \sin^2(c + dx)}{2d} + \frac{b^3 \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]

[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)

Maple [A] time = 0.027, size = 64, normalized size = 1.

$$\frac{a^3 \ln(\sin(dx + c))}{d} + 3 \frac{a^2 b \sin(dx + c)}{d} + \frac{3 (\sin(dx + c))^2 ab^2}{2d} + \frac{(\sin(dx + c))^3 b^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^3,x)

[Out] a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*sin(d*x+c)^3/d

Maxima [A] time = 1.56722, size = 77, normalized size = 1.15

$$\frac{2b^3 \sin(dx + c)^3 + 9ab^2 \sin(dx + c)^2 + 6a^3 \log(\sin(dx + c)) + 18a^2b \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)) + 18*a^2*b*sin(d*x + c))/d

Fricas [A] time = 1.49174, size = 158, normalized size = 2.36

$$\frac{9ab^2 \cos(dx + c)^2 - 6a^3 \log\left(\frac{1}{2} \sin(dx + c)\right) + 2(b^3 \cos(dx + c)^2 - 9a^2b - b^3) \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/6*(9*a*b^2*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(b^3*cos(d*x + c)^2 - 9*a^2*b - b^3)*sin(d*x + c))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^3 \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**3,x)

[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x), x)

Giac [A] time = 1.99966, size = 78, normalized size = 1.16

$$\frac{2b^3 \sin(dx + c)^3 + 9ab^2 \sin(dx + c)^2 + 6a^3 \log(|\sin(dx + c)|) + 18a^2b \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x + c))) + 18*a^2*b*sin(d*x + c))/d

3.163 $\int \cot^3(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=116

$$\frac{b(3a^2 - b^2) \sin(c + dx)}{d} - \frac{a(a^2 - 3b^2) \log(\sin(c + dx))}{d} - \frac{3a^2 b \csc(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d}$$

[Out] $(-3*a^2*b*\text{Csc}[c + d*x])/d - (a^3*\text{Csc}[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*\text{Log}[\text{Sin}[c + d*x]])/d - (b*(3*a^2 - b^2)*\text{Sin}[c + d*x])/d - (3*a*b^2*\text{Sin}[c + d*x]^2)/(2*d) - (b^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0943571, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$\frac{b(3a^2 - b^2) \sin(c + dx)}{d} - \frac{a(a^2 - 3b^2) \log(\sin(c + dx))}{d} - \frac{3a^2 b \csc(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} - \frac{3ab^2 \sin^2(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^3*(a + b*\text{Sin}[c + d*x])^3, x]$

[Out] $(-3*a^2*b*\text{Csc}[c + d*x])/d - (a^3*\text{Csc}[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*\text{Log}[\text{Sin}[c + d*x]])/d - (b*(3*a^2 - b^2)*\text{Sin}[c + d*x])/d - (3*a*b^2*\text{Sin}[c + d*x]^2)/(2*d) - (b^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 2721

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*\text{tan}[(e_.) + (f_.)*(x_.)]^{(p_.)}, x_Symbol] :> \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p + 1)/2}], x], x, b*\text{Sin}[e + f*x], x] /;$ FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((f_.) + (g_.)*(x_.)^{(n_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + b \sin(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3(b^2-x^2)}{x^3} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-3a^2\left(1 - \frac{b^2}{3a^2}\right) + \frac{a^3b^2}{x^3} + \frac{3a^2b^2}{x^2} + \frac{-a^3+3ab^2}{x} - 3ax - x^2\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= -\frac{3a^2b \csc(c + dx)}{d} - \frac{a^3 \csc^2(c + dx)}{2d} - \frac{a(a^2 - 3b^2) \log(\sin(c + dx))}{d} - \frac{b(3a^2 - b^2) \sin^2(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.320514, size = 97, normalized size = 0.84

$$\frac{-6b(b^2 - 3a^2) \sin(c + dx) + 6a(a^2 - 3b^2) \log(\sin(c + dx)) + 18a^2b \csc(c + dx) + 3a^3 \csc^2(c + dx) + 9ab^2 \sin^2(c + dx) - b^3 \sin^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]

[Out] $-(18*a^2*b*\text{Csc}[c + d*x] + 3*a^3*\text{Csc}[c + d*x]^2 + 6*a*(a^2 - 3*b^2)*\text{Log}[\text{Sin}[c + d*x]]) - 6*b*(-3*a^2 + b^2)*\text{Sin}[c + d*x] + 9*a*b^2*\text{Sin}[c + d*x]^2 + 2*b^3*\text{Sin}[c + d*x]^3)/(6*d)$

Maple [A] time = 0.065, size = 165, normalized size = 1.4

$$\frac{a^3 (\cot(dx + c))^2}{2d} - \frac{a^3 \ln(\sin(dx + c))}{d} - 3 \frac{a^2 b (\cos(dx + c))^4}{d \sin(dx + c)} - 3 \frac{a^2 b \sin(dx + c) (\cos(dx + c))^2}{d} - 6 \frac{a^2 b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x)

[Out] $-1/2/d*a^3*\cot(d*x+c)^2 - a^3*\ln(\sin(d*x+c))/d - 3/d*a^2*b/\sin(d*x+c)*\cos(d*x+c)^4 - 3/d*a^2*b*\sin(d*x+c)*\cos(d*x+c)^2 - 6*a^2*b*\sin(d*x+c)/d + 3/2/d*a*b^2*\cos(d*x+c)^2 + 3/d*a*b^2*\ln(\sin(d*x+c)) + 1/3/d*b^3*\cos(d*x+c)^2*\sin(d*x+c) + 2/3/d*b^3*\sin(d*x+c)$

Maxima [A] time = 1.89972, size = 132, normalized size = 1.14

$$\frac{2b^3 \sin(dx + c)^3 + 9ab^2 \sin(dx + c)^2 + 6(a^3 - 3ab^2) \log(\sin(dx + c)) + 6(3a^2b - b^3) \sin(dx + c) + \frac{3(6a^2b \sin(dx + c))}{\sin(dx + c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/6*(2*b^3*\sin(d*x + c)^3 + 9*a*b^2*\sin(d*x + c)^2 + 6*(a^3 - 3*a*b^2)*\log(\sin(d*x + c)) + 6*(3*a^2*b - b^3)*\sin(d*x + c) + 3*(6*a^2*b*\sin(d*x + c) + a^3)/\sin(d*x + c)^2)/d$

Fricas [A] time = 1.56308, size = 358, normalized size = 3.09

$$\frac{18ab^2 \cos(dx + c)^4 - 27ab^2 \cos(dx + c)^2 + 6a^3 + 9ab^2 + 12(a^3 - 3ab^2 - (a^3 - 3ab^2) \cos(dx + c)^2) \log\left(\frac{1}{2} \sin(dx + c)\right)}{12(d \cos(dx + c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $1/12*(18*a*b^2*\cos(d*x + c)^4 - 27*a*b^2*\cos(d*x + c)^2 + 6*a^3 + 9*a*b^2 + 12*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*\cos(d*x + c)^2)*\log(1/2*\sin(d*x + c)) + 4*(b^3*\cos(d*x + c)^4 + 18*a^2*b - 2*b^3 - (9*a^2*b - b^3)*\cos(d*x + c)^2)*\sin(d*x + c))/(d*\cos(d*x + c)^2 - d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^3 \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**3,x)

[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**3, x)

Giac [A] time = 2.0221, size = 177, normalized size = 1.53

$$\frac{2b^3 \sin(dx + c)^3 + 9ab^2 \sin(dx + c)^2 + 18a^2b \sin(dx + c) - 6b^3 \sin(dx + c) + 6(a^3 - 3ab^2) \log(|\sin(dx + c)|) - \frac{3(3a^3}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 18*a^2*b*sin(d*x + c) - 6*b^3*sin(d*x + c) + 6*(a^3 - 3*a*b^2)*log(abs(sin(d*x + c)))) - 3*(3*a^3*sin(d*x + c)^2 - 9*a*b^2*sin(d*x + c)^2 - 6*a^2*b*sin(d*x + c) - a^3)/sin(d*x + c)^2/d

3.164 $\int \cot^5(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=165

$$\frac{b(3a^2 - 2b^2) \sin(c + dx)}{d} + \frac{a(2a^2 - 3b^2) \csc^2(c + dx)}{2d} + \frac{b(6a^2 - b^2) \csc(c + dx)}{d} + \frac{a(a^2 - 6b^2) \log(\sin(c + dx))}{d} - \frac{a^2}{d}$$

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
+ d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.14103, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 948}

$$\frac{b(3a^2 - 2b^2) \sin(c + dx)}{d} + \frac{a(2a^2 - 3b^2) \csc^2(c + dx)}{2d} + \frac{b(6a^2 - b^2) \csc(c + dx)}{d} + \frac{a(a^2 - 6b^2) \log(\sin(c + dx))}{d} - \frac{a^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
+ d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p
_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 948

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^
2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + b \sin(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3(b^2-x^2)^2}{x^5} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(3a^2\left(1 - \frac{2b^2}{3a^2}\right) + \frac{a^3b^4}{x^5} + \frac{3a^2b^4}{x^4} + \frac{-2a^3b^2+3ab^4}{x^3} + \frac{-6a^2b^2+b^4}{x^2} + \frac{a^3-6ab^2}{x} + 3\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{b(6a^2 - b^2) \csc(c + dx)}{d} + \frac{a(2a^2 - 3b^2) \csc^2(c + dx)}{2d} - \frac{a^2b \csc^3(c + dx)}{d} - \frac{a^3}{d} \end{aligned}$$

Mathematica [A] time = 1.16383, size = 144, normalized size = 0.87

$$\frac{6a(2a^2 - 3b^2) \csc^2(c + dx) - 12b(b^2 - 6a^2) \csc(c + dx) + 2(6b(3a^2 - 2b^2) \sin(c + dx) + 6a(a^2 - 6b^2) \log(\sin(c + dx)))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]

[Out] (-12*b*(-6*a^2 + b^2)*Csc[c + d*x] + 6*a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2 - 12*a^2*b*Csc[c + d*x]^3 - 3*a^3*Csc[c + d*x]^4 + 2*(6*a*(a^2 - 6*b^2)*Log[Sin[c + d*x]] + 6*b*(3*a^2 - 2*b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c + d*x]^3))/(12*d)

Maple [B] time = 0.062, size = 316, normalized size = 1.9

$$-\frac{a^3 (\cot(dx + c))^4}{4d} + \frac{a^3 (\cot(dx + c))^2}{2d} + \frac{a^3 \ln(\sin(dx + c))}{d} - \frac{a^2 b (\cos(dx + c))^6}{d (\sin(dx + c))^3} + 3 \frac{a^2 b (\cos(dx + c))^6}{d \sin(dx + c)} + 8 \frac{a^2 b \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x)

[Out] -1/4/d*a^3*cot(d*x+c)^4+1/2/d*a^3*cot(d*x+c)^2+a^3*ln(sin(d*x+c))/d-1/d*a^2*b/sin(d*x+c)^3*cos(d*x+c)^6+3/d*a^2*b/sin(d*x+c)*cos(d*x+c)^6+8*a^2*b*sin(d*x+c)/d+3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^4+4/d*a^2*b*sin(d*x+c)*cos(d*x+c)^2-3/2/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^6-3/2/d*a*b^2*cos(d*x+c)^4-3/d*a*b^2*cos(d*x+c)^2-6/d*a*b^2*ln(sin(d*x+c))-1/d*b^3/sin(d*x+c)*cos(d*x+c)^6-8/3/d*b^3*sin(d*x+c)-1/d*b^3*sin(d*x+c)*cos(d*x+c)^4-4/3/d*b^3*cos(d*x+c)^2*sin(d*x+c)

Maxima [A] time = 1.35412, size = 192, normalized size = 1.16

$$\frac{4b^3 \sin(dx + c)^3 + 18ab^2 \sin(dx + c)^2 + 12(a^3 - 6ab^2) \log(\sin(dx + c)) + 12(3a^2b - 2b^3) \sin(dx + c) - \frac{3(4a^2b \sin(dx + c))}{12d}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 12*(a^3 - 6*a*b^2)*log(sin(d*x + c)) + 12*(3*a^2*b - 2*b^3)*sin(d*x + c) - 3*(4*a^2*b*sin(d*x + c) - 4*(6*a^2*b - b^3)*sin(d*x + c)^3 + a^3 - 2*(2*a^3 - 3*a*b^2)*sin(d*x + c)^2)/sin(d*x + c)^4/d

Fricas [A] time = 1.60026, size = 536, normalized size = 3.25

$$18ab^2 \cos(dx + c)^6 - 45ab^2 \cos(dx + c)^4 - 9a^3 + 9ab^2 + 6(2a^3 + 3ab^2) \cos(dx + c)^2 - 12((a^3 - 6ab^2) \cos(dx + c))^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/12*(18*a*b^2*\cos(d*x + c)^6 - 45*a*b^2*\cos(d*x + c)^4 - 9*a^3 + 9*a*b^2 \\ & + 6*(2*a^3 + 3*a*b^2)*\cos(d*x + c)^2 - 12*((a^3 - 6*a*b^2)*\cos(d*x + c)^4 + \\ & a^3 - 6*a*b^2 - 2*(a^3 - 6*a*b^2)*\cos(d*x + c)^2)*\log(1/2*\sin(d*x + c)) + \\ & 4*(b^3*\cos(d*x + c)^6 - 3*(3*a^2*b - b^3)*\cos(d*x + c)^4 - 24*a^2*b + 8*b^3 \\ & + 12*(3*a^2*b - b^3)*\cos(d*x + c)^2)*\sin(d*x + c))/(d*\cos(d*x + c)^4 - 2*d \\ & *\cos(d*x + c)^2 + d) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 2.1406, size = 250, normalized size = 1.52

$$\frac{4b^3 \sin(dx + c)^3 + 18ab^2 \sin(dx + c)^2 + 36a^2b \sin(dx + c) - 24b^3 \sin(dx + c) + 12(a^3 - 6ab^2) \log(|\sin(dx + c)|) - 12d}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/12*(4*b^3*\sin(d*x + c)^3 + 18*a*b^2*\sin(d*x + c)^2 + 36*a^2*b*\sin(d*x + c) \\ &) - 24*b^3*\sin(d*x + c) + 12*(a^3 - 6*a*b^2)*\log(\text{abs}(\sin(d*x + c))) - (25*a \\ & ^3*\sin(d*x + c)^4 - 150*a*b^2*\sin(d*x + c)^4 - 72*a^2*b*\sin(d*x + c)^3 + 12 \\ & *b^3*\sin(d*x + c)^3 - 12*a^3*\sin(d*x + c)^2 + 18*a*b^2*\sin(d*x + c)^2 + 12* \\ & a^2*b*\sin(d*x + c) + 3*a^3)/\sin(d*x + c)^4)/d \end{aligned}$$

3.165 $\int (a + b \sin(c + dx))^3 \tan^4(c + dx) dx$

Optimal. Leaf size=220

$$-\frac{3a^2b \cos(c + dx)}{d} + \frac{a^2b \sec^3(c + dx)}{d} - \frac{6a^2b \sec(c + dx)}{d} + \frac{a^3 \tan^3(c + dx)}{3d} - \frac{a^3 \tan(c + dx)}{d} + a^3x + \frac{5ab^2 \tan^3(c + dx)}{2d}$$

[Out] $a^3x + (15ab^2x)/2 - (3a^2b \cos[c + dx])/d - (3b^3 \cos[c + dx])/d + (b^3 \cos[c + dx]^3)/(3d) - (6a^2b \sec[c + dx])/d - (3b^3 \sec[c + dx])/d + (a^2b \sec[c + dx]^3)/d + (b^3 \sec[c + dx]^3)/(3d) - (a^3 \tan[c + dx])/d - (15ab^2 \tan[c + dx])/(2d) + (a^3 \tan[c + dx]^3)/(3d) + (5ab^2 \tan[c + dx]^3)/(2d) - (3ab^2 \sin[c + dx]^2 \tan[c + dx]^3)/(2d)$

Rubi [A] time = 0.224299, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$-\frac{3a^2b \cos(c + dx)}{d} + \frac{a^2b \sec^3(c + dx)}{d} - \frac{6a^2b \sec(c + dx)}{d} + \frac{a^3 \tan^3(c + dx)}{3d} - \frac{a^3 \tan(c + dx)}{d} + a^3x + \frac{5ab^2 \tan^3(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]

[Out] $a^3x + (15ab^2x)/2 - (3a^2b \cos[c + dx])/d - (3b^3 \cos[c + dx])/d + (b^3 \cos[c + dx]^3)/(3d) - (6a^2b \sec[c + dx])/d - (3b^3 \sec[c + dx])/d + (a^2b \sec[c + dx]^3)/d + (b^3 \sec[c + dx]^3)/(3d) - (a^3 \tan[c + dx])/d - (15ab^2 \tan[c + dx])/(2d) + (a^3 \tan[c + dx]^3)/(3d) + (5ab^2 \tan[c + dx]^3)/(2d) - (3ab^2 \sin[c + dx]^2 \tan[c + dx]^3)/(2d)$

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2590

Int[sin[(e_) + (f_)*(x_)^(m_)]*tan[(e_) + (f_)*(x_)^(n_)], x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + b \sin(c + dx))^3 \tan^4(c + dx) dx &= \int (a^3 \tan^4(c + dx) + 3a^2b \sin(c + dx) \tan^4(c + dx) + 3ab^2 \sin^2(c + dx) \tan^4(c + dx) + b^3 \sin^3(c + dx) \tan^4(c + dx)) dx \\
&= a^3 \int \tan^4(c + dx) dx + (3a^2b) \int \sin(c + dx) \tan^4(c + dx) dx + (3ab^2) \int \sin^2(c + dx) \tan^4(c + dx) dx + b^3 \int \sin^3(c + dx) \tan^4(c + dx) dx \\
&= \frac{a^3 \tan^3(c + dx)}{3d} - a^3 \int \tan^2(c + dx) dx - \frac{(3a^2b) \text{Subst}\left(\int \frac{(1-x^2)^2}{x^4} dx, x, \cos(c + dx)\right)}{d} \\
&= -\frac{a^3 \tan(c + dx)}{d} + \frac{a^3 \tan^3(c + dx)}{3d} - \frac{3ab^2 \sin^2(c + dx) \tan^3(c + dx)}{2d} + a^3 \int \tan^2(c + dx) dx \\
&= a^3 x - \frac{3a^2b \cos(c + dx)}{d} - \frac{3b^3 \cos(c + dx)}{d} + \frac{b^3 \cos^3(c + dx)}{3d} - \frac{6a^2b \sec(c + dx)}{d} \\
&= a^3 x - \frac{3a^2b \cos(c + dx)}{d} - \frac{3b^3 \cos(c + dx)}{d} + \frac{b^3 \cos^3(c + dx)}{3d} - \frac{6a^2b \sec(c + dx)}{d} \\
&= a^3 x + \frac{15}{2} ab^2 x - \frac{3a^2b \cos(c + dx)}{d} - \frac{3b^3 \cos(c + dx)}{d} + \frac{b^3 \cos^3(c + dx)}{3d} - \frac{6a^2b \sec(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.678412, size = 226, normalized size = 1.03

$$\frac{\sec^3(c + dx) (36a (2a^2 + 15b^2) (c + dx) \cos(c + dx) - 3 (144a^2b + 91b^3) \cos(2(c + dx)) - 36a^2b \cos(4(c + dx)) - 300a^2)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*SIN[c + d*x])^3*TAN[c + d*x]^4,x]

[Out] (Sec[c + d*x]^3*(-300*a^2*b - 210*b^3 + 36*a*(2*a^2 + 15*b^2)*(c + d*x)*Cos[c + d*x] - 3*(144*a^2*b + 91*b^3)*Cos[2*(c + d*x)] + 24*a^3*c*Cos[3*(c + d*x)] + 180*a*b^2*c*Cos[3*(c + d*x)] + 24*a^3*d*x*Cos[3*(c + d*x)] + 180*a*b^2*d*x*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos[4*(c + d*x)] + b^3*Cos[6*(c + d*x)] - 90*a*b^2*SIN[c + d*x] - 32*a^3*SIN[3*(c + d*x)] - 195*a*b^2*SIN[3*(c + d*x)] - 9*a*b^2*SIN[5*(c + d*x)]))/(96*d)

Maple [A] time = 0.058, size = 268, normalized size = 1.2

$$\frac{1}{d} \left(a^3 \left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c) + dx+c \right) + 3a^2b \left(\frac{1}{3} \frac{(\sin(dx+c))^6}{(\cos(dx+c))^3} - \frac{(\sin(dx+c))^6}{\cos(dx+c)} - (8/3 + (\sin(dx+c))^4 + 4) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x)

[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^2*b*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+b^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))

Maxima [A] time = 1.82353, size = 225, normalized size = 1.02

$$\frac{2 \left(\tan(dx+c)^3 + 3dx + 3c - 3 \tan(dx+c) \right) a^3 + 3 \left(2 \tan(dx+c)^3 + 15dx + 15c - \frac{3 \tan(dx+c)}{\tan(dx+c)^2 + 1} - 12 \tan(dx+c) \right) ab^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")

[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c))/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^3 - 6*a^2*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d

Fricas [A] time = 1.59267, size = 369, normalized size = 1.68

$$\frac{2b^3 \cos(dx+c)^6 + 3(2a^3 + 15ab^2)dx \cos(dx+c)^3 - 18(a^2b + b^3) \cos(dx+c)^4 + 6a^2b + 2b^3 - 18(2a^2b + b^3) \cos(dx+c)}{6d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")

[Out] 1/6*(2*b^3*cos(d*x + c)^6 + 3*(2*a^3 + 15*a*b^2)*d*x*cos(d*x + c)^3 - 18*(a^2*b + b^3)*cos(d*x + c)^4 + 6*a^2*b + 2*b^3 - 18*(2*a^2*b + b^3)*cos(d*x + c))

$$c)^2 - (9*a*b^2*\cos(d*x + c)^4 - 2*a^3 - 6*a*b^2 + 2*(4*a^3 + 21*a*b^2)*\cos(d*x + c)^2*\sin(d*x + c))/(d*\cos(d*x + c)^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**4,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

3.166 $\int (a + b \sin(c + dx))^3 \tan^2(c + dx) dx$

Optimal. Leaf size=146

$$\frac{3a^2b \cos(c + dx)}{d} + \frac{3a^2b \sec(c + dx)}{d} + \frac{a^3 \tan(c + dx)}{d} + a^3(-x) + \frac{9ab^2 \tan(c + dx)}{2d} - \frac{3ab^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{9}{2}$$

[Out] $-(a^3x) - (9ab^2x)/2 + (3a^2b \cos[c + dx])/d + (2b^3 \cos[c + dx])/d - (b^3 \cos[c + dx]^3)/(3d) + (3a^2b \sec[c + dx])/d + (b^3 \sec[c + dx])/d + (a^3 \tan[c + dx])/d + (9ab^2 \tan[c + dx])/(2d) - (3ab^2 \sin[c + dx]^2 \tan[c + dx])/(2d)$

Rubi [A] time = 0.169452, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{3a^2b \cos(c + dx)}{d} + \frac{3a^2b \sec(c + dx)}{d} + \frac{a^3 \tan(c + dx)}{d} + a^3(-x) + \frac{9ab^2 \tan(c + dx)}{2d} - \frac{3ab^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{9}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b \sin[c + dx])^3 \tan[c + dx]^2, x]$

[Out] $-(a^3x) - (9ab^2x)/2 + (3a^2b \cos[c + dx])/d + (2b^3 \cos[c + dx])/d - (b^3 \cos[c + dx]^3)/(3d) + (3a^2b \sec[c + dx])/d + (b^3 \sec[c + dx])/d + (a^3 \tan[c + dx])/d + (9ab^2 \tan[c + dx])/(2d) - (3ab^2 \sin[c + dx]^2 \tan[c + dx])/(2d)$

Rule 2722

$\text{Int}[(a + b \sin[e + fx])^m (g \tan[e + fx])^p, x] \text{ ; FreeQ}\{a, b, e, f, g, p, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 3473

$\text{Int}[(b \tan[c + dx])^n, x] \text{ ; FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a x, x] \text{ ; FreeQ}[a, x]$

Rule 2590

$\text{Int}[\sin[e + fx]^m \tan[e + fx]^n, x] \text{ ; FreeQ}\{e, f, x\} \ \&\& \ \text{IntegersQ}[m, n, (m + n - 1)/2]$

Rule 14

$\text{Int}[u (c + dx)^m, x] \text{ ; FreeQ}\{c, m, x\} \ \&\& \ \text{SumQ}[u] \ \&\& \ \text{!LinearQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (a + b \sin[v])]$

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]]
/; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (a + b \sin(c + dx))^3 \tan^2(c + dx) dx &= \int (a^3 \tan^2(c + dx) + 3a^2b \sin(c + dx) \tan^2(c + dx) + 3ab^2 \sin^2(c + dx) \tan^2(c + dx) + b^3 \sin^3(c + dx) \tan^2(c + dx)) dx \\ &= a^3 \int \tan^2(c + dx) dx + (3a^2b) \int \sin(c + dx) \tan^2(c + dx) dx + (3ab^2) \int \sin^2(c + dx) \tan^2(c + dx) dx + b^3 \int \sin^3(c + dx) \tan^2(c + dx) dx \\ &= \frac{a^3 \tan(c + dx)}{d} - a^3 \int 1 dx - \frac{(3a^2b) \text{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} + \frac{(3ab^2) \text{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -a^3x + \frac{a^3 \tan(c + dx)}{d} - \frac{3ab^2 \sin^2(c + dx) \tan(c + dx)}{2d} - \frac{(3a^2b) \text{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \cos(c + dx)\right)}{d} \\ &= -a^3x + \frac{3a^2b \cos(c + dx)}{d} + \frac{2b^3 \cos(c + dx)}{d} - \frac{b^3 \cos^3(c + dx)}{3d} + \frac{3a^2b \sec(c + dx)}{d} \\ &= -a^3x - \frac{9}{2}ab^2x + \frac{3a^2b \cos(c + dx)}{d} + \frac{2b^3 \cos(c + dx)}{d} - \frac{b^3 \cos^3(c + dx)}{3d} + \frac{3a^2b \sec(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.729084, size = 113, normalized size = 0.77

$$\frac{3a \left((8a^2 + 27b^2) \tan(c + dx) - 4(2a^2 + 9b^2)(c + dx) \right) + b \sec(c + dx) \left(4(9a^2 + 5b^2) \cos(2(c + dx)) + 108a^2 + 9ab \sin(2(c + dx)) \right)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]

[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c + d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)

Maple [A] time = 0.047, size = 169, normalized size = 1.2

$$\frac{1}{d} \left(a^3 (\tan(dx+c) - dx - c) + 3a^2b \left(\frac{(\sin(dx+c))^4}{\cos(dx+c)} + (2 + (\sin(dx+c))^2) \cos(dx+c) \right) + 3ab^2 \left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + ((\sin(dx+c))^2 \cos(dx+c)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x)

[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))

Maxima [A] time = 3.01256, size = 161, normalized size = 1.1

$$\frac{6(dx+c-\tan(dx+c))a^3+9\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)ab^2+2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)b^3}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")

[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d

Fricas [A] time = 1.48544, size = 271, normalized size = 1.86

$$\frac{2b^3 \cos(dx+c)^4 + 3(2a^3 + 9ab^2)dx \cos(dx+c) - 18a^2b - 6b^3 - 6(3a^2b + 2b^3) \cos(dx+c)^2 - 3(3ab^2 \cos(dx+c) + 2a^3 + 6ab^2) \sin(dx+c)}{6d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")

[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2 + 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^3 \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**2,x)

[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**2, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")

[Out] Timed out

3.167 $\int \cot^2(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=102

$$\frac{3a^2b \cos(c + dx)}{d} - \frac{3a^2b \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^3 \cot(c + dx)}{d} + a^3(-x) + \frac{3ab^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3}{2}ab^2x - \frac{b^3 \cos(c + dx)}{3d}$$

[Out] $-(a^3x) + (3a^2b \cos(c + dx))/d - (3a^2b \operatorname{ArcTanh}[\cos(c + dx)])/d + (3a^2b \operatorname{Cos}[c + dx])/d - (b^3 \operatorname{Cos}[c + dx]^3)/(3d) - (a^3 \operatorname{Cot}[c + dx])/d + (3a^2b \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx])/(2d)$

Rubi [A] time = 0.111823, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473, 2565, 30}

$$\frac{3a^2b \cos(c + dx)}{d} - \frac{3a^2b \tanh^{-1}(\cos(c + dx))}{d} - \frac{a^3 \cot(c + dx)}{d} + a^3(-x) + \frac{3ab^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3}{2}ab^2x - \frac{b^3 \cos(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cot}[c + dx]^2(a + b \operatorname{Sin}[c + dx])^3, x]$

[Out] $-(a^3x) + (3a^2b \cos(c + dx))/d - (3a^2b \operatorname{ArcTanh}[\cos(c + dx)])/d + (3a^2b \operatorname{Cos}[c + dx])/d - (b^3 \operatorname{Cos}[c + dx]^3)/(3d) - (a^3 \operatorname{Cot}[c + dx])/d + (3a^2b \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx])/(2d)$

Rule 2722

$\operatorname{Int}[(a + b \sin(e + f x))^m ((g + f \tan(e + f x))^p), x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(g + f \tan(e + f x))^p, (a + b \sin(e + f x))^m, x], x] /;$ FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2635

$\operatorname{Int}[(b \sin(c + d x))^n], x] \rightarrow -\operatorname{Simp}[(b \cos(c + d x))(b \sin(c + d x))^{n-1}]/(d n), x] + \operatorname{Dist}[(b^2)^{n-1}/n, \operatorname{Int}[(b \sin(c + d x))^{n-2}], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\operatorname{Int}[a x], x] \rightarrow \operatorname{Simp}[a x], x] /;$ FreeQ[a, x]

Rule 2592

$\operatorname{Int}[(a \sin(e + f x))^m \tan(e + f x)^n], x] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin(e + f x)], x\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(ff x)^{m+n}/(a^2 - ff^2 x^2)^{(n+1)/2}], x], x, (a \sin(e + f x))/ff], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2]

Rule 321

$\operatorname{Int}[(c + b x)^m (a + b x)^n], x] \rightarrow \operatorname{Simp}[(c + b x)^{m-1} (c x)^{m-n+1} (a + b x)^{n+1}]/(b(m+n+1)), x] - \operatorname{Dist}[(a c)^{m-n+1}/(b(m+n+1)), \operatorname{Int}[(c x)^{m-n} (a + b x)^n], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p]

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 2565

Int[(cos[(e_) + (f_)*(x_)])*(a_)^(m_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + b \sin(c + dx))^3 dx &= \int (3ab^2 \cos^2(c + dx) + 3a^2b \cos(c + dx) \cot(c + dx) + a^3 \cot^2(c + dx) + b^3 \csc^2(c + dx)) dx \\ &= a^3 \int \cot^2(c + dx) dx + (3a^2b) \int \cos(c + dx) \cot(c + dx) dx + (3ab^2) \int \cos^2(c + dx) dx \\ &= -\frac{a^3 \cot(c + dx)}{d} + \frac{3ab^2 \cos(c + dx) \sin(c + dx)}{2d} - a^3 \int 1 dx + \frac{1}{2} (3ab^2) \int 1 dx \\ &= -a^3 x + \frac{3}{2} ab^2 x + \frac{3a^2b \cos(c + dx)}{d} - \frac{b^3 \cos^3(c + dx)}{3d} - \frac{a^3 \cot(c + dx)}{d} + \frac{3ab^2}{2d} \\ &= -a^3 x + \frac{3}{2} ab^2 x - \frac{3a^2b \tanh^{-1}(\cos(c + dx))}{d} + \frac{3a^2b \cos(c + dx)}{d} - \frac{b^3 \cos^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 1.29539, size = 143, normalized size = 1.4

$$\frac{(36a^2b - 3b^3) \cos(c + dx) + 6a \left(a^2 \tan\left(\frac{1}{2}(c + dx)\right) - 2a^2c - 2a^2dx + 6ab \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 6ab \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]

[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*Tan[(c + d*x)/2]))/(12*d)

Maple [A] time = 0.052, size = 125, normalized size = 1.2

$$-a^3x - \frac{a^3 \cot(dx+c)}{d} - \frac{a^3c}{d} + 3 \frac{a^2b \ln(\csc(dx+c) - \cot(dx+c))}{d} + 3 \frac{a^2b \cos(dx+c)}{d} + \frac{3ab^2 \cos(dx+c) \sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x)

[Out] $-a^3x - a^3 \cot(dx+c)/d - 1/d a^3 c + 3/d a^2 b \ln(\csc(dx+c) - \cot(dx+c)) + 3 a^2 b \cos(dx+c)/d + 3/2 a b^2 \cos(dx+c) \sin(dx+c)/d + 3/2 a b^2 x + 3/2 d a b^2 c - 1/3 b^3 \cos(dx+c)^3/d$

Maxima [A] time = 1.51545, size = 128, normalized size = 1.25

$$\frac{4b^3 \cos(dx+c)^3 + 12 \left(dx+c + \frac{1}{\tan(dx+c)} \right) a^3 - 9(2dx+2c + \sin(2dx+2c))ab^2 - 18a^2b(2\cos(dx+c) - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/12*(4*b^3*\cos(dx+c)^3 + 12*(dx+c + 1/\tan(dx+c))*a^3 - 9*(2*dx + 2*c + \sin(2*dx + 2*c))*a*b^2 - 18*a^2*b*(2*\cos(dx+c) - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1)))/d$

Fricas [A] time = 1.6047, size = 370, normalized size = 3.63

$$\frac{9ab^2 \cos(dx+c)^3 + 9a^2b \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 9a^2b \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 3(2a^3 - 3a^2b \cos(dx+c) + (2b^3 \cos(dx+c)^3 - 18a^2b \cos(dx+c) + 3(2a^3 - 3a^2b \cos(dx+c)) * dx) * \sin(dx+c))}{6d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/6*(9*a*b^2*\cos(dx+c)^3 + 9*a^2*b*\log(1/2*\cos(dx+c) + 1/2)*\sin(dx+c) - 9*a^2*b*\log(-1/2*\cos(dx+c) + 1/2)*\sin(dx+c) + 3*(2*a^3 - 3*a*b^2*\cos(dx+c) + (2*b^3*\cos(dx+c)^3 - 18*a^2*b*\cos(dx+c) + 3*(2*a^3 - 3*a*b^2)*dx)*\sin(dx+c))/(d*\sin(dx+c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sin(c + dx))^3 \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**3,x)

[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**2, x)

Giac [B] time = 2.62788, size = 269, normalized size = 2.64

$$18 a^2 b \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right| \right) + 3 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 3 (2 a^3 - 3 a b^2) (dx + c) - \frac{3 (6 a^2 b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a^3)}{\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)} - \frac{2 (9 a b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a^3)}{\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}$$

$6d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{6} (18 a^2 b \log(\text{abs}(\tan(\frac{1}{2} d x + \frac{1}{2} c))) + 3 a^3 \tan(\frac{1}{2} d x + \frac{1}{2} c) - 3 (2 a^3 - 3 a b^2) (d x + c) - 3 (6 a^2 b \tan(\frac{1}{2} d x + \frac{1}{2} c) + a^3) / \tan(\frac{1}{2} d x + \frac{1}{2} c) - 2 (9 a b^2 \tan(\frac{1}{2} d x + \frac{1}{2} c) + a^3) / \tan(\frac{1}{2} d x + \frac{1}{2} c) - 2 (9 a b^2 \tan(\frac{1}{2} d x + \frac{1}{2} c)^5 - 18 a^2 b \tan(\frac{1}{2} d x + \frac{1}{2} c)^4 + 6 b^3 \tan(\frac{1}{2} d x + \frac{1}{2} c)^4 - 36 a^2 b \tan(\frac{1}{2} d x + \frac{1}{2} c)^2 - 9 a b^2 \tan(\frac{1}{2} d x + \frac{1}{2} c) - 18 a^2 b + 2 b^3) / (\tan(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1)^3) / d$

3.168 $\int \cot^4(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=194

$$-\frac{9a^2b \cos(c + dx)}{2d} - \frac{3a^2b \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{9a^2b \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^3 \cot^3(c + dx)}{3d} + \frac{a^3 \cot(c + dx)}{d} + a^3$$

[Out] $a^3x - (9ab^2x)/2 + (9a^2b \operatorname{ArcTanh}[\cos(c + dx)])/(2d) - (b^3 \operatorname{ArcTanh}[\cos(c + dx)]/d - (9a^2b \cos(c + dx))/(2d) + (b^3 \cos(c + dx))/d + (b^3 \cos(c + dx)^3)/(3d) + (a^3 \cot(c + dx))/d - (9ab^2 \cot(c + dx))/(2d) + (3ab^2 \cos(c + dx)^2 \cot(c + dx))/(2d) - (3a^2b \cos(c + dx) \cot(c + dx)^2)/(2d) - (a^3 \cot(c + dx)^3)/(3d)$

Rubi [A] time = 0.183918, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 2592, 302, 206, 2591, 288, 321, 203, 3473, 8}

$$-\frac{9a^2b \cos(c + dx)}{2d} - \frac{3a^2b \cos(c + dx) \cot^2(c + dx)}{2d} + \frac{9a^2b \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^3 \cot^3(c + dx)}{3d} + \frac{a^3 \cot(c + dx)}{d} + a^3$$

Antiderivative was successfully verified.

[In] $\text{Int}[\cot(c + dx)^4(a + b \sin(c + dx))^3, x]$

[Out] $a^3x - (9ab^2x)/2 + (9a^2b \operatorname{ArcTanh}[\cos(c + dx)])/(2d) - (b^3 \operatorname{ArcTanh}[\cos(c + dx)]/d - (9a^2b \cos(c + dx))/(2d) + (b^3 \cos(c + dx))/d + (b^3 \cos(c + dx)^3)/(3d) + (a^3 \cot(c + dx))/d - (9ab^2 \cot(c + dx))/(2d) + (3ab^2 \cos(c + dx)^2 \cot(c + dx))/(2d) - (3a^2b \cos(c + dx) \cot(c + dx)^2)/(2d) - (a^3 \cot(c + dx)^3)/(3d)$

Rule 2722

$\text{Int}[(a + b \sin(e + f x))^m (g \tan(e + f x) + h)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g \tan(e + f x))^p, (a + b \sin(e + f x))^m, x], x] /;$ FreeQ[{a, b, e, f, g, h, x} && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

$\text{Int}[(a \sin(e + f x) + b)^m \tan(e + f x)^n, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\sin(e + f x), x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(ff x)^{m+n}/(a^2 - ff^2 x^2)^{(n+1)/2}, x], x, (a \sin(e + f x))/ff], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

$\text{Int}[x^m / ((a + b x)^n), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b x^n, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 206

$\text{Int}[(a + b x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \operatorname{ArcTanh}[\text{Rt}[-b, 2] x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]]
/; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1]/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x]
/; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol]
:> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + b \sin(c + dx))^3 dx &= \int (b^3 \cos^3(c + dx) \cot(c + dx) + 3ab^2 \cos^2(c + dx) \cot^2(c + dx) + 3a^2b \cos(c + dx) \cot^3(c + dx)) dx \\
&= a^3 \int \cot^4(c + dx) dx + (3a^2b) \int \cos(c + dx) \cot^3(c + dx) dx + (3ab^2) \int \cos^2(c + dx) \cot^2(c + dx) dx \\
&= -\frac{a^3 \cot^3(c + dx)}{3d} - a^3 \int \cot^2(c + dx) dx - \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2} dx, x, \cos(c + dx)\right)}{d} \\
&= \frac{a^3 \cot(c + dx)}{d} + \frac{3ab^2 \cos^2(c + dx) \cot(c + dx)}{2d} - \frac{3a^2b \cos(c + dx) \cot^2(c + dx)}{2d} \\
&= a^3 x - \frac{9a^2b \cos(c + dx)}{2d} + \frac{b^3 \cos(c + dx)}{d} + \frac{b^3 \cos^3(c + dx)}{3d} + \frac{a^3 \cot(c + dx)}{d} \\
&= a^3 x - \frac{9}{2} ab^2 x + \frac{9a^2b \tanh^{-1}(\cos(c + dx))}{2d} - \frac{b^3 \tanh^{-1}(\cos(c + dx))}{d} - \frac{9a^2b \cos(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 6.23632, size = 355, normalized size = 1.83

$$\frac{a(2a^2 - 9b^2)(c + dx)}{2d} + \frac{b(5b^2 - 12a^2)\cos(c + dx)}{4d} + \frac{(2b^3 - 9a^2b)\log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{(9a^2b - 2b^3)\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]

[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*Log[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)

Maple [A] time = 0.062, size = 264, normalized size = 1.4

$$-\frac{a^3(\cot(dx+c))^3}{3d} + \frac{a^3\cot(dx+c)}{d} + a^3x + \frac{a^3c}{d} - \frac{3a^2b(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{3a^2b(\cos(dx+c))^3}{2d} - \frac{9a^2b\cos(dx+c)}{2d} - \frac{9a^2b\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x)

[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-3/2/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^5-3/2/d*a^2*b*cos(d*x+c)^3-9/2*a^2*b*cos(d*x+c)/d-9/2/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/d*a*b^2/sin(d*x+c)*cos(d*x+c)^5-3/d*a*b^2*sin(d*x+c)*cos(d*x+c)^3-9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d*a*b^2*c+1/3*b^3*cos(d*x+c)^3/d+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 2.21021, size = 252, normalized size = 1.3

$$4\left(3dx + 3c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3 - 18\left(3dx + 3c + \frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2 + 2\left(2\cos(dx+c)^3 + 6\cos(dx+c) - 3\log(\cos(dx+c) + 1) + 3\log(\cos(dx+c) - 1)\right)b^3 + 9a^2b(2\cos(dx+c)/(\cos(dx+c)^2 - 1) - 4\cos(dx+c) + 3\log(\cos(dx+c) + 1) - 3\log(\cos(dx+c) - 1))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 + 2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d

Fricas [A] time = 1.65166, size = 699, normalized size = 3.6

$$18 ab^2 \cos(dx + c)^5 + 8(2a^3 - 9ab^2) \cos(dx + c)^3 - 3(9a^2b - 2b^3 - (9a^2b - 2b^3) \cos(dx + c)^2) \log\left(\frac{1}{2} \cos(dx + c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) + 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**3,x)

[Out] Timed out

Giac [B] time = 2.6525, size = 568, normalized size = 2.93

$$3a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 27a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 45a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 108ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 36(2a^3 - 9ab^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))) + (198*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^3)/d

3.169 $\int \cot^6(c + dx)(a + b \sin(c + dx))^3 dx$

Optimal. Leaf size=291

$$\frac{45a^2b \cos(c + dx)}{8d} - \frac{3a^2b \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{15a^2b \cos(c + dx) \cot^2(c + dx)}{8d} - \frac{45a^2b \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^3 \cot^5(c + dx)}{5d}$$

[Out] $-(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.2309, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 2592, 288, 302, 206, 2591, 203, 321, 3473, 8}

$$\frac{45a^2b \cos(c + dx)}{8d} - \frac{3a^2b \cos(c + dx) \cot^4(c + dx)}{4d} + \frac{15a^2b \cos(c + dx) \cot^2(c + dx)}{8d} - \frac{45a^2b \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^3 \cot^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]

[Out] $-(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*d)$

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I

$\text{LtQ}[(m + n*(p + 1) + 1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 302

$\text{Int}[(x_)^{(m_)} / ((a_) + (b_.) * (x_)^{(n_)}), x_Symbol] \ :> \ \text{Int}[\text{PolynomialDivide}[x^{m_}, a + b*x^{n_}, x], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 206

$\text{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2591

$\text{Int}[\sin[(e_.) + (f_.) * (x_)]^{(m_)} * ((b_.) * \tan[(e_.) + (f_.) * (x_)]^{(n_.)}), x_Symbol] \ :> \ \text{With}[\{ff = \text{FreeFactors}[\tan[e + f*x], x]\}, \text{Dist}[(b*ff)/f, \text{Subst}[\text{Int}[(ff*x)^{(m+n)} / (b^2 + ff^2*x^2)^{(m/2+1)}, x], x, (b*\tan[e + f*x])/ff], x]] \ /; \ \text{FreeQ}[\{b, e, f, n\}, x] \ \&\& \ \text{IntegerQ}[m/2]$

Rule 203

$\text{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1 * \text{ArcTan}[(\text{Rt}[b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 321

$\text{Int}[(c_) * (x_)^{(m_)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x_Symbol] \ :> \ \text{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^{n_})^{(p+1)}) / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^{n_} * (m-n+1)) / (b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)} * (a + b*x^{n_})^p, x], x] \ /; \ \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 3473

$\text{Int}[(b_.) * \tan[(c_.) + (d_.) * (x_)]^{(n_)}, x_Symbol] \ :> \ \text{Simp}[(b * (b * \tan[c + d*x])^{(n-1)}) / (d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b * \tan[c + d*x])^{(n-2)}, x], x] \ /; \ \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a_, x_Symbol] \ :> \ \text{Simp}[a*x, x] \ /; \ \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\sin(c+dx))^3 dx &= \int (b^3 \cos^3(c+dx) \cot^3(c+dx) + 3ab^2 \cos^2(c+dx) \cot^4(c+dx) + 3a^2b \cos(c+dx) \cot^5(c+dx) + a^3 \cot^6(c+dx)) dx \\
&= a^3 \int \cot^6(c+dx) dx + (3a^2b) \int \cos(c+dx) \cot^5(c+dx) dx + (3ab^2) \int \cos^2(c+dx) \cot^4(c+dx) dx \\
&= -\frac{a^3 \cot^5(c+dx)}{5d} - a^3 \int \cot^4(c+dx) dx - \frac{(3a^2b) \text{Subst}\left(\int \frac{x^6}{(1-x^2)^3} dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{b^3 \cos^3(c+dx) \cot^2(c+dx)}{2d} + \frac{a^3 \cot^3(c+dx)}{3d} + \frac{3ab^2 \cos^2(c+dx) \cot^3(c+dx)}{2d} \\
&= -\frac{a^3 \cot(c+dx)}{d} + \frac{15a^2b \cos(c+dx) \cot^2(c+dx)}{8d} - \frac{b^3 \cos^3(c+dx) \cot^2(c+dx)}{2d} \\
&= -a^3x + \frac{45a^2b \cos(c+dx)}{8d} - \frac{5b^3 \cos(c+dx)}{2d} - \frac{5b^3 \cos^3(c+dx)}{6d} - \frac{a^3 \cot(c+dx)}{d} \\
&= -a^3x + \frac{15}{2}ab^2x - \frac{45a^2b \tanh^{-1}(\cos(c+dx))}{8d} + \frac{5b^3 \tanh^{-1}(\cos(c+dx))}{2d} + \frac{45a^2b^3}{8d}
\end{aligned}$$

Mathematica [A] time = 2.57765, size = 346, normalized size = 1.19

$$\frac{-600a(2a^2 - 15b^2)(c+dx) \csc^4(c+dx) + 1200b(4b^2 - 9a^2) \left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \right) + \csc^5(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]

[Out] (-600*a*(2*a^2 - 15*b^2)*(c + d*x)*Csc[c + d*x]^4 + 1200*b*(-9*a^2 + 4*b^2)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])) + 5*Cot[c + d*x]*Csc[c + d*x]^4*(-80*a^3 + 285*a*b^2 + 12*b*(60*a^2 - 29*b^2)*Sin[c + d*x]) + Csc[c + d*x]^5*(5*(40*a^3 - 489*a*b^2)*Cos[3*(c + d*x)] + (-184*a^3 + 1065*a*b^2)*Cos[5*(c + d*x)] + 5*(-9*a*b^2*Cos[7*(c + d*x)] + 60*a*(2*a^2 - 15*b^2)*(c + d*x)*Sin[3*(c + d*x)] - 306*a^2*b*Sin[4*(c + d*x)] + 122*b^3*Sin[4*(c + d*x)] - 24*a^3*c*Sin[5*(c + d*x)] + 180*a*b^2*c*Sin[5*(c + d*x)] - 24*a^3*d*x*Sin[5*(c + d*x)] + 180*a*b^2*d*x*Sin[5*(c + d*x)] + 36*a^2*b*Sin[6*(c + d*x)] - 22*b^3*Sin[6*(c + d*x)] - b^3*Sin[8*(c + d*x)])))/(1920*d)

Maple [A] time = 0.063, size = 415, normalized size = 1.4

$$-\frac{a^3 (\cot(dx+c))^5}{5d} + \frac{a^3 (\cot(dx+c))^3}{3d} - \frac{a^3 \cot(dx+c)}{d} - a^3x - \frac{a^3c}{d} - \frac{3a^2b (\cos(dx+c))^7}{4d (\sin(dx+c))^4} + \frac{9a^2b (\cos(dx+c))^7}{8d (\sin(dx+c))^2} + \frac{9a^2b^3}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x)

[Out] -1/5*a^3*cot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d-a^3*x-1/d*a^3*c-3/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^7+9/8/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^7+9/8/d*a^2*b*cos(d*x+c)^5+15/8/d*a^2*b*cos(d*x+c)^3+45/8*a^2*b*cos(d*x+c)/d+45/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/d*a*b^2/sin(d*x+c)*cos(d*x+c)^7+4/d*a*b^2*sin(d*x+c)*cos(d*x+c)^5+5/d*a*b^2*sin(d*x+c)*cos(d*x+c)^3+15/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+15/2*a*

$$b^2*x+15/2/d*a*b^2*c-1/2/d*b^3/\sin(d*x+c)^2*\cos(d*x+c)^7-1/2/d*b^3*\cos(d*x+c)^5-5/6*b^3*\cos(d*x+c)^3/d-5/2*b^3*\cos(d*x+c)/d-5/2/d*b^3*\ln(\csc(d*x+c)-\cot(d*x+c))$$

Maxima [A] time = 2.64117, size = 340, normalized size = 1.17

$$16 \left(15 dx + 15 c + \frac{15 \tan(dx+c)^4 - 5 \tan(dx+c)^2 + 3}{\tan(dx+c)^5} \right) a^3 - 120 \left(15 dx + 15 c + \frac{15 \tan(dx+c)^4 + 10 \tan(dx+c)^2 - 2}{\tan(dx+c)^5 + \tan(dx+c)^3} \right) ab^2 + 20 \left(4 \cos(dx+c)^3 - 6 \cos(dx+c) / (\cos(dx+c)^2 - 1) + 24 \cos(dx+c) - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1) \right) b^3 + 45 a^2 b (2 (9 \cos(dx+c)^3 - 7 \cos(dx+c)) / (\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1) - 16 \cos(dx+c) + 15 \log(\cos(dx+c) + 1) - 15 \log(\cos(dx+c) - 1)) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out]
$$-1/240*(16*(15*d*x + 15*c + (15*\tan(d*x + c)^4 - 5*\tan(d*x + c)^2 + 3)/\tan(d*x + c)^5)*a^3 - 120*(15*d*x + 15*c + (15*\tan(d*x + c)^4 + 10*\tan(d*x + c)^2 - 2)/(\tan(d*x + c)^5 + \tan(d*x + c)^3))*a*b^2 + 20*(4*\cos(d*x + c)^3 - 6*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) + 24*\cos(d*x + c) - 15*\log(\cos(d*x + c) + 1) + 15*\log(\cos(d*x + c) - 1))*b^3 + 45*a^2*b*(2*(9*\cos(d*x + c)^3 - 7*\cos(d*x + c))/(\cos(d*x + c)^4 - 2*\cos(d*x + c)^2 + 1) - 16*\cos(d*x + c) + 15*\log(\cos(d*x + c) + 1) - 15*\log(\cos(d*x + c) - 1)))/d$$

Fricas [A] time = 1.83246, size = 1014, normalized size = 3.48

$$360 ab^2 \cos(dx+c)^7 + 184 (2 a^3 - 15 ab^2) \cos(dx+c)^5 - 280 (2 a^3 - 15 ab^2) \cos(dx+c)^3 + 75 ((9 a^2 b - 4 b^3) \cos(dx+c)^4 + 9 a^2 b - 4 b^3 - 2 (9 a^2 b - 4 b^3) \cos(dx+c)^2) \log(1/2 \cos(dx+c) + 1/2) \sin(dx+c) - 75 ((9 a^2 b - 4 b^3) \cos(dx+c)^4 + 9 a^2 b - 4 b^3 - 2 (9 a^2 b - 4 b^3) \cos(dx+c)^2) \log(-1/2 \cos(dx+c) + 1/2) \sin(dx+c) + 120 (2 a^3 - 15 a b^2) \cos(dx+c) + 10 (8 b^3 \cos(dx+c)^7 + 12 (2 a^3 - 15 a b^2) d x \cos(dx+c)^4 - 8 (9 a^2 b - 4 b^3) \cos(dx+c)^5 - 24 (2 a^3 - 15 a b^2) d x \cos(dx+c)^2 + 25 (9 a^2 b - 4 b^3) \cos(dx+c)^3 + 12 (2 a^3 - 15 a b^2) d x - 15 (9 a^2 b - 4 b^3) \cos(dx+c)) \sin(dx+c) / ((d \cos(dx+c)^4 - 2 d \cos(dx+c)^2 + d) \sin(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out]
$$-1/240*(360*a*b^2*\cos(d*x + c)^7 + 184*(2*a^3 - 15*a*b^2)*\cos(d*x + c)^5 - 280*(2*a^3 - 15*a*b^2)*\cos(d*x + c)^3 + 75*((9*a^2*b - 4*b^3)*\cos(d*x + c)^4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*\cos(d*x + c)^2)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - 75*((9*a^2*b - 4*b^3)*\cos(d*x + c)^4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*\cos(d*x + c)^2)*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 120*(2*a^3 - 15*a*b^2)*\cos(d*x + c) + 10*(8*b^3*\cos(d*x + c)^7 + 12*(2*a^3 - 15*a*b^2)*d*x*\cos(d*x + c)^4 - 8*(9*a^2*b - 4*b^3)*\cos(d*x + c)^5 - 24*(2*a^3 - 15*a*b^2)*d*x*\cos(d*x + c)^2 + 25*(9*a^2*b - 4*b^3)*\cos(d*x + c)^3 + 12*(2*a^3 - 15*a*b^2)*d*x - 15*(9*a^2*b - 4*b^3)*\cos(d*x + c))*\sin(d*x + c) / ((d*\cos(d*x + c)^4 - 2*d*\cos(d*x + c)^2 + d)*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 2.01324, size = 636, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out]
$$\frac{1}{960} \left(6a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 45a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 70a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 120ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 720a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 120b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 660a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3240ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 480(2a^3 - 15ab^2)(dx + c) + 600(9a^2b - 4b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - 320(9ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 18a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 18b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 36a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 9ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 18a^2b + 14b^3) / \left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^3 - (12330a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 5480b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 660a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 3240ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 720a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 120b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 70a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 120ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 45a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 6a^3) / \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 \right) / d$$

$$3.170 \quad \int \frac{\tan^5(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=204

$$\frac{a^5 \log(a + b \sin(c + dx))}{d(a^2 - b^2)^3} - \frac{(8a^2 + 9ab + 3b^2) \log(1 - \sin(c + dx))}{16d(a + b)^3} - \frac{(8a^2 - 9ab + 3b^2) \log(\sin(c + dx) + 1)}{16d(a - b)^3} + \frac{\sec^4(c + dx)}{d}$$

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.363019, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 801}

$$\frac{a^5 \log(a + b \sin(c + dx))}{d(a^2 - b^2)^3} - \frac{(8a^2 + 9ab + 3b^2) \log(1 - \sin(c + dx))}{16d(a + b)^3} - \frac{(8a^2 - 9ab + 3b^2) \log(\sin(c + dx) + 1)}{16d(a - b)^3} + \frac{\sec^4(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)^(p_)], x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)(b^2-x^2)^3} dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2} - \frac{b^4(4a^2-b^2)x}{a^2-b^2} - 4b^2x^3}{(a+x)(b^2-x^2)^2} dx, x, b\sin(c+dx)\right)}{4b^2d} \\
&= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)(4a(2a^2-b^2) - b(9a^2-5b^2)\sin(c+dx))}{8(a^2-b^2)^2d} + \dots \\
&= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)(4a(2a^2-b^2) - b(9a^2-5b^2)\sin(c+dx))}{8(a^2-b^2)^2d} + \dots \\
&= -\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3d}
\end{aligned}$$

Mathematica [A] time = 1.46175, size = 184, normalized size = 0.9

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{(8a^2-9ab+3b^2)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{7a+5b}{(a+b)^2(\sin(c+dx)-1)} + \frac{5b-7a}{(a-b)^2(\sin(c+dx)+1)} + \frac{a^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]

[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)

Maple [A] time = 0.066, size = 304, normalized size = 1.5

$$\frac{a^5 \ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} + \frac{7a}{16d(a+b)^2(\sin(dx+c)-1)} + \frac{5b}{16d(a+b)^2(\sin(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c)),x)

[Out] 1/d*a^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+7/16/d/(a+b)^2/(sin(d*x+c)-1)*a+5/16/d/(a+b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+sin(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+c))*a^2+9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2

Maxima [A] time = 1.8256, size = 389, normalized size = 1.91

$$\frac{16 a^5 \log(b \sin(dx+c)+a)}{a^6-3 a^4 b^2+3 a^2 b^4-b^6} - \frac{(8 a^2-9 a b+3 b^2) \log(\sin(dx+c)+1)}{a^3-3 a^2 b+3 a b^2-b^3} - \frac{(8 a^2+9 a b+3 b^2) \log(\sin(dx+c)-1)}{a^3+3 a^2 b+3 a b^2+b^3} - \frac{2((9 a^2 b-5 b^3) \sin(dx+c)^3+6 a^3-2 a b^2-4(2 a^3-a^2 b) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4}{(a^4-2 a^2 b^2+b^4) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4}$$

16 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3 - a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c)))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d

Fricas [A] time = 2.47436, size = 594, normalized size = 2.91

$$16 a^5 \cos(dx+c)^4 \log(b \sin(dx+c)+a) - (8 a^5 + 15 a^4 b - 10 a^2 b^3 + 3 b^5) \cos(dx+c)^4 \log(\sin(dx+c)+1) - (8 a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{a+b \sin(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)), x)

Giac [A] time = 5.11783, size = 463, normalized size = 2.27

$$\frac{16 a^5 b \log(b \sin(dx+c)+a)}{a^6 b-3 a^4 b^3+3 a^2 b^5-b^7} - \frac{(8 a^2-9 a b+3 b^2) \log(|\sin(dx+c)+1|)}{a^3-3 a^2 b+3 a b^2-b^3} - \frac{(8 a^2+9 a b+3 b^2) \log(|\sin(dx+c)-1|)}{a^3+3 a^2 b+3 a b^2+b^3} + \frac{2(6 a^5 \sin(dx+c)^4-9 a^4 b \sin(dx+c)^3+14 a^2 b^3 \sin(dx+c)^2-6 a^2 b^2 \sin(dx+c)+3 b^5) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4}{(a^4-2 a^2 b^2+b^4) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x + c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

$$3.171 \quad \int \frac{\tan^3(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=126

$$-\frac{a^3 \log(a + b \sin(c + dx))}{d(a^2 - b^2)^2} + \frac{\sec^2(c + dx)(a - b \sin(c + dx))}{2d(a^2 - b^2)} + \frac{(2a + b) \log(1 - \sin(c + dx))}{4d(a + b)^2} + \frac{(2a - b) \log(\sin(c + dx))}{4d(a - b)^2}$$

[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)

Rubi [A] time = 0.190922, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 801}

$$-\frac{a^3 \log(a + b \sin(c + dx))}{d(a^2 - b^2)^2} + \frac{\sec^2(c + dx)(a - b \sin(c + dx))}{2d(a^2 - b^2)} + \frac{(2a + b) \log(1 - \sin(c + dx))}{4d(a + b)^2} + \frac{(2a - b) \log(\sin(c + dx))}{4d(a - b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]

[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 1647

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)^2} dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2} - \frac{b^2(2a^2-b^2)x}{a^2-b^2}}{(a+x)(b^2-x^2)} dx, x, b\sin(c+dx)\right)}{2b^2d} \\
&= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right) dx, x, b\sin(c+dx)\right)}{2b^2d} \\
&= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2d} + \frac{\sec^2(c+dx)}{2(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] time = 0.484179, size = 117, normalized size = 0.93

$$\frac{-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]

[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)

Maple [A] time = 0.059, size = 164, normalized size = 1.3

$$-\frac{a^3 \ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)a}{2d(a+b)^2} + \frac{\ln(\sin(dx+c)-1)b}{4d(a+b)^2} + \frac{1}{d(4a-4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)),x)

[Out] -1/d*a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-4*b)/(1+sin(d*x+c))+1/2/d/(a-b)^2*ln(1+sin(d*x+c))*a-1/4/d/(a-b)^2*ln(1+sin(d*x+c))*b

Maxima [A] time = 1.53645, size = 192, normalized size = 1.52

$$\frac{\frac{4a^3\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} - \frac{(2a-b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2} - \frac{(2a+b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2} - \frac{2(b\sin(dx+c)-a)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out]
$$\frac{-1/4*(4*a^3*\log(b*\sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*\log(\sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*\log(\sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) - 2*(b*\sin(d*x + c) - a)/((a^2 - b^2)*\sin(d*x + c)^2 - a^2 + b^2))/d}$$

Fricas [A] time = 1.96149, size = 367, normalized size = 2.91

$$\frac{4a^3 \cos(dx + c)^2 \log(b \sin(dx + c) + a) - (2a^3 + 3a^2b - b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (2a^3 - 3a^2b + b^3) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) - 2a^3 + 2ab^2 + 2(a^2b - b^3) \sin(dx + c)}{4(a^4 - 2a^2b^2 + b^4)d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out]
$$\frac{-1/4*(4*a^3*\cos(d*x + c)^2*\log(b*\sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3)*\cos(d*x + c)^2*\log(\sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*\cos(d*x + c)^2*\log(-\sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*\sin(d*x + c)}{((a^4 - 2*a^2*b^2 + b^4)*d*\cos(d*x + c)^2)}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)), x)

Giac [A] time = 1.98003, size = 239, normalized size = 1.9

$$\frac{\frac{4a^3b \log(|b \sin(dx+c)+a|)}{a^4b-2a^2b^3+b^5} - \frac{(2a-b) \log(|\sin(dx+c)+1|)}{a^2-2ab+b^2} - \frac{(2a+b) \log(|\sin(dx+c)-1|)}{a^2+2ab+b^2} + \frac{2(a^3 \sin(dx+c)^2 - a^2b \sin(dx+c) + b^3 \sin(dx+c) - ab^2)}{(a^4 - 2a^2b^2 + b^4)(\sin(dx+c)^2 - 1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out]
$$\frac{-1/4*(4*a^3*b*\log(\text{abs}(b*\sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a - b)*\log(\text{abs}(\sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*\log(\text{abs}(\sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*\sin(d*x + c)^2 - a^2*b*\sin(d*x + c) + b^3*\sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(\sin(d*x + c)^2 - 1)))/d}$$

$$3.172 \quad \int \frac{\tan(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=74

$$\frac{a \log(a + b \sin(c + dx))}{d(a^2 - b^2)} - \frac{\log(1 - \sin(c + dx))}{2d(a + b)} - \frac{\log(\sin(c + dx) + 1)}{2d(a - b)}$$

[Out] $-\text{Log}[1 - \text{Sin}[c + d*x]]/(2*(a + b)*d) - \text{Log}[1 + \text{Sin}[c + d*x]]/(2*(a - b)*d) + (a*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)*d)$

Rubi [A] time = 0.0661064, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 801}

$$\frac{a \log(a + b \sin(c + dx))}{d(a^2 - b^2)} - \frac{\log(1 - \sin(c + dx))}{2d(a + b)} - \frac{\log(\sin(c + dx) + 1)}{2d(a - b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[c + d*x]/(a + b*\text{Sin}[c + d*x]), x]$

[Out] $-\text{Log}[1 - \text{Sin}[c + d*x]]/(2*(a + b)*d) - \text{Log}[1 + \text{Sin}[c + d*x]]/(2*(a - b)*d) + (a*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)*d)$

Rule 2721

$\text{Int}[(a + b*\sin[e + (f_*)*(x_*)])^{m_*}*\tan[(e + (f_*)*(x_*)]^{p_*}), x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p + 1)/2}], x], x, b*\text{Sin}[e + f*x], x] /;$ FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 801

$\text{Int}[(d + e*x)^m*(f + g*x)/((a + c*x^2)^2), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\tan(c + dx)}{a + b \sin(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+x)(b^2-x^2)} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)} + \frac{a}{(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)(b+x)}\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= -\frac{\log(1 - \sin(c + dx))}{2(a + b)d} - \frac{\log(1 + \sin(c + dx))}{2(a - b)d} + \frac{a \log(a + b \sin(c + dx))}{(a^2 - b^2)d} \end{aligned}$$

Mathematica [A] time = 0.0863228, size = 87, normalized size = 1.18

$$\frac{a \log(a + b \sin(c + dx)) + (b - a) \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - (a + b) \log\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)}{d(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]

[Out] $((-a + b) \cdot \text{Log}[\text{Cos}[(c + d \cdot x)/2] - \text{Sin}[(c + d \cdot x)/2]] - (a + b) \cdot \text{Log}[\text{Cos}[(c + d \cdot x)/2] + \text{Sin}[(c + d \cdot x)/2]]) + a \cdot \text{Log}[a + b \cdot \text{Sin}[c + d \cdot x]] / ((a - b) \cdot (a + b) \cdot d)$

Maple [A] time = 0.049, size = 76, normalized size = 1.

$$\frac{a \ln(a + b \sin(dx + c))}{d(a + b)(a - b)} - \frac{\ln(\sin(dx + c) - 1)}{d(2a + 2b)} - \frac{\ln(1 + \sin(dx + c))}{d(2a - 2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] $1/d \cdot a/(a+b)/(a-b) \cdot \ln(a+b \cdot \sin(d \cdot x+c)) - 1/d/(2 \cdot a+2 \cdot b) \cdot \ln(\sin(d \cdot x+c)-1) - 1/d/(2 \cdot a-2 \cdot b) \cdot \ln(1+\sin(d \cdot x+c))$

Maxima [A] time = 1.55148, size = 88, normalized size = 1.19

$$\frac{\frac{2a \log(b \sin(dx+c)+a)}{a^2-b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} - \frac{\log(\sin(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] $1/2 \cdot (2 \cdot a \cdot \log(b \cdot \sin(d \cdot x + c) + a) / (a^2 - b^2) - \log(\sin(d \cdot x + c) + 1) / (a - b) - \log(\sin(d \cdot x + c) - 1) / (a + b)) / d$

Fricas [A] time = 1.61071, size = 157, normalized size = 2.12

$$\frac{2a \log(b \sin(dx + c) + a) - (a + b) \log(\sin(dx + c) + 1) - (a - b) \log(-\sin(dx + c) + 1)}{2(a^2 - b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] $1/2 \cdot (2 \cdot a \cdot \log(b \cdot \sin(d \cdot x + c) + a) - (a + b) \cdot \log(\sin(d \cdot x + c) + 1) - (a - b) \cdot \log(-\sin(d \cdot x + c) + 1)) / ((a^2 - b^2) \cdot d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.3001, size = 96, normalized size = 1.3

$$\frac{\frac{2ab \log(|b \sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} - \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

$$3.173 \quad \int \frac{\cot(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b \sin(c+dx))}{ad}$$

[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)

Rubi [A] time = 0.0397367, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b \sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]

[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{a+b \sin(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, b \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sin(c+dx)\right)}{ad} \\ &= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b \sin(c+dx))}{ad} \end{aligned}$$

Mathematica [A] time = 0.0203458, size = 34, normalized size = 1.

$$\frac{\log(\sin(c + dx))}{ad} - \frac{\log(a + b \sin(c + dx))}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]

[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)

Maple [A] time = 0.026, size = 35, normalized size = 1.

$$\frac{\ln(\sin(dx + c))}{da} - \frac{\ln(a + b \sin(dx + c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] ln(sin(d*x+c))/a/d-1/d/a*ln(a+b*sin(d*x+c))

Maxima [A] time = 1.46505, size = 45, normalized size = 1.32

$$-\frac{\frac{\log(b \sin(dx+c)+a)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d

Fricas [A] time = 1.56974, size = 80, normalized size = 2.35

$$-\frac{\log(b \sin(dx + c) + a) - \log\left(-\frac{1}{2} \sin(dx + c)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.6945, size = 47, normalized size = 1.38

$$-\frac{\frac{\log(|b \sin(dx+c)+a|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

$$3.174 \quad \int \frac{\cot^3(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=84

$$-\frac{(a^2 - b^2) \log(\sin(c + dx))}{a^3 d} + \frac{(a^2 - b^2) \log(a + b \sin(c + dx))}{a^3 d} + \frac{b \csc(c + dx)}{a^2 d} - \frac{\csc^2(c + dx)}{2ad}$$

[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c + d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)

Rubi [A] time = 0.0888076, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$-\frac{(a^2 - b^2) \log(\sin(c + dx))}{a^3 d} + \frac{(a^2 - b^2) \log(a + b \sin(c + dx))}{a^3 d} + \frac{b \csc(c + dx)}{a^2 d} - \frac{\csc^2(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]

[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c + d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c + dx)}{a + b \sin(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{b^2 - x^2}{x^3(a+x)} dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3} - \frac{b^2}{a^2x^2} + \frac{-a^2+b^2}{a^3x} + \frac{a^2-b^2}{a^3(a+x)}\right) dx, x, b \sin(c + dx)\right)}{d} \\ &= \frac{b \csc(c + dx)}{a^2 d} - \frac{\csc^2(c + dx)}{2ad} - \frac{(a^2 - b^2) \log(\sin(c + dx))}{a^3 d} + \frac{(a^2 - b^2) \log(a + b \sin(c + dx))}{a^3 d} \end{aligned}$$

Mathematica [A] time = 0.1613, size = 65, normalized size = 0.77

$$\frac{2(a^2 - b^2) (\log(\sin(c + dx)) - \log(a + b \sin(c + dx))) + a^2 \csc^2(c + dx) - 2ab \csc(c + dx)}{2a^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]

[Out] $-(-2*a*b*\text{Csc}[c + d*x] + a^2*\text{Csc}[c + d*x]^2 + 2*(a^2 - b^2)*(\text{Log}[\text{Sin}[c + d*x]] - \text{Log}[a + b*\text{Sin}[c + d*x]])) / (2*a^3*d)$

Maple [A] time = 0.063, size = 106, normalized size = 1.3

$$\frac{\ln(a + b \sin(dx + c))}{da} - \frac{\ln(a + b \sin(dx + c))b^2}{da^3} - \frac{1}{2da(\sin(dx + c))^2} - \frac{\ln(\sin(dx + c))}{da} + \frac{\ln(\sin(dx + c))b^2}{da^3} + \frac{1}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)),x)

[Out] $1/d/a*\ln(a+b*\sin(d*x+c))-1/d/a^3*\ln(a+b*\sin(d*x+c))*b^2-1/2/d/a/\sin(d*x+c)^2-\ln(\sin(d*x+c))/a/d+1/d/a^3*\ln(\sin(d*x+c))*b^2+1/d/a^2*b/\sin(d*x+c)$

Maxima [A] time = 1.61607, size = 104, normalized size = 1.24

$$\frac{\frac{2(a^2-b^2)\log(b\sin(dx+c)+a)}{a^3} - \frac{2(a^2-b^2)\log(\sin(dx+c))}{a^3} + \frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] $1/2*(2*(a^2 - b^2)*\log(b*\sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*\log(\sin(d*x + c))/a^3 + (2*b*\sin(d*x + c) - a)/(a^2*\sin(d*x + c)^2))/d$

Fricas [A] time = 1.5958, size = 271, normalized size = 3.23

$$\frac{2ab\sin(dx+c) - a^2 - 2((a^2 - b^2)\cos(dx+c)^2 - a^2 + b^2)\log(b\sin(dx+c) + a) + 2((a^2 - b^2)\cos(dx+c)^2 - a^2 + b^2)\log(\sin(dx+c))}{2(a^3d\cos(dx+c)^2 - a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] $-1/2*(2*a*b*\sin(d*x + c) - a^2 - 2*((a^2 - b^2)*\cos(d*x + c)^2 - a^2 + b^2)*\log(b*\sin(d*x + c) + a) + 2*((a^2 - b^2)*\cos(d*x + c)^2 - a^2 + b^2)*\log(-1/2*\sin(d*x + c)))/(a^3*d*\cos(d*x + c)^2 - a^3*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)), x)

Giac [A] time = 2.0115, size = 154, normalized size = 1.83

$$\frac{\frac{2(a^2-b^2)\log(|\sin(dx+c)|)}{a^3} - \frac{2(a^2b-b^3)\log(|b\sin(dx+c)+a|)}{a^3b} - \frac{3a^2\sin(dx+c)^2-3b^2\sin(dx+c)^2+2ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out]
$$-1/2*(2*(a^2 - b^2)*\log(\text{abs}(\sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*\log(\text{abs}(b*\sin(d*x + c) + a))/(a^3*b) - (3*a^2*\sin(d*x + c)^2 - 3*b^2*\sin(d*x + c)^2 + 2*a*b*\sin(d*x + c) - a^2)/(a^3*\sin(d*x + c)^2))/d$$

3.175 $\int \frac{\cot^5(c+dx)}{a+b \sin(c+dx)} dx$

Optimal. Leaf size=148

$$\frac{(2a^2 - b^2) \csc^2(c + dx)}{2a^3d} - \frac{b(2a^2 - b^2) \csc(c + dx)}{a^4d} + \frac{(a^2 - b^2)^2 \log(\sin(c + dx))}{a^5d} - \frac{(a^2 - b^2)^2 \log(a + b \sin(c + dx))}{a^5d} +$$

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^5*d)
```

Rubi [A] time = 0.138352, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$\frac{(2a^2 - b^2) \csc^2(c + dx)}{2a^3d} - \frac{b(2a^2 - b^2) \csc(c + dx)}{a^4d} + \frac{(a^2 - b^2)^2 \log(\sin(c + dx))}{a^5d} - \frac{(a^2 - b^2)^2 \log(a + b \sin(c + dx))}{a^5d} +$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^5*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{\cot^5(c+dx)}{a+b \sin(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5} - \frac{b^4}{a^2x^4} + \frac{-2a^2b^2+b^4}{a^3x^3} + \frac{2a^2b^2-b^4}{a^4x^2} + \frac{(a^2-b^2)^2}{a^5x} - \frac{(a^2-b^2)^2}{a^5(a+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= -\frac{b(2a^2 - b^2) \csc(c + dx)}{a^4d} + \frac{(2a^2 - b^2) \csc^2(c + dx)}{2a^3d} + \frac{b \csc^3(c + dx)}{3a^2d} - \frac{\csc^4(c + dx)}{4ad} + \frac{(a^2 - b^2)^2 \log(\sin(c + dx))}{a^5d} - \frac{(a^2 - b^2)^2 \log(a + b \sin(c + dx))}{a^5d} \end{aligned}$$

Mathematica [A] time = 3.84689, size = 115, normalized size = 0.78

$$\frac{6a^2(2a^2 - b^2)\csc^2(c + dx) + 12ab(b^2 - 2a^2)\csc(c + dx) + 12(a^2 - b^2)^2(\log(\sin(c + dx)) - \log(a + b\sin(c + dx))) + 4a^3}{12a^5d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]

[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 + 4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)

Maple [A] time = 0.063, size = 216, normalized size = 1.5

$$-\frac{\ln(a + b\sin(dx + c))}{da} + 2\frac{\ln(a + b\sin(dx + c))b^2}{da^3} - \frac{\ln(a + b\sin(dx + c))b^4}{da^5} - \frac{1}{4da(\sin(dx + c))^4} + \frac{1}{da(\sin(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)),x)

[Out] -1/d/a*ln(a+b*sin(d*x+c))+2/d/a^3*ln(a+b*sin(d*x+c))*b^2-1/d/a^5*ln(a+b*sin(d*x+c))*b^4-1/4/d/a/sin(d*x+c)^4+1/d/a/sin(d*x+c)^2-1/2/d/a^3/sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a/d-2/d/a^3*ln(sin(d*x+c))*b^2+1/d/a^5*ln(sin(d*x+c))*b^4-2/d/a^2*b/sin(d*x+c)+1/d/a^4*b^3/sin(d*x+c)+1/3/d/a^2*b/sin(d*x+c)^3

Maxima [A] time = 1.46289, size = 188, normalized size = 1.27

$$\frac{12(a^4 - 2a^2b^2 + b^4)\log(b\sin(dx+c)+a)}{a^5} - \frac{12(a^4 - 2a^2b^2 + b^4)\log(\sin(dx+c))}{a^5} - \frac{4a^2b\sin(dx+c) - 12(2a^2b - b^3)\sin(dx+c)^3 - 3a^3 + 6(2a^3 - ab^2)\sin(dx+c)^2}{a^4\sin(dx+c)^4}$$

12d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin(d*x + c)^4))/d

Fricas [A] time = 1.663, size = 629, normalized size = 4.25

$$9a^4 - 6a^2b^2 - 6(2a^4 - a^2b^2)\cos(dx + c)^2 - 12((a^4 - 2a^2b^2 + b^4)\cos(dx + c)^4 + a^4 - 2a^2b^2 + b^4 - 2(a^4 - 2a^2b^2 + b^4))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.53765, size = 271, normalized size = 1.83

$$\frac{12(a^4 - 2a^2b^2 + b^4) \log(|\sin(dx+c)|)}{a^5} - \frac{12(a^4b - 2a^2b^3 + b^5) \log(|b \sin(dx+c) + a|)}{a^5b} - \frac{25a^4 \sin(dx+c)^4 - 50a^2b^2 \sin(dx+c)^4 + 25b^4 \sin(dx+c)^4 + 24a^3b \sin(dx+c)^4 - 12a^2b^3 \sin(dx+c)^4 + 12a^4 \sin(dx+c)^2 + 6a^2b^2 \sin(dx+c)^2 - 4a^3b \sin(dx+c) + 3a^4}{a^5 \sin(dx+c)^4} + \frac{12d}{a^5 \sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x + c)^4 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

$$3.176 \quad \int \frac{\tan^4(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=177

$$\frac{2a^4 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} + \frac{a \tan^3(c+dx)}{3d(a^2-b^2)} - \frac{a^3 \tan(c+dx)}{d(a^2-b^2)^2} - \frac{b \sec^3(c+dx)}{3d(a^2-b^2)} + \frac{a^2 b \sec(c+dx)}{d(a^2-b^2)^2} + \frac{b \sec(c+dx)}{d(a^2-b^2)}$$

[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(5/2)*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)

Rubi [A] time = 0.239069, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 204}

$$\frac{2a^4 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} + \frac{a \tan^3(c+dx)}{3d(a^2-b^2)} - \frac{a^3 \tan(c+dx)}{d(a^2-b^2)^2} - \frac{b \sec^3(c+dx)}{3d(a^2-b^2)} + \frac{a^2 b \sec(c+dx)}{d(a^2-b^2)^2} + \frac{b \sec(c+dx)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]

[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(5/2)*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)

Rule 2727

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e + f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*p] && GtQ[p, 1]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_.)]^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_.)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)} dx &= \frac{a \int \sec^2(c+dx) \tan^2(c+dx) dx}{a^2-b^2} - \frac{a^2 \int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)} dx}{a^2-b^2} - \frac{b \int \sec(c+dx) \tan^3(c+dx) dx}{a^2-b^2} \\ &= -\frac{a^3 \int \sec^2(c+dx) dx}{(a^2-b^2)^2} + \frac{a^4 \int \frac{1}{a+b\sin(c+dx)} dx}{(a^2-b^2)^2} + \frac{(a^2b) \int \sec(c+dx) \tan(c+dx) dx}{(a^2-b^2)^2} + \frac{a \text{Subst}(\int \sec^2(x) dx, x, \tan(c+dx))}{(a^2-b^2)^2} \\ &= \frac{b \sec(c+dx)}{(a^2-b^2)d} - \frac{b \sec^3(c+dx)}{3(a^2-b^2)d} + \frac{a \tan^3(c+dx)}{3(a^2-b^2)d} + \frac{a^3 \text{Subst}(\int 1 dx, x, -\tan(c+dx))}{(a^2-b^2)^2 d} + \frac{(2a^4) \text{Subst}(\int \frac{1}{1-u^2} du, u, \tan(c+dx))}{(a^2-b^2)^2 d} \\ &= \frac{a^2b \sec(c+dx)}{(a^2-b^2)^2 d} + \frac{b \sec(c+dx)}{(a^2-b^2)d} - \frac{b \sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^3 \tan(c+dx)}{(a^2-b^2)^2 d} + \frac{a \tan^3(c+dx)}{3(a^2-b^2)d} - \frac{(4a^4) \text{Subst}(\int \frac{1}{1-u^2} du, u, \tan(c+dx))}{(a^2-b^2)^2 d} \\ &= \frac{2a^4 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{a^2b \sec(c+dx)}{(a^2-b^2)^2 d} + \frac{b \sec(c+dx)}{(a^2-b^2)d} - \frac{b \sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^3 \tan(c+dx)}{(a^2-b^2)^2 d} \end{aligned}$$

Mathematica [A] time = 1.41876, size = 195, normalized size = 1.1

$$\frac{48a^4 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec^3(c+dx)(3b(11a^2-5b^2)\cos(c+dx)+12b(b^2-2a^2)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-16a^2b+8a^3\sin(3(c+dx))+6ab^2\sin(c+dx))}{(a-b)^2(a+b)^2}$$

$24d$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]

[Out] $((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)}) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x] + 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2)/(24*d)$

Maple [A] time = 0.067, size = 269, normalized size = 1.5

$$-\frac{32}{3d(32a+32b)}\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^{-3} - 16\frac{1}{d(32a+32b)(\tan(1/2dx+c/2)-1)^2} + \frac{a}{d(a+b)^2}\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c)),x)

[Out] $-32/3/d/(\tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/d/(32*a+32*b)/(\tan(1/2*d*x+1/2*c)-1)^2+1/d/(a+b)^2/(\tan(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^2/(\tan(1/2*d*x+1/2*c)-1)*b+2/d*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^{(1/2)}*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-32/3/d/(\tan(1/2*d*x+1/2*c)+1)^3/(32*a-32*b)+16/d/(32*a-32*b)/(\tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^2/(\tan(1/2*d*x+1/2*c)+1)*a-1/2/d/(a-b)^2/(\tan(1/2*d*x+1/2*c)+1)*b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.63934, size = 1054, normalized size = 5.95

$$\left[\frac{3\sqrt{-a^2+b^2}a^4\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b-4a^2b^2}{6(a^6-3a^4b^2+3a^2b^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*(3*\sqrt{-a^2 + b^2})*a^4*\cos(d*x + c)^3*\log(((2*a^2 - b^2)*\cos(d*x + c) \\ &)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2 + 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x + c))*\sqrt{-a^2 + b^2}))/ (b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*\cos(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*\cos(d*x + c)^3), -1/3*(3*\sqrt{a^2 - b^2})*a^4*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c)))*\cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b - 3*a^2*b^3 + b^5)*\cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*\cos(d*x + c)^3)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)),x)

[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)), x)

Giac [A] time = 3.30637, size = 325, normalized size = 1.84

$$2 \left[\frac{3 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right) a^4}{(a^4 - 2a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{3a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 3a^2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^4 - 10a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 4ab^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 12a^2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 3b^3}{(a^4 - 2a^2b^2 + b^4) \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} \right] \frac{1}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out]
$$\begin{aligned} & 2/3*(3*(\pi*\operatorname{floor}(1/2*(d*x + c)/\pi + 1/2)*\operatorname{sgn}(a) + \arctan((a*\tan(1/2*d*x + 1/2*c) + b)/\sqrt{a^2 - b^2}))*a^4/((a^4 - 2*a^2*b^2 + b^4)*\sqrt{a^2 - b^2}) \\ & + (3*a^3*\tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*\tan(1/2*d*x + 1/2*c)^4 - 10*a^3*\tan(1/2*d*x + 1/2*c)^3 + 4*a*b^2*\tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*\tan(1/2*d*x + 1/2*c) - 6*b^3*\tan(1/2*d*x + 1/2*c)^2 + 3*a^3*\tan(1/2*d*x + 1/2*c) - 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3) \\ & /d \end{aligned}$$

$$3.177 \quad \int \frac{\tan^2(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=96

$$-\frac{2a^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}} + \frac{a \tan(c+dx)}{d(a^2-b^2)} - \frac{b \sec(c+dx)}{d(a^2-b^2)}$$

[Out] $(-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^{(3/2)}*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)$

Rubi [A] time = 0.0988934, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}} + \frac{a \tan(c+dx)}{d(a^2-b^2)} - \frac{b \sec(c+dx)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]

[Out] $(-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^{(3/2)}*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)$

Rule 2727

Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e + f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*p] && GtQ[p, 1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)} dx &= \frac{a \int \sec^2(c+dx) dx}{a^2-b^2} - \frac{a^2 \int \frac{1}{a+b\sin(c+dx)} dx}{a^2-b^2} - \frac{b \int \sec(c+dx) \tan(c+dx) dx}{a^2-b^2} \\ &= -\frac{a \operatorname{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{(a^2-b^2)d} - \frac{(2a^2) \operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)d} - \frac{b \int \sec(c+dx) \tan(c+dx) dx}{a^2-b^2} \\ &= -\frac{b \sec(c+dx)}{(a^2-b^2)d} + \frac{a \tan(c+dx)}{(a^2-b^2)d} + \frac{(4a^2) \operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2} dx, x, 2b+2a \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)d} \\ &= -\frac{2a^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}d} - \frac{b \sec(c+dx)}{(a^2-b^2)d} + \frac{a \tan(c+dx)}{(a^2-b^2)d} \end{aligned}$$

Mathematica [A] time = 0.202612, size = 152, normalized size = 1.58

$$\frac{\sqrt{a^2-b^2}(a \sin(c+dx) + b \cos(c+dx) - b) - 2a^2 \cos(c+dx) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2} \left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]), x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.053, size = 117, normalized size = 1.2

$$-2 \frac{a^2}{d(a-b)(a+b)\sqrt{a^2-b^2}} \arctan\left(\frac{1}{2} \frac{2a \tan(1/2 dx + c/2) + 2b}{\sqrt{a^2-b^2}}\right) - 8 \frac{1}{d(8a-8b)(\tan(1/2 dx + c/2) + 1)} - 8 \frac{1}{d(8a-8b)(\tan(1/2 dx + c/2) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2/(a+b*sin(d*x+c)),x)`

[Out] $-2/d*a^2/(a-b)/(a+b)/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-8/d/(8*a-8*b)/(\tan(1/2*d*x+1/2*c)+1)-8/d/(8*a+8*b)/(\tan(1/2*d*x+1/2*c)-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.67045, size = 684, normalized size = 7.12

$$\left[\frac{\sqrt{-a^2 + b^2} a^2 \cos(dx + c) \log\left(\frac{(2a^2 - b^2) \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2 + 2(a \cos(dx + c) \sin(dx + c) + b \cos(dx + c)) \sqrt{-a^2 + b^2}}{b^2 \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2}\right) - 2a^2b + 2b^3 + 2(a^3 - a^2b^2 + b^4)d \cos(dx + c)}{2(a^4 - 2a^2b^2 + b^4)d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")`

[Out] $[1/2*(\sqrt{-a^2 + b^2})*a^2*\cos(d*x + c)*\log(((2*a^2 - b^2)*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2 + 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x + c))*\sqrt{-a^2 + b^2}))/((b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*\sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*\cos(d*x + c)), (\sqrt{a^2 - b^2})*a^2*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2})*\cos(d*x + c)))*\cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*\sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*\cos(d*x + c))]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)),x)`

[Out] `Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)), x)`

Giac [A] time = 2.05295, size = 144, normalized size = 1.5

$$\frac{2 \left(\frac{\left(\left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + b}{\sqrt{a^2 - b^2}}\right)\right) a^2}{(a^2 - b^2)^{\frac{3}{2}}} + \frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b}{(a^2 - b^2) \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) - b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1))/d

$$3.178 \quad \int \frac{\cot^2(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b \tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

[Out] $(-2*\text{Sqrt}[a^2 - b^2]*\text{ArcTan}[(b + a*\text{Tan}[(c + d*x)/2]]/\text{Sqrt}[a^2 - b^2])/(a^2*d) + (b*\text{ArcTanh}[\text{Cos}[c + d*x]])/(a^2*d) - \text{Cot}[c + d*x]/(a*d)$

Rubi [A] time = 0.242757, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b \tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^2/(a + b*\text{Sin}[c + d*x]),x]$

[Out] $(-2*\text{Sqrt}[a^2 - b^2]*\text{ArcTan}[(b + a*\text{Tan}[(c + d*x)/2]]/\text{Sqrt}[a^2 - b^2])/(a^2*d) + (b*\text{ArcTanh}[\text{Cos}[c + d*x]])/(a^2*d) - \text{Cot}[c + d*x]/(a*d)$

Rule 2723

$\text{Int}[(a + b*\sin(e + f*x))^m/\tan(e + f*x)^2, x_Symbol] \rightarrow \text{Int}[(a + b*\sin(e + f*x))^m*(1 - \sin(e + f*x)^2)/\sin(e + f*x)^2, x] /;$ $\text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3056

$\text{Int}[(a + b*\sin(e + f*x))^m*((c + d*\sin(e + f*x))^n*((A + C)*\sin(e + f*x))^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 + a^2*C)*\text{Cos}[e + f*x]*(a + b*\sin(e + f*x))^{m+1}*(c + d*\sin(e + f*x))^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \text{Dist}[1/(m+1)*(b*c - a*d)*(a^2 - b^2), \text{Int}[(a + b*\sin(e + f*x))^{m+1}*(c + d*\sin(e + f*x))^n*\text{Simp}[a*(m+1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m+n+2) - (c*(A*b^2 + a^2*C) + b*(m+1)*(b*c - a*d)*(A + C))*\sin(e + f*x) - d*(A*b^2 + a^2*C)*(m+n+3)*\sin(e + f*x)^2, x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, C, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ ((\text{EqQ}[a, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n]) \ || \ !(\text{IntegerQ}[2*n] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ ((\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m]) \ || \ \text{EqQ}[a, 0])))$

Rule 3001

$\text{Int}[(A + B*\sin(e + f*x))/((a + b*\sin(e + f*x))^2), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/(b*c - a*d), \text{Int}[1/(a + b*\sin(e + f*x)), x], x] + \text{Dist}[(B*c - A*d)/(b*c - a*d), \text{Int}[1/(c + d*\sin(e + f*x)), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)} dx &= \int \frac{\csc^2(c+dx)(1-\sin^2(c+dx))}{a+b\sin(c+dx)} dx \\
 &= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)} dx}{a} \\
 &= -\frac{\cot(c+dx)}{ad} - \frac{b \int \csc(c+dx) dx}{a^2} + \frac{(-a^2+b^2) \int \frac{1}{a+b\sin(c+dx)} dx}{a^2} \\
 &= \frac{b \tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} - \frac{(2(a^2-b^2)) \text{Subst}\left(\int \frac{1}{a+2bx+ax^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d} \\
 &= \frac{b \tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} + \frac{(4(a^2-b^2)) \text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2} dx, x, 2b+2a \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d} \\
 &= -\frac{2\sqrt{a^2-b^2} \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2 d} + \frac{b \tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad}
 \end{aligned}$$

Mathematica [A] time = 0.235631, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + a \tan\left(\frac{1}{2}(c+dx)\right) - a \cot\left(\frac{1}{2}(c+dx)\right) - 2b \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]

[Out] (-4*sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/sqrt[a^2 - b^2]] - a*Cot[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*

Tan[(c + d*x)/2]/(2*a^2*d)

Maple [B] time = 0.06, size = 155, normalized size = 1.9

$$\frac{1}{2da} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \frac{1}{d\sqrt{a^2 - b^2}} \arctan\left(\frac{1}{2} \frac{2a \tan(1/2 dx + c/2) + 2b}{\sqrt{a^2 - b^2}}\right) + 2 \frac{b^2}{da^2\sqrt{a^2 - b^2}} \arctan\left(\frac{1}{2} \frac{2a \tan(1/2 dx + c/2) + 2b}{\sqrt{a^2 - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)),x)

[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.74227, size = 801, normalized size = 10.01

$$\frac{b \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - b \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) + \sqrt{-a^2 + b^2} \log\left(\frac{(2a^2 - b^2) \cos(dx + c)^2 - 2ab \sin(dx + c) + a^2}{2a^2 d \sin(dx + c)}\right)}{2a^2 d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)),x)

[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)), x)

Giac [A] time = 1.86967, size = 174, normalized size = 2.17

$$\frac{\frac{2b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a} + \frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + b}{\sqrt{a^2 - b^2}}\right)\right)\sqrt{a^2 - b^2}}{a^2} - \frac{2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - a}{a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] $-1/2*(2*b*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c))))/a^2 - \tan(1/2*d*x + 1/2*c)/a + 4*(\pi*\operatorname{floor}(1/2*(d*x + c)/\pi + 1/2)*\operatorname{sgn}(a) + \arctan((a*\tan(1/2*d*x + 1/2*c) + b)/\sqrt{a^2 - b^2}))*\sqrt{a^2 - b^2}/a^2 - (2*b*\tan(1/2*d*x + 1/2*c) - a)/(a^2*\tan(1/2*d*x + 1/2*c))/d$

$$3.179 \quad \int \frac{\cot^4(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=154

$$\frac{2(a^2 - b^2)^{3/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^4 d} + \frac{(4a^2 - 3b^2) \cot(c + dx)}{3a^3 d} - \frac{b(3a^2 - 2b^2) \tanh^{-1}(\cos(c + dx))}{2a^4 d} + \frac{b \cot(c + dx) \operatorname{csc}(c + dx)}{2a^2 d}$$

[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d)

Rubi [A] time = 0.425707, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(a^2 - b^2)^{3/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^4 d} + \frac{(4a^2 - 3b^2) \cot(c + dx)}{3a^3 d} - \frac{b(3a^2 - 2b^2) \tanh^{-1}(\cos(c + dx))}{2a^4 d} + \frac{b \cot(c + dx) \operatorname{csc}(c + dx)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]

[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d)

Rule 2725

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)^4, x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1] && IntegerQ[2*m]

Rule 3055

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3001

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2660

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)} dx &= \frac{b \cot(c+dx) \csc(c+dx)}{2a^2d} - \frac{\cot(c+dx) \csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)(2(4a^2-3b^2)-ab\sin(c+dx)-3(2a^2-ab\sin(c+dx)))}{a+b\sin(c+dx)} dx}{6a^2} \\ &= \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b \cot(c+dx) \csc(c+dx)}{2a^2d} - \frac{\cot(c+dx) \csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc(c+dx)}{a+b\sin(c+dx)} dx}{6a^2} \\ &= \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b \cot(c+dx) \csc(c+dx)}{2a^2d} - \frac{\cot(c+dx) \csc^2(c+dx)}{3ad} + \frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b \cot(c+dx) \csc(c+dx)}{2a^2d} \\ &= -\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b \cot(c+dx) \csc(c+dx)}{2a^2d} \\ &= \frac{2(a^2-b^2)^{3/2} \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} \end{aligned}$$

Mathematica [B] time = 6.11882, size = 350, normalized size = 2.27

$$\frac{(3a^2b-2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(2b^3-3a^2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\right)}{6a^3d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]

[Out] $(2*(a^2 - b^2)^{3/2}*\text{ArcTan}[(\text{Sec}[(c + d*x)/2]*(b*\text{Cos}[(c + d*x)/2] + a*\text{Sin}[(c + d*x)/2]))/\text{Sqrt}[a^2 - b^2]])/(a^4*d) + ((4*a^2*\text{Cos}[(c + d*x)/2] - 3*b^2*\text{Cos}[(c + d*x)/2])*\text{Csc}[(c + d*x)/2])/(6*a^3*d) + (b*\text{Csc}[(c + d*x)/2]^2)/(8*a^2*d) - (\text{Cot}[(c + d*x)/2]*\text{Csc}[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3)*\text{Log}[\text{Cos}[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*\text{Log}[\text{Sin}[(c + d*x)/2]])/(2*a^4*d) - (b*\text{Sec}[(c + d*x)/2]^2)/(8*a^2*d) + (\text{Sec}[(c + d*x)/2]*(-4*a^2*\text{Sin}[(c + d*x)/2] + 3*b^2*\text{Sin}[(c + d*x)/2]))/(6*a^3*d) + (\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/(24*a*d)$

Maple [B] time = 0.069, size = 348, normalized size = 2.3

$$\frac{1}{24da} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 - \frac{b}{8da^2} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{5}{8da} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{b^2}{2da^3} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \frac{1}{d\sqrt{a^2 - b^2}} \arctan\left(\frac{1}{2} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{b}{a} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c)),x)

[Out] $1/24/d/a*\tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*b*\tan(1/2*d*x+1/2*c)^2-5/8/d/a*\tan(1/2*d*x+1/2*c)+1/2/d/a^3*b^2*\tan(1/2*d*x+1/2*c)+2/d/(a^2-b^2)^{1/2}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{1/2})-4/d/a^2/(a^2-b^2)^{1/2}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{1/2})*b^2+2/d/a^4/(a^2-b^2)^{1/2}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{1/2})*b^4-1/24/d/a/\tan(1/2*d*x+1/2*c)^3+5/8/d/a/\tan(1/2*d*x+1/2*c)-1/2/d/a^3/\tan(1/2*d*x+1/2*c)*b^2+1/8/d/a^2*b/\tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*b*\ln(\tan(1/2*d*x+1/2*c))-1/d/a^4*b^3*\ln(\tan(1/2*d*x+1/2*c))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.92888, size = 1507, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] $[-1/12*(6*a^2*b*\cos(d*x + c)*\sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*\cos(d*x + c)^3 + 6*((a^2 - b^2)*\cos(d*x + c)^2 - a^2 + b^2)*\text{sqrt}(-a^2 + b^2)*\log(((2*a$

$$\begin{aligned} &^2 - b^2) \cos(dx + c)^2 - 2ab \sin(dx + c) - a^2 - b^2 + 2(a \cos(dx + c) \\ &+ b \sin(dx + c)) \sqrt{-a^2 + b^2} / (b^2 \cos(dx + c)^2 - 2 \\ &ab \sin(dx + c) - a^2 - b^2) \sin(dx + c) - 3(3a^2b - 2b^3 - (3a^2b \\ &b - 2b^3) \cos(dx + c)^2) \log(1/2 \cos(dx + c) + 1/2) \sin(dx + c) + 3(3a^2b \\ &- 2b^3 - (3a^2b - 2b^3) \cos(dx + c)^2) \log(-1/2 \cos(dx + c) + 1 \\ &/2) \sin(dx + c) + 12(a^3 - ab^2) \cos(dx + c) / ((a^4 d \cos(dx + c)^2 - \\ &a^4 d) \sin(dx + c)), -1/12(6a^2b \cos(dx + c) \sin(dx + c) - 4(4a^3 - \\ &3ab^2) \cos(dx + c)^3 + 12((a^2 - b^2) \cos(dx + c)^2 - a^2 + b^2) \sqrt{ \\ &(a^2 - b^2) \arctan(-(a \sin(dx + c) + b) / (\sqrt{a^2 - b^2} \cos(dx + c)))} \sin \\ &(dx + c) - 3(3a^2b - 2b^3 - (3a^2b - 2b^3) \cos(dx + c)^2) \log(1/2 \\ &\cos(dx + c) + 1/2) \sin(dx + c) + 3(3a^2b - 2b^3 - (3a^2b - 2b^3) \\ &\cos(dx + c)^2) \log(-1/2 \cos(dx + c) + 1/2) \sin(dx + c) + 12(a^3 - ab^2 \\ &) \cos(dx + c) / ((a^4 d \cos(dx + c)^2 - a^4 d) \sin(dx + c))] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**4/(a+b*sin(dx+c)),x)

[Out] Integral(cot(c + dx)**4/(a + b*sin(c + dx)), x)

Giac [A] time = 1.91054, size = 369, normalized size = 2.4

$$\frac{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 15a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 12b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3} + \frac{12(3a^2b - 2b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^4} + \frac{48(a^4 - 2a^2b^2 + b^4) \left(\pi \left\lfloor \frac{dx}{2} \right\rfloor + \frac{dx}{2} - \frac{c}{2} \right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^4/(a+b*sin(dx+c)),x, algorithm="giac")

[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x + 1/2*c)^3))/d

$$3.180 \quad \int \frac{\cot^6(c+dx)}{a+b \sin(c+dx)} dx$$

Optimal. Leaf size=307

$$\frac{2(a^2 - b^2)^{5/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^6 d} - \frac{(-35a^2 b^2 + 23a^4 + 15b^4) \cot(c+dx)}{15a^5 d} + \frac{b(-20a^2 b^2 + 15a^4 + 8b^4) \tanh^{-1}(\cos(c+dx))}{8a^6 d}$$

[Out] $(-2*(a^2 - b^2)^{(5/2)}*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) - ((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Csc[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)$

Rubi [A] time = 1.10601, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(a^2 - b^2)^{5/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^6 d} - \frac{(-35a^2 b^2 + 23a^4 + 15b^4) \cot(c+dx)}{15a^5 d} + \frac{b(-20a^2 b^2 + 15a^4 + 8b^4) \tanh^{-1}(\cos(c+dx))}{8a^6 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]

[Out] $(-2*(a^2 - b^2)^{(5/2)}*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) - ((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Csc[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)$

Rule 2726

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^6, x_Symbol] := -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4, x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m - 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]

Rule 3055

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]

```

*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3001

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rule 2660

```

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]

```

Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)} dx &= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{6a^3d} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{(15a^4-22a^2b^2+10b^4)\cot(c+dx)\csc^3(c+dx)}{30a^3b^2d} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} \\
&= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{8a^4bd} \\
&= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{8a^4bd} \\
&= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc^2(c+dx)}{2b^2d} \\
&= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc^2(c+dx)}{2b^2d} \\
&= -\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d}
\end{aligned}$$

Mathematica [A] time = 1.38587, size = 504, normalized size = 1.64

$$-1920(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-1120a^3b^2\tan\left(\frac{1}{2}(c+dx)\right)-32(-35a^3b^2+23a^5+15ab^4)\cot\left(\frac{1}{2}(c+dx)\right)+1$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]

[Out] $(-1920*(a^2-b^2)^{5/2}*ArcTan[(b+a*Tan[(c+d*x)/2])/Sqrt[a^2-b^2]] - 32*(23*a^5-35*a^3*b^2+15*a*b^4)*Cot[(c+d*x)/2] - 270*a^4*b*Csc[(c+d*x)/2]^2 + 120*a^2*b^3*Csc[(c+d*x)/2]^2 + 15*a^4*b*Csc[(c+d*x)/2]^4 + 1800*a^4*b*Log[Cos[(c+d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c+d*x)/2]] + 960*b^5*Log[Cos[(c+d*x)/2]] - 1800*a^4*b*Log[Sin[(c+d*x)/2]] + 2400*a^2*b^3*Log[Sin[(c+d*x)/2]] - 960*b^5*Log[Sin[(c+d*x)/2]] + 270*a^4*b*Sec[(c+d*x)/2]^2 - 120*a^2*b^3*Sec[(c+d*x)/2]^2 - 15*a^4*b*Sec[(c+d*x)/2]^4 - 656*a^5*Csc[c+d*x]^3*Sin[(c+d*x)/2]^4 + 320*a^3*b^2*Csc[c+d*x]^3*Sin[(c+d*x)/2]^4 + 41*a^5*Csc[(c+d*x)/2]^4*Sin[c+d*x] - 20*a^3*b^2*Csc[(c+d*x)/2]^4*Sin[c+d*x] - 3*a^5*Csc[(c+d*x)/2]^6*Sin[c+d*x] + 736*a^5*Tan[(c+d*x)/2] - 1120*a^3*b^2*Tan[(c+d*x)/2] + 480*a*b^4*Tan[(c+d*x)/2] + 6*a^5*Sec[(c+d*x)/2]^4*Tan[(c+d*x)/2])/(960*a^6*d)$

Maple [B] time = 0.073, size = 629, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -15/8/d/a^2*b*ln(tan(1/2*d*x+1/2*c))+5/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))+1/4/d/a^2*b*tan(1/2*d*x+1/2*c)^2-9/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+9/8/d/a^3/tan(1/2*d*x+1/2*c)*b^2-1/4/d/a^2*b/tan(1/2*d*x+1/2*c)^2+6/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/160/d/a*tan(1/2*d*x+1/2*c)^5-7/96/d/a*tan(1/2*d*x+1/2*c)^3+7/96/d/a/tan(1/2*d*x+1/2*c)^3-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4-1/64/d/a^2*b*tan(1/2*d*x+1/2*c)^4+1/24/d/a^3*b^2*tan(1/2*d*x+1/2*c)^3-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)+1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^3/tan(1/2*d*x+1/2*c)^2-1/d/a^6*b^5*ln(tan(1/2*d*x+1/2*c))-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/16/d/a*tan(1/2*d*x+1/2*c)-11/16/d/a/tan(1/2*d*x+1/2*c)+2/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-6/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.83409, size = 2566, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
```

```
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^6(c + dx)}{a + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 2.21212, size = 662, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

$$3.181 \quad \int \frac{\tan^5(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=242

$$-\frac{a^5}{d(a^2-b^2)^3(a+b \sin(c+dx))} + \frac{a^4(a^2+5b^2)\log(a+b \sin(c+dx))}{d(a^2-b^2)^4} + \frac{\sec^4(c+dx)(a^2-2ab \sin(c+dx)+b^2)}{4d(a^2-b^2)^2}$$

```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 + Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.632621, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{d(a^2-b^2)^3(a+b \sin(c+dx))} + \frac{a^4(a^2+5b^2)\log(a+b \sin(c+dx))}{d(a^2-b^2)^4} + \frac{\sec^4(c+dx)(a^2-2ab \sin(c+dx)+b^2)}{4d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 + Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*(a^2 - b^2)^3*d)
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int \frac{\tan^5(c + dx)}{(a + b \sin(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^2(b^2-x^2)^3} dx, x, b \sin(c + dx)\right)}{d}$$

$$= \frac{\sec^4(c + dx)(a^2 + b^2 - 2ab \sin(c + dx))}{4(a^2 - b^2)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^6}{(a^2-b^2)^2} - \frac{4a^4b^4x}{(a^2-b^2)^2} - \frac{6ab^6x^2}{(a^2-b^2)^2} - 4b^2x^3}{(a+x)^2(b^2-x^2)^2} dx, x, b \sin(c + dx)\right)}{4b^2d}$$

$$= \frac{\sec^4(c + dx)(a^2 + b^2 - 2ab \sin(c + dx))}{4(a^2 - b^2)^2 d} - \frac{\sec^2(c + dx)(2(2a^4 + 3a^2b^2 - b^4) - ab(9a^2 - b^2))}{4(a^2 - b^2)^3 d}$$

$$= \frac{\sec^4(c + dx)(a^2 + b^2 - 2ab \sin(c + dx))}{4(a^2 - b^2)^2 d} - \frac{\sec^2(c + dx)(2(2a^4 + 3a^2b^2 - b^4) - ab(9a^2 - b^2))}{4(a^2 - b^2)^3 d}$$

$$= -\frac{a(4a + b) \log(1 - \sin(c + dx))}{8(a + b)^4 d} - \frac{a(4a - b) \log(1 + \sin(c + dx))}{8(a - b)^4 d} + \frac{a^4(a^2 + 5b^2) \log(a + b \sin(c + dx))}{(a^2 - b^2)^4 d}$$

Mathematica [A] time = 6.25186, size = 240, normalized size = 0.99

$$-\frac{a^5}{d(a^2 - b^2)^3(a + b \sin(c + dx))} + \frac{a^4(a^2 + 5b^2) \log(a + b \sin(c + dx))}{d(a^2 - b^2)^4} - \frac{7a + 3b}{16d(a + b)^3(1 - \sin(c + dx))} - \frac{7a - 3b}{16d(a - b)^3(1 + \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]

[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 + Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) + 1/(16*(a + b)^2*d*(1 - Sin[c + d*x])^2) - (7*a + 3*b)/(16*(a + b)^3*d*(1 - Sin[c + d*x])) + 1/(16*(a - b)^2*d*(1 + Sin[c + d*x])^2) - (7*a - 3*b)/(16*(a - b)^3*d*(1 + Sin[c + d*x])) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))

Maple [A] time = 0.093, size = 318, normalized size = 1.3

$$-\frac{a^5}{d(a + b)^3(a - b)^3(a + b \sin(dx + c))} + \frac{a^6 \ln(a + b \sin(dx + c))}{d(a + b)^4(a - b)^4} + 5 \frac{a^4 \ln(a + b \sin(dx + c)) b^2}{d(a + b)^4(a - b)^4} + \frac{1}{16d(a + b)^2(\sin(dx + c) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x)

[Out] -1/d*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+1/d*a^6/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))+5/d*a^4/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^2+1/16/d/(a+b)^2/(sin(dx+c)+1)

$$\frac{n(d*x+c)-1)^2+3/16/d/(a+b)^3/(\sin(d*x+c)-1)*b+7/16/d/(a+b)^3/(\sin(d*x+c)-1)*a-1/2/d*a^2/(a+b)^4*\ln(\sin(d*x+c)-1)-1/8/d*a/(a+b)^4*\ln(\sin(d*x+c)-1)*b+1/16/d/(a-b)^2/(1+\sin(d*x+c))^2+3/16/d/(a-b)^3/(1+\sin(d*x+c))*b-7/16/d/(a-b)^3/(1+\sin(d*x+c))*a-1/2/d*a^2/(a-b)^4*\ln(1+\sin(d*x+c))+1/8/d*a/(a-b)^4*\ln(1+\sin(d*x+c))*b}$$

Maxima [B] time = 1.8279, size = 682, normalized size = 2.82

$$\frac{8(a^6+5a^4b^2)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{(4a^2-ab)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(4a^2+ab)\log(\sin(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{2(7a^5+6a^3b^2-ab^4+(4a^6-3a^4b^2+3a^2b^4-b^6)\sin(dx+c))}{a^7-3a^5b^2+3a^3b^4-ab^6+(a^6b-3a^4b^3+3a^2b^5-b^7)\sin(dx+c)}$$

8

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{8} \frac{(8(a^6 + 5a^4b^2) \log(b \sin(dx + c) + a) / (a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8) - (4a^2 - ab) \log(\sin(dx + c) + 1) / (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4) - (4a^2 + ab) \log(\sin(dx + c) - 1) / (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) - 2(7a^5 + 6a^3b^2 - ab^4 + (4a^6 + 9a^4b^2 - a^2b^4) \sin(dx + c)^4 + (5a^4b - 7a^2b^3 + 2b^5) \sin(dx + c)^3 - (12a^5 + 13a^3b^2 - a^2b^4) \sin(dx + c)^2 - (4a^4b - 5a^2b^3 + b^5) \sin(dx + c)) / (a^7 - 3a^5b^2 + 3a^3b^4 - ab^6 + (a^6b - 3a^4b^3 + 3a^2b^5 - b^7) \sin(dx + c)^5 + (a^7 - 3a^5b^2 + 3a^3b^4 - ab^6) \sin(dx + c)^4 - 2(a^6b - 3a^4b^3 + 3a^2b^5 - b^7) \sin(dx + c)^3 - 2(a^7 - 3a^5b^2 + 3a^3b^4 - ab^6) \sin(dx + c)^2 + (a^6b - 3a^4b^3 + 3a^2b^5 - b^7) \sin(dx + c)))}{d}$

Fricas [B] time = 3.38575, size = 1231, normalized size = 5.09

$$\frac{2a^7 - 6a^5b^2 + 6a^3b^4 - 2ab^6 - 2(4a^7 + 5a^5b^2 - 10a^3b^4 + ab^6) \cos(dx + c)^4 - 2(4a^7 - 9a^5b^2 + 6a^3b^4 - ab^6) \cos(dx + c)^3 + 8((a^6b + 5a^4b^3) \cos(dx + c)^4 \sin(dx + c) + (a^7 + 5a^5b^2) \cos(dx + c)^4 \log(b \sin(dx + c) + a) - ((4a^6b + 15a^5b^2 + 20a^4b^3 + 10a^3b^4 - ab^6) \cos(dx + c)^4 \sin(dx + c) + (4a^7 + 15a^6b + 20a^5b^2 + 10a^4b^3 - a^2b^5) \cos(dx + c)^4) \log(\sin(dx + c) + 1) - ((4a^6b - 15a^5b^2 + 20a^4b^3 - 10a^3b^4 + ab^6) \cos(dx + c)^4 \sin(dx + c) + (4a^7 - 15a^6b + 20a^5b^2 - 10a^4b^3 + a^2b^5) \cos(dx + c)^4) \log(-\sin(dx + c) + 1) - 2(a^6b - 3a^4b^3 + 3a^2b^5 - b^7 - (5a^6b - 12a^4b^3 + 9a^2b^5 - 2b^7) \cos(dx + c)^2) \sin(dx + c)}{(a^8b - 4a^6b^3 + 6a^4b^5 - 4a^2b^7 + b^9) d \cos(dx + c)^4 \sin(dx + c) + (a^9 - 4a^7b^2 + 6a^5b^4 - 4a^3b^6 + ab^8) d \cos(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $\frac{1}{8} \frac{(2a^7 - 6a^5b^2 + 6a^3b^4 - 2ab^6 - 2(4a^7 + 5a^5b^2 - 10a^3b^4 + ab^6) \cos(dx + c)^4 - 2(4a^7 - 9a^5b^2 + 6a^3b^4 - ab^6) \cos(dx + c)^3 + 8((a^6b + 5a^4b^3) \cos(dx + c)^4 \sin(dx + c) + (a^7 + 5a^5b^2) \cos(dx + c)^4 \log(b \sin(dx + c) + a) - ((4a^6b + 15a^5b^2 + 20a^4b^3 + 10a^3b^4 - ab^6) \cos(dx + c)^4 \sin(dx + c) + (4a^7 + 15a^6b + 20a^5b^2 + 10a^4b^3 - a^2b^5) \cos(dx + c)^4) \log(\sin(dx + c) + 1) - ((4a^6b - 15a^5b^2 + 20a^4b^3 - 10a^3b^4 + ab^6) \cos(dx + c)^4 \sin(dx + c) + (4a^7 - 15a^6b + 20a^5b^2 - 10a^4b^3 + a^2b^5) \cos(dx + c)^4) \log(-\sin(dx + c) + 1) - 2(a^6b - 3a^4b^3 + 3a^2b^5 - b^7 - (5a^6b - 12a^4b^3 + 9a^2b^5 - 2b^7) \cos(dx + c)^2) \sin(dx + c))}{(a^8b - 4a^6b^3 + 6a^4b^5 - 4a^2b^7 + b^9) d \cos(dx + c)^4 \sin(dx + c) + (a^9 - 4a^7b^2 + 6a^5b^4 - 4a^3b^6 + ab^8) d \cos(dx + c)^4}$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**2, x)

Giac [B] time = 4.67247, size = 667, normalized size = 2.76

$$\frac{8(a^6b+5a^4b^3)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9} - \frac{(4a^2-ab)\log(|\sin(dx+c)+1|)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(4a^2+ab)\log(|\sin(dx+c)-1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{8(a^6b\sin(dx+c)+5a^4b^3\sin(dx+c)+2a^7+4a^5b^2)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)(b\sin(dx+c)+a)} + \frac{2}{b\sin(dx+c)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/8*(8*(a^6*b + 5*a^4*b^3)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (4*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(abs(sin(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^6*b*sin(d*x + c) + 5*a^4*b^3*sin(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(3*a^6*sin(d*x + c)^4 + 15*a^4*b^2*sin(d*x + c)^4 - 9*a^5*b*sin(d*x + c)^3 + 10*a^3*b^3*sin(d*x + c)^3 - a*b^5*sin(d*x + c)^3 - 2*a^6*sin(d*x + c)^2 - 28*a^4*b^2*sin(d*x + c)^2 - 8*a^2*b^4*sin(d*x + c)^2 + 2*b^6*sin(d*x + c)^2 + 7*a^5*b*sin(d*x + c) - 6*a^3*b^3*sin(d*x + c) - a*b^5*sin(d*x + c) + 12*a^4*b^2 + 7*a^2*b^4 - b^6)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2)/d

$$3.182 \quad \int \frac{\tan^3(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=161

$$\frac{a^3}{d(a^2 - b^2)^2 (a + b \sin(c + dx))} - \frac{a^2 (a^2 + 3b^2) \log(a + b \sin(c + dx))}{d(a^2 - b^2)^3} + \frac{\sec^2(c + dx) (a^2 - 2ab \sin(c + dx) + b^2)}{2d(a^2 - b^2)^2} + \frac{a}{d(a^2 - b^2)}$$

[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)

Rubi [A] time = 0.311696, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{d(a^2 - b^2)^2 (a + b \sin(c + dx))} - \frac{a^2 (a^2 + 3b^2) \log(a + b \sin(c + dx))}{d(a^2 - b^2)^3} + \frac{\sec^2(c + dx) (a^2 - 2ab \sin(c + dx) + b^2)}{2d(a^2 - b^2)^2} + \frac{a}{d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]

[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 1647

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2} dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2(b^2-x^2)^2} dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a^2+b^2-2ab\sin(c+dx))}{2(a^2-b^2)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{(a^2-b^2)^2} - \frac{2a^2b^2x}{a^2-b^2} - \frac{2ab^4x^2}{(a^2-b^2)^2}}{(a+x)^2(b^2-x^2)} dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a^2+b^2-2ab\sin(c+dx))}{2(a^2-b^2)^2 d} + \frac{\text{Subst}\left(\int \left(-\frac{ab^2}{(a+b)^3(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)^2} - \frac{2a^2b^2}{(a-b)^3(a+x)}\right) dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{a \log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{a \log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{a^2(a^2+3b^2) \log(a+b\sin(c+dx))}{(a^2-b^2)^3 d} + \frac{1}{a^2}$$

Mathematica [A] time = 0.719396, size = 145, normalized size = 0.9

$$\frac{4a^3}{(a^2-b^2)^2(a+b\sin(c+dx))} - \frac{4a^2(a^2+3b^2) \log(a+b\sin(c+dx))}{(a^2-b^2)^3} - \frac{1}{(a+b)^2(\sin(c+dx)-1)} + \frac{1}{(a-b)^2(\sin(c+dx)+1)} + \frac{2a \log(1-\sin(c+dx))}{(a+b)^3} + \frac{2a \log(\sin(c+dx)+1)}{(a-b)^3}$$

$$4d$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]

[Out] ((2*a*Log[1 - Sin[c + d*x]])/(a + b)^3 + (2*a*Log[1 + Sin[c + d*x]])/(a - b)^3 - (4*a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]]/(a^2 - b^2)^3 - 1/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])) + (4*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])))/(4*d)

Maple [A] time = 0.087, size = 182, normalized size = 1.1

$$\frac{a^3}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))} - \frac{a^4 \ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} - 3 \frac{a^2 \ln(a+b\sin(dx+c))b^2}{d(a+b)^3(a-b)^3} - \frac{1}{4d(a+b)^2(\sin(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x)

[Out] 1/d*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))-1/d*a^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-3/d*a^2/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*b^2-1/4/d/(a+b)^2/(sin(d*x+c)-1)+1/2/d*a/(a+b)^3*ln(sin(d*x+c)-1)+1/4/d/(a-b)^2/(1+sin(d*x+c))+1/2*a*ln(1+sin(d*x+c))/(a-b)^3/d

Maxima [A] time = 1.75269, size = 370, normalized size = 2.3

$$\frac{2(a^4+3a^2b^2) \log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{a \log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} - \frac{a \log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a^3+ab^2-2(a^3+ab^2) \sin(dx+c)^2-(a^2b-b^3) \sin(dx+c)}{a^5-2a^3b^2+ab^4-(a^4b-2a^2b^3+b^5) \sin(dx+c)^3-(a^5-2a^3b^2+ab^4) \sin(dx+c)^2}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out]
$$-1/2*(2*(a^4 + 3*a^2*b^2)*\log(b*\sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - a*\log(\sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - a*\log(\sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3 + a*b^2 - 2*(a^3 + a*b^2)*\sin(d*x + c)^2 - (a^2*b - b^3)*\sin(d*x + c)))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*\sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^4)*\sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*\sin(d*x + c))/d$$

Fricas [B] time = 2.3941, size = 869, normalized size = 5.4

$$\frac{a^5 - 2a^3b^2 + ab^4 + 2(a^5 - ab^4)\cos(dx + c)^2 - 2((a^4b + 3a^2b^3)\cos(dx + c)^2 \sin(dx + c) + (a^5 + 3a^3b^2)\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out]
$$1/2*(a^5 - 2*a^3*b^2 + a*b^4 + 2*(a^5 - a*b^4)*\cos(d*x + c)^2 - 2*((a^4*b + 3*a^2*b^3)*\cos(d*x + c)^2*\sin(d*x + c) + (a^5 + 3*a^3*b^2)*\cos(d*x + c)^2)*\log(b*\sin(d*x + c) + a) + ((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*\cos(d*x + c)^2*\sin(d*x + c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) + ((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*\cos(d*x + c)^2*\sin(d*x + c) + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5)*\sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*\cos(d*x + c)^2*\sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*\cos(d*x + c)^2)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 1.84256, size = 335, normalized size = 2.08

$$\frac{2(a^4b+3a^2b^3)\log(b\sin(dx+c)+a)}{a^6b-3a^4b^3+3a^2b^5-b^7} - \frac{a\log(|\sin(dx+c)+1|)}{a^3-3a^2b+3ab^2-b^3} - \frac{a\log(|\sin(dx+c)-1|)}{a^3+3a^2b+3ab^2+b^3} - \frac{2a^3\sin(dx+c)^2+2ab^2\sin(dx+c)^2+a^2b\sin(dx+c)-b^3\sin(dx+c)-3a^4b+3a^2b^3}{(a^4-2a^2b^2+b^4)(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out]
$$-1/2*(2*(a^4*b + 3*a^2*b^3)*\log(\text{abs}(b*\sin(d*x + c) + a)))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - a*\log(\text{abs}(\sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2$$

$$\begin{aligned}
& - b^3) - a \log(\abs{\sin(dx + c) - 1}) / (a^3 + 3a^2b + 3ab^2 + b^3) - (2 \\
& * a^3 \sin(dx + c)^2 + 2ab^2 \sin(dx + c)^2 + a^2b \sin(dx + c) - b^3 \sin \\
& (dx + c) - 3a^3 - ab^2) / ((a^4 - 2a^2b^2 + b^4) * (b \sin(dx + c)^3 + a \sin \\
& (dx + c)^2 - b \sin(dx + c) - a)) / d
\end{aligned}$$

$$3.183 \quad \int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=109

$$-\frac{a}{d(a^2-b^2)(a+b \sin(c+dx))} + \frac{(a^2+b^2) \log(a+b \sin(c+dx))}{d(a^2-b^2)^2} - \frac{\log(1-\sin(c+dx))}{2d(a+b)^2} - \frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.0956814, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 801}

$$-\frac{a}{d(a^2-b^2)(a+b \sin(c+dx))} + \frac{(a^2+b^2) \log(a+b \sin(c+dx))}{d(a^2-b^2)^2} - \frac{\log(1-\sin(c+dx))}{2d(a+b)^2} - \frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 801

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2(b^2-x^2)} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^2(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^2} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{1}{2(a-b)^2(b+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{(a^2+b^2) \log(a+b \sin(c+dx))}{(a^2-b^2)^2 d} - \frac{1}{(a^2-b^2)^2 d} \end{aligned}$$

Mathematica [A] time = 0.286521, size = 162, normalized size = 1.49

$$\frac{a \left(-2 \left((a^2 + b^2) \log(a + b \sin(c + dx)) - a^2 + b^2 \right) + (a - b)^2 \log(1 - \sin(c + dx)) + (a + b)^2 \log(\sin(c + dx) + 1) \right) + b \sin(c + dx)}{2d(a - b)^2(a + b)^2(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] -(a*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(-a^2 + b^2 + (a^2 + b^2)*Log[a + b*Sin[c + d*x]])) + b*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(a^2 + b^2)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x]/(2*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x]))

Maple [A] time = 0.078, size = 132, normalized size = 1.2

$$-\frac{a}{d(a+b)(a-b)(a+b\sin(dx+c))} + \frac{\ln(a+b\sin(dx+c))a^2}{d(a+b)^2(a-b)^2} + \frac{\ln(a+b\sin(dx+c))b^2}{d(a+b)^2(a-b)^2} - \frac{\ln(\sin(dx+c)-1)}{2d(a+b)^2} - \frac{\ln(1+\sin(dx+c))}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^2,x)

[Out] -1/d*a/(a+b)/(a-b)/(a+b*sin(d*x+c))+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*a^2+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*b^2-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)-1/2*ln(1+sin(d*x+c))/(a-b)^2/d

Maxima [A] time = 1.7641, size = 167, normalized size = 1.53

$$\frac{\frac{2(a^2+b^2)\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} - \frac{2a}{a^3-ab^2+(a^2b-b^3)\sin(dx+c)} - \frac{\log(\sin(dx+c)+1)}{a^2-2ab+b^2} - \frac{\log(\sin(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(2*(a^2 + b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*a/(a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d

Fricas [A] time = 1.69463, size = 463, normalized size = 4.25

$$\frac{2a^3 - 2ab^2 - 2(a^3 + ab^2 + (a^2b + b^3)\sin(dx+c))\log(b\sin(dx+c)+a) + (a^3 + 2a^2b + ab^2 + (a^2b + 2ab^2 + b^3)\sin(dx+c))\log(b\sin(dx+c)-1)}{2((a^4b - 2a^2b^3 + b^5)d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(2*a^3 - 2*a*b^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*x + c))*log(b*sin(d*x + c) - 1))/d

$x + c)) \cdot \log(\sin(dx + c) + 1) + (a^3 - 2a^2b + ab^2 + (a^2b - 2ab^2 + b^3) \cdot \sin(dx + c)) \cdot \log(-\sin(dx + c) + 1) / ((a^4b - 2a^2b^3 + b^5) \cdot d \cdot \sin(dx + c) + (a^5 - 2a^3b^2 + ab^4) \cdot d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 1.28426, size = 211, normalized size = 1.94

$$\frac{\frac{2(a^2b+b^3) \log(|b \sin(dx+c)+a|)}{a^4b-2a^2b^3+b^5} - \frac{\log(|\sin(dx+c)+1|)}{a^2-2ab+b^2} - \frac{\log(|\sin(dx+c)-1|)}{a^2+2ab+b^2} - \frac{2(a^2b \sin(dx+c)+b^3 \sin(dx+c)+2a^3)}{(a^4-2a^2b^2+b^4)(b \sin(dx+c)+a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{2} \cdot \frac{2(a^2b + b^3) \cdot \log(\text{abs}(b \cdot \sin(dx + c) + a)) / (a^4b - 2a^2b^3 + b^5) - \log(\text{abs}(\sin(dx + c) + 1)) / (a^2 - 2ab + b^2) - \log(\text{abs}(\sin(dx + c) - 1)) / (a^2 + 2ab + b^2) - 2(a^2b \cdot \sin(dx + c) + b^3 \cdot \sin(dx + c) + 2a^3)}{((a^4 - 2a^2b^2 + b^4) \cdot (b \cdot \sin(dx + c) + a)) / d}$

$$3.184 \quad \int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=53

$$-\frac{\log(a+b \sin(c+dx))}{a^2d} + \frac{\log(\sin(c+dx))}{a^2d} + \frac{1}{ad(a+b \sin(c+dx))}$$

[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.0526635, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 44}

$$-\frac{\log(a+b \sin(c+dx))}{a^2d} + \frac{\log(\sin(c+dx))}{a^2d} + \frac{1}{ad(a+b \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b*Sin[c + d*x]))

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(a+b \sin(c+dx))}{a^2d} + \frac{1}{ad(a+b \sin(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.0760863, size = 42, normalized size = 0.79

$$\frac{\frac{a}{a+b \sin(c+dx)} - \log(a+b \sin(c+dx)) + \log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] (Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]] + a/(a + b*Sin[c + d*x]))/(a^2*d)

Maple [A] time = 0.038, size = 54, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{a^2d} - \frac{\ln(a+b\sin(dx+c))}{a^2d} + \frac{1}{da(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^2,x)

[Out] ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))

Maxima [A] time = 1.47039, size = 63, normalized size = 1.19

$$\frac{1}{ab\sin(dx+c)+a^2} - \frac{\log(b\sin(dx+c)+a)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] (1/(a*b*sin(d*x + c) + a^2) - log(b*sin(d*x + c) + a)/a^2 + log(sin(d*x + c))/a^2)/d

Fricas [A] time = 1.63667, size = 176, normalized size = 3.32

$$\frac{(b\sin(dx+c)+a)\log(b\sin(dx+c)+a) - (b\sin(dx+c)+a)\log\left(-\frac{1}{2}\sin(dx+c)\right) - a}{a^2bd\sin(dx+c) + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -((b*sin(d*x + c) + a)*log(b*sin(d*x + c) + a) - (b*sin(d*x + c) + a)*log(-1/2*sin(d*x + c)) - a)/(a^2*b*d*sin(d*x + c) + a^3*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 1.49888, size = 69, normalized size = 1.3

$$\frac{b \left(\frac{\log\left(\left| -\frac{a}{b \sin(dx+c)+a} + 1 \right| \right)}{a^2 b} + \frac{1}{(b \sin(dx+c)+a)ab} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] b*(log(abs(-a/(b*sin(d*x + c) + a) + 1)))/(a^2*b) + 1/((b*sin(d*x + c) + a)*a*b))/d

$$3.185 \quad \int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=114

$$\frac{a^2 - b^2}{a^3 d (a + b \sin(c + dx))} - \frac{(a^2 - 3b^2) \log(\sin(c + dx))}{a^4 d} + \frac{(a^2 - 3b^2) \log(a + b \sin(c + dx))}{a^4 d} + \frac{2b \csc(c + dx)}{a^3 d} - \frac{\csc^2(c + dx)}{2a^2 d}$$

[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) - (a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.114796, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$\frac{a^2 - b^2}{a^3 d (a + b \sin(c + dx))} - \frac{(a^2 - 3b^2) \log(\sin(c + dx))}{a^4 d} + \frac{(a^2 - 3b^2) \log(a + b \sin(c + dx))}{a^4 d} + \frac{2b \csc(c + dx)}{a^3 d} - \frac{\csc^2(c + dx)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]

[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) - (a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^2} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2 x^3} - \frac{2b^2}{a^3 x^2} + \frac{-a^2+3b^2}{a^4 x} + \frac{a^2-b^2}{a^3(a+x)^2} + \frac{a^2-3b^2}{a^4(a+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{2b \csc(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)}{2a^2 d} - \frac{(a^2-3b^2) \log(\sin(c+dx))}{a^4 d} + \frac{(a^2-3b^2) \log(a+b \sin(c+dx))}{a^4 d} \end{aligned}$$

Mathematica [A] time = 0.607729, size = 96, normalized size = 0.84

$$\frac{2(a^2 - 3b^2) \log(\sin(c + dx)) - 2(a^2 - 3b^2) \log(a + b \sin(c + dx)) + a^2 \csc^2(c + dx) + \frac{2a(a-b)(a+b)}{a+b \sin(c+dx)} - 4ab \csc(c + dx)}{2a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]

[Out] $-(4ab \operatorname{Csc}[c + dx] + a^2 \operatorname{Csc}[c + dx]^2 + 2(a^2 - 3b^2) \operatorname{Log}[\operatorname{Sin}[c + dx]] - 2(a^2 - 3b^2) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] + (2a(a - b)(a + b))/(a + b \operatorname{Sin}[c + dx]))/(2a^4d)$

Maple [A] time = 0.089, size = 150, normalized size = 1.3

$$\frac{\ln(a + b \sin(dx + c))}{a^2d} - 3 \frac{\ln(a + b \sin(dx + c))b^2}{da^4} - \frac{1}{da(a + b \sin(dx + c))} + \frac{b^2}{da^3(a + b \sin(dx + c))} - \frac{1}{2a^2d(\sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x)

[Out] $\ln(a+b \sin(dx+c))/a^2/d - 3/d/a^4 \ln(a+b \sin(dx+c)) * b^2 - 1/a/d/(a+b \sin(dx+c)) + 1/d/a^3/(a+b \sin(dx+c)) * b^2 - 1/2/d/a^2/\sin(dx+c)^2 - \ln(\sin(dx+c))/a^2/d + 3/d/a^4 \ln(\sin(dx+c)) * b^2 + 2/d/a^3 * b/\sin(dx+c)$

Maxima [A] time = 1.36351, size = 157, normalized size = 1.38

$$\frac{\frac{3ab \sin(dx+c) - 2(a^2 - 3b^2) \sin(dx+c)^2 - a^2}{a^3b \sin(dx+c)^3 + a^4 \sin(dx+c)^2} + \frac{2(a^2 - 3b^2) \log(b \sin(dx+c) + a)}{a^4} - \frac{2(a^2 - 3b^2) \log(\sin(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $1/2 * ((3ab \sin(dx + c) - 2(a^2 - 3b^2) \sin(dx + c)^2 - a^2)/(a^3b \sin(dx + c)^3 + a^4 \sin(dx + c)^2) + 2(a^2 - 3b^2) \log(b \sin(dx + c) + a)/a^4 - 2(a^2 - 3b^2) \log(\sin(dx + c))/a^4)/d$

Fricas [B] time = 1.61861, size = 599, normalized size = 5.25

$$\frac{3a^2b \sin(dx + c) - 3a^3 + 6ab^2 + 2(a^3 - 3ab^2) \cos(dx + c)^2 + 2(a^3 - 3ab^2 - (a^3 - 3ab^2) \cos(dx + c)^2) + (a^2b - 3b^3 - \dots)}{2(\dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/2 * (3a^2b \sin(dx + c) - 3a^3 + 6a^2b^2 + 2(a^3 - 3a^2b^2) \cos(dx + c)^2 + 2(a^3 - 3a^2b^2 - (a^3 - 3a^2b^2) \cos(dx + c)^2) + (a^2b - 3b^3 - \dots)$

$$\frac{(a^2b - 3b^3)\cos(dx + c)^2\sin(dx + c)\log(b\sin(dx + c) + a) - 2(a^3 - 3ab^2 - (a^3 - 3ab^2)\cos(dx + c)^2 + (a^2b - 3b^3 - (a^2b - 3b^3)\cos(dx + c)^2)\sin(dx + c))\log(-1/2\sin(dx + c))}{a^5d\cos(dx + c)^2 - a^5d + (a^4bd\cos(dx + c)^2 - a^4bd)\sin(dx + c)}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c + dx)}{(a + b\sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**3/(a+b*sin(dx+c))**2,x)

[Out] Integral(cot(c + dx)**3/(a + b*sin(c + dx))**2, x)

Giac [A] time = 1.97623, size = 223, normalized size = 1.96

$$\frac{2(a^2-3b^2)\log(|\sin(dx+c)|)}{a^4} - \frac{2(a^2b-3b^3)\log(|b\sin(dx+c)+a|)}{a^4b} + \frac{2(a^2b\sin(dx+c)-3b^3\sin(dx+c)+2a^3-4ab^2)}{(b\sin(dx+c)+a)a^4} - \frac{3a^2\sin(dx+c)^2-9b^2\sin(dx+c)^2+4a^2b\sin(dx+c)}{a^4\sin(dx+c)^2}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^3/(a+b*sin(dx+c))^2,x, algorithm="giac")

[Out]
$$\frac{-1/2*(2*(a^2 - 3*b^2)*\log(\text{abs}(\sin(dx + c)))/a^4 - 2*(a^2*b - 3*b^3)*\log(\text{abs}(b*\sin(dx + c) + a))/(a^4*b) + 2*(a^2*b*\sin(dx + c) - 3*b^3*\sin(dx + c) + 2*a^3 - 4*a*b^2)/((b*\sin(dx + c) + a)*a^4) - (3*a^2*\sin(dx + c)^2 - 9*b^2*\sin(dx + c)^2 + 4*a*b*\sin(dx + c) - a^2)/(a^4*\sin(dx + c)^2))/d$$

$$3.186 \quad \int \frac{\cot^5(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=188

$$\frac{(a^2 - b^2)^2}{a^5 d (a + b \sin(c + dx))} + \frac{(2a^2 - 3b^2) \csc^2(c + dx)}{2a^4 d} - \frac{4b(a^2 - b^2) \csc(c + dx)}{a^5 d} + \frac{(-6a^2 b^2 + a^4 + 5b^4) \log(\sin(c + dx))}{a^6 d} - \dots$$

[Out] $(-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c + d*x]))$

Rubi [A] time = 0.181468, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$\frac{(a^2 - b^2)^2}{a^5 d (a + b \sin(c + dx))} + \frac{(2a^2 - 3b^2) \csc^2(c + dx)}{2a^4 d} - \frac{4b(a^2 - b^2) \csc(c + dx)}{a^5 d} + \frac{(-6a^2 b^2 + a^4 + 5b^4) \log(\sin(c + dx))}{a^6 d} - \dots$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]

[Out] $(-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c + d*x]))$

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2} dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^2} dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2(a^2-b^2)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{(a^2-b^2)^2}{a^5(a+x)^2} + \frac{-a^4+6a^2b^2-5b^4}{a^6(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \dots$$

Mathematica [A] time = 6.14389, size = 187, normalized size = 0.99

$$\frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{(-6a^2b^2+a^4+5b^4)\log(\sin(c+dx))}{a^6d} - \frac{(-6a^2b^2+a^4+5b^4)\log(a+b\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2, x]

[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c + d*x]))

Maple [A] time = 0.098, size = 282, normalized size = 1.5

$$-\frac{\ln(a+b\sin(dx+c))}{a^2d} + 6\frac{\ln(a+b\sin(dx+c))b^2}{da^4} - 5\frac{\ln(a+b\sin(dx+c))b^4}{da^6} + \frac{1}{da(a+b\sin(dx+c))} - 2\frac{1}{da^3(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^2, x)

[Out] -ln(a+b*sin(d*x+c))/a^2/d+6/d/a^4*ln(a+b*sin(d*x+c))*b^2-5/d/a^6*ln(a+b*sin(d*x+c))*b^4+1/a/d/(a+b*sin(d*x+c))-2/d/a^3/(a+b*sin(d*x+c))*b^2+1/d/a^5/(a+b*sin(d*x+c))*b^4-1/4/d/a^2/sin(d*x+c)^4+1/d/a^2/sin(d*x+c)^2-3/2/d/a^4/sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a^2/d-6/d/a^4*ln(sin(d*x+c))*b^2+5/d/a^6*ln(sin(d*x+c))*b^4+2/3/d/a^3*b/sin(d*x+c)^3-4/d/a^3*b/sin(d*x+c)+4/d*b^3/a^5/sin(d*x+c)

Maxima [A] time = 1.73361, size = 255, normalized size = 1.36

$$\frac{5a^3b\sin(dx+c)+12(a^4-6a^2b^2+5b^4)\sin(dx+c)^4-3a^4-6(6a^3b-5ab^3)\sin(dx+c)^3+2(6a^4-5a^2b^2)\sin(dx+c)^2}{a^5b\sin(dx+c)^5+a^6\sin(dx+c)^4} - \frac{12(a^4-6a^2b^2+5b^4)\log(b\sin(dx+c)+a)}{a^6}$$

12 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2, x, algorithm="maxima")

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

Fricas [B] time = 2.29355, size = 1241, normalized size = 6.6

$$21 a^5 - 82 a^3 b^2 + 60 a b^4 + 12 (a^5 - 6 a^3 b^2 + 5 a b^4) \cos(dx + c)^4 - 2 (18 a^5 - 77 a^3 b^2 + 60 a b^4) \cos(dx + c)^2 - 12 (a^5 - 6 a^3 b^2 + 5 a b^4) \sin(dx + c)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.81061, size = 375, normalized size = 1.99

$$\frac{12(a^4 - 6a^2b^2 + 5b^4) \log(|\sin(dx+c)|)}{a^6} - \frac{12(a^4b - 6a^2b^3 + 5b^5) \log(|b \sin(dx+c) + a|)}{a^6b} + \frac{12(a^4b \sin(dx+c) - 6a^2b^3 \sin(dx+c) + 5b^5 \sin(dx+c) + 2a^5 - 8a^3b^2 + 6ab^4)}{(b \sin(dx+c) + a)a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
```

$$\begin{aligned} & x + c) - 6*a^2*b^3*\sin(d*x + c) + 5*b^5*\sin(d*x + c) + 2*a^5 - 8*a^3*b^2 + \\ & 6*a*b^4)/((b*\sin(d*x + c) + a)*a^6) - (25*a^4*\sin(d*x + c)^4 - 150*a^2*b^2* \\ & \sin(d*x + c)^4 + 125*b^4*\sin(d*x + c)^4 + 48*a^3*b*\sin(d*x + c)^3 - 48*a*b^ \\ & 3*\sin(d*x + c)^3 - 12*a^4*\sin(d*x + c)^2 + 18*a^2*b^2*\sin(d*x + c)^2 - 8*a^ \\ & 3*b*\sin(d*x + c) + 3*a^4)/(a^6*\sin(d*x + c)^4))/d \end{aligned}$$

$$3.187 \quad \int \frac{\tan^4(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=333

$$\frac{2a^5 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} + \frac{8a^3b^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} + \frac{a^4b \cos(c+dx)}{d(a^2-b^2)^3(a+b \sin(c+dx))} - \frac{(3a+b) \cos(c+dx)}{4d(a+b)^3(1-\sin(c+dx))}$$

[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(7/2)*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.626607, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2731, 2650, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^5 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} + \frac{8a^3b^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} + \frac{a^4b \cos(c+dx)}{d(a^2-b^2)^3(a+b \sin(c+dx))} - \frac{(3a+b) \cos(c+dx)}{4d(a+b)^3(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]

[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(7/2)*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))

Rule 2731

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m]/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, p/2]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2648


```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2} dx &= \int \left(\frac{1}{4(a+b)^2(-1+\sin(c+dx))^2} + \frac{3a+b}{4(a+b)^3(-1+\sin(c+dx))} + \frac{1}{4(a-b)^2(1+\sin(c+dx))} \right) dx \\
&= \frac{\int \frac{1}{(1+\sin(c+dx))^2} dx}{4(a-b)^2} - \frac{(3a-b) \int \frac{1}{1+\sin(c+dx)} dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2} dx}{4(a+b)^2} + \frac{(3a+b) \int \frac{1}{-1+\sin(c+dx)} dx}{4(a+b)^3} \\
&= \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))^2} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^2 d(1+\sin(c+dx))^2} \\
&= \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3 d(1-\sin(c+dx))} \\
&= \frac{8a^3 b^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))} \\
&= \frac{8a^3 b^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))} \\
&= \frac{2a^5 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{8a^3 b^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{12(a+b)^2 d(1-\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 1.96747, size = 341, normalized size = 1.02

$$\frac{24a^3(a^2+4b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{12a^4 b \cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))} - \frac{4(4a+b) \sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2 \sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]

[Out] ((24*a^3*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - (4*(4*a + b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(-4*a + b)*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*a^4*b*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x]))/(12*d)

Maple [A] time = 0.098, size = 382, normalized size = 1.2

$$-\frac{1}{3d(a+b)^2} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-3} - \frac{1}{2d(a+b)^2} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-2} + \frac{a}{d(a+b)^3} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} + 2 \frac{1}{d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2+1/d*a/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+2/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)+2/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b+2/d*a^5/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+8/d*a^3/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d*a/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50435, size = 1790, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - 2*(7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 - 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c)^4 - (7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**2,x)

[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 2.7838, size = 548, normalized size = 1.65

$$2 \left(\frac{3(a^5 + 4a^3b^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sqrt{a^2 - b^2}} + \frac{3(a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a^4b)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a \right)} + \frac{3a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{2}{3} * (3 * (a^5 + 4 * a^3 * b^2) * (\pi * \text{floor}(1/2 * (d * x + c) / \pi + 1/2) * \text{sgn}(a) + \arctan((a * \tan(1/2 * d * x + 1/2 * c) + b) / \sqrt{a^2 - b^2}))) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * \sqrt{a^2 - b^2}) + 3 * (a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c) + a^4 * b) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * (a * \tan(1/2 * d * x + 1/2 * c)^2 + 2 * b * \tan(1/2 * d * x + 1/2 * c) + a)) + (3 * a^4 * \tan(1/2 * d * x + 1/2 * c)^5 + 9 * a^2 * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 - 6 * a^3 * b * \tan(1/2 * d * x + 1/2 * c)^4 - 6 * a * b^3 * \tan(1/2 * d * x + 1/2 * c)^4 - 10 * a^4 * \tan(1/2 * d * x + 1/2 * c)^3 - 18 * a^2 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 + 4 * b^4 * \tan(1/2 * d * x + 1/2 * c)^3 + 24 * a^3 * b * \tan(1/2 * d * x + 1/2 * c)^2 + 3 * a^4 * \tan(1/2 * d * x + 1/2 * c) + 9 * a^2 * b^2 * \tan(1/2 * d * x + 1/2 * c) - 10 * a^3 * b - 2 * a * b^3) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * (\tan(1/2 * d * x + 1/2 * c)^2 - 1)^3) / d$

$$3.188 \quad \int \frac{\tan^2(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=200

$$\frac{2a^3 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} - \frac{4ab^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} - \frac{a^2b \cos(c+dx)}{d(a^2-b^2)^2(a+b \sin(c+dx))} + \frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}$$

[Out] $(-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)}*d - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)}*d + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))$

Rubi [A] time = 0.301028, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2731, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^3 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} - \frac{4ab^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}} - \frac{a^2b \cos(c+dx)}{d(a^2-b^2)^2(a+b \sin(c+dx))} + \frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]

[Out] $(-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)}*d - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)}*d + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))$

Rule 2731

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m]/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, p/2]

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2} dx &= \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{a^2}{(a^2-b^2)(a+b\sin(c+dx))} \right) dx \\
 &= \frac{\int \frac{1}{1+\sin(c+dx)} dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)} dx}{2(a+b)^2} - \frac{(2ab^2) \int \frac{1}{a+b\sin(c+dx)} dx}{(a^2-b^2)^2} - \frac{a^2 \int \frac{1}{(a+b\sin(c+dx))^2} dx}{a^2-b^2} \\
 &= \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} - \frac{a^2 b \cos(c+dx)}{(a^2-b^2)^2 d(a+b\sin(c+dx))} \\
 &= \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} - \frac{a^2 b \cos(c+dx)}{(a^2-b^2)^2 d(a+b\sin(c+dx))} \\
 &= -\frac{4ab^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} \\
 &= -\frac{4ab^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} \\
 &= -\frac{2a^3 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} - \frac{4ab^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))}
 \end{aligned}$$

Mathematica [A] time = 1.01231, size = 169, normalized size = 0.84

$$\frac{2a(a^2+2b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^2 b \cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]

[Out]
$$\frac{((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^{(5/2)} + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))}{d}$$

Maple [A] time = 0.088, size = 282, normalized size = 1.4

$$-\frac{1}{d(a+b)^2} \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} - 2 \frac{ab^2 \tan(1/2 dx + c/2)}{d(a-b)^2(a+b)^2 \left((\tan(1/2 dx + c/2))^2 a + 2 \tan(1/2 dx + c/2) b + a \right)} - 2 \frac{1}{d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x)

[Out]
$$-1/d/(a+b)^2/(\tan(1/2*d*x+1/2*c)-1)-2/d*a/(a-b)^2/(a+b)^2/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^2*\tan(1/2*d*x+1/2*c)-2/d*a^2/(a-b)^2/(a+b)^2/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b-2/d*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-4/d*a/(a-b)^2/(a+b)^2/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^2-1/d/(a-b)^2/(\tan(1/2*d*x+1/2*c)+1)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.95834, size = 1261, normalized size = 6.3

$$\frac{2a^4b - 4a^2b^3 + 2b^5 + 2(2a^4b - a^2b^3 - b^5)\cos(dx+c)^2 + ((a^3b + 2ab^3)\cos(dx+c)\sin(dx+c) + (a^4 + 2a^2b^2)\cos(dx+c)\sqrt{-a^2+b^2}\log(-((2a^2-b^2)\cos(dx+c)^2 - 2a*b*\sin(dx+c) - a^2 - b^2 - 2(a*\cos(dx+c)*\sin(dx+c) + b*\cos(dx+c))*\sqrt{-a^2+b^2}))/b^2*\cos(dx+c)^2 - 2a*b*\sin(dx+c) - a^2 - b^2)}{2((a^6b - 3a^4b^3 + 3a^2b^5 - b^7)d\cos(dx+c) + (a^4 + 2a^2b^2)\cos(dx+c)\sqrt{-a^2+b^2}\log(-((2a^2-b^2)\cos(dx+c)^2 - 2a*b*\sin(dx+c) - a^2 - b^2 - 2(a*\cos(dx+c)*\sin(dx+c) + b*\cos(dx+c))*\sqrt{-a^2+b^2}))/b^2*\cos(dx+c)^2 - 2a*b*\sin(dx+c) - a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out]
$$[-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*\cos(d*x + c)^2 + ((a^3*b + 2*a*b^3)*\cos(d*x + c)*\sin(d*x + c) + (a^4 + 2*a^2*b^2)*\cos(d*x + c))*\sqrt{-a^2 + b^2}*\log(-((2*a^2 - b^2)*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2 - 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x + c))*\sqrt{-a^2 + b^2}))/b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2) - 2*($$

$$a^5 - 2a^3b^2 + ab^4) \sin(dx + c) / ((a^6b - 3a^4b^3 + 3a^2b^5 - b^7) d \cos(dx + c) \sin(dx + c) + (a^7 - 3a^5b^2 + 3a^3b^4 - ab^6) d \cos(dx + c)), -(a^4b - 2a^2b^3 + b^5 + (2a^4b - a^2b^3 - b^5) \cos(dx + c))^2 - ((a^3b + 2ab^3) \cos(dx + c) \sin(dx + c) + (a^4 + 2a^2b^2) \cos(dx + c)) \sqrt{a^2 - b^2} \arctan(-(a \sin(dx + c) + b) / (\sqrt{a^2 - b^2} \cos(dx + c))) - (a^5 - 2a^3b^2 + ab^4) \sin(dx + c) / ((a^6b - 3a^4b^3 + 3a^2b^5 - b^7) d \cos(dx + c) \sin(dx + c) + (a^7 - 3a^5b^2 + 3a^3b^4 - ab^6) d \cos(dx + c))]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**2/(a+b*sin(dx+c))**2,x)

[Out] Integral(tan(c + dx)**2/(a + b*sin(c + dx))**2, x)

Giac [A] time = 2.15159, size = 339, normalized size = 1.7

$$2 \left(\frac{(a^3 + 2ab^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^4 - 2a^2b^2 + b^4) \sqrt{a^2 - b^2}} + \frac{a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 2ab^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + a^2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2b^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a^3}{\left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^4 + 2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - a} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2/(a+b*sin(dx+c))^2,x, algorithm="giac")

[Out] -2*((a^3 + 2a*b^2)*(pi*floor(1/2*(dx + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*a^2*b^2 + b^4)))/d

$$3.189 \quad \int \frac{\cot^2(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=115

$$\frac{2(a^2 - 2b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^3 d \sqrt{a^2 - b^2}} + \frac{2b \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{2 \cot(c + dx)}{a^2 d} + \frac{\cot(c + dx)}{ad(a + b \sin(c + dx))}$$

[Out] $(-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))$

Rubi [A] time = 0.452534, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$\frac{2(a^2 - 2b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^3 d \sqrt{a^2 - b^2}} + \frac{2b \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{2 \cot(c + dx)}{a^2 d} + \frac{\cot(c + dx)}{ad(a + b \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[c + d*x]^2/(a + b*\text{Sin}[c + d*x])^2, x]$

[Out] $(-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))$

Rule 2723

$\text{Int}[(a + b*\sin[e + f*x])^m/\tan[e + f*x]^2, x_Symbol] :> \text{Int}[(a + b*\sin[e + f*x])^m*(1 - \sin[e + f*x]^2)/\sin[e + f*x]^2, x] /;$ FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]

Rule 3056

$\text{Int}[(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^n*((A + C)*\sin[e + f*x]^2), x_Symbol] :> -\text{Simp}[(A*b^2 + a^2*C)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^n*\text{Simp}[a*(m+1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m+n+2) - (c*(A*b^2 + a^2*C) + b*(m+1)*(b*c - a*d)*(A + C))*\sin[e + f*x] - d*(A*b^2 + a^2*C)*(m+n+3)*\sin[e + f*x]^2, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3001

$\text{Int}[(A + B*\sin[e + f*x])/((a + b*\sin[e + f*x])*(c + d*\sin[e + f*x])), x_Symbol] :> \text{Dist}[(A*b - a*B)/(b*c - a*d), \text{Int}[1/(a + b*\sin[e + f*x]), x], x] + \text{Dist}[(B*c - A*d)/(b*c - a*d), \text{Int}[1/(c + d*\sin[e + f*x]), x], x] /;$ FreeQ[{a, b, c, d, e, f,

A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2660

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2} dx &= \int \frac{\csc^2(c+dx)(1-\sin^2(c+dx))}{(a+b\sin(c+dx))^2} dx \\
 &= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)(2(a^2-b^2)-(a^2-b^2)\sin^2(c+dx))}{a+b\sin(c+dx)} dx}{a(a^2-b^2)} \\
 &= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)(-2b(a^2-b^2)-a(a^2-b^2)\sin(c+dx))}{a+b\sin(c+dx)} dx}{a^2(a^2-b^2)} \\
 &= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx) dx}{a^3} - \frac{(a^2-2b^2)\int \frac{1}{a+b\sin(c+dx)} dx}{a^3} \\
 &= \frac{2b \tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2(a^2-2b^2)) \text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)} dx\right)}{a^3} \\
 &= \frac{2b \tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4(a^2-2b^2)) \text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)} dx\right)}{a^3} \\
 &= -\frac{2(a^2-2b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}d} + \frac{2b \tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}
 \end{aligned}$$

Mathematica [A] time = 0.745506, size = 139, normalized size = 1.21

$$\frac{4(a^2-2b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ab \cos(c+dx)}{a+b\sin(c+dx)} - a \tan\left(\frac{1}{2}(c+dx)\right) + a \cot\left(\frac{1}{2}(c+dx)\right) + 4b \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)$$

$2a^3d$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Sin[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*x)/2])/(2*a^3*d)
```

Maple [B] time = 0.086, size = 245, normalized size = 2.1

$$\frac{1}{2da^2} \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \frac{b^2 \tan(1/2 dx + c/2)}{da^3 \left((\tan(1/2 dx + c/2))^2 a + 2 \tan(1/2 dx + c/2) b + a \right)} - 2 \frac{b}{da^2 \left((\tan(1/2 dx + c/2))^2 a + 2 \tan(1/2 dx + c/2) b + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.81611, size = 1681, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) - 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)]/(
```

$$(a^5*b - a^3*b^3)*d*\cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*\sin(d*x + c) - (a^5*b - a^3*b^3)*d, (2*(a^3*b - a*b^3)*\cos(d*x + c)*\sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*\cos(d*x + c)^2 + (a^3 - 2*a*b^2)*\sin(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c))) + (a^4 - a^2*b^2)*\cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*\cos(d*x + c)^2 + (a^3*b - a*b^3)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) + (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*\cos(d*x + c)^2 + (a^3*b - a*b^3)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*\cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*\sin(d*x + c) - (a^5*b - a^3*b^3)*d)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 1.71518, size = 294, normalized size = 2.56

$$\frac{12b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^3} - \frac{3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^2} + \frac{12 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + b}{\sqrt{a^2 - b^2}}\right) \right) (a^2 - 2b^2)}{\sqrt{a^2 - b^2} a^3} - \frac{4ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^3 + 2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2 + 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d

$$3.190 \quad \int \frac{\cot^4(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=238

$$\frac{2(-5a^2b^2 + a^4 + 4b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2-b^2}}\right)}{a^5 d \sqrt{a^2-b^2}} + \frac{(7a^2 - 12b^2) \cot(c+dx)}{3a^4 d} - \frac{b(3a^2 - 4b^2) \tanh^{-1}(\cos(c+dx))}{a^5 d} - \frac{(a^2 - b^2) \cot(c+dx)}{a^5 d}$$

[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.695298, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(-5a^2b^2 + a^4 + 4b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2-b^2}}\right)}{a^5 d \sqrt{a^2-b^2}} + \frac{(7a^2 - 12b^2) \cot(c+dx)}{3a^4 d} - \frac{b(3a^2 - 4b^2) \tanh^{-1}(\cos(c+dx))}{a^5 d} - \frac{(a^2 - b^2) \cot(c+dx)}{a^5 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]

[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x]))

Rule 2724

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^4, x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 + b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 3055

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)]

```

2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3001

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*SIN[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rule 2660

```

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]

```

Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2} dx &= \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)(6(a^2-2b^2)-ab\sin(c+dx))}{a+b\sin(c+dx)} dx}{3ad(a+b\sin(c+dx))} \\
&= -\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{3ad(a+b\sin(c+dx))} \\
&= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)}{3a^2bd(a+b\sin(c+dx))} \\
&= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)}{3a^2bd(a+b\sin(c+dx))} \\
&= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)}{a^3bd} \\
&= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)}{a^3bd} \\
&= \frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}d} - \frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}
\end{aligned}$$

Mathematica [A] time = 6.29471, size = 403, normalized size = 1.69

$$\frac{(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{(4b^3-3a^2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)}{a+b\sin(c+dx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]

[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)

Maple [B] time = 0.096, size = 527, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x)

[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^2*tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/2*c))

$$2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^2*\tan(1/2*d*x+1/2*c)-2/d/a^5/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*\tan(1/2*d*x+1/2*c)*b^4+2/d/a^2/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b-2/d/a^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^3+2/d/a/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-10/d/a^3/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^2+8/d/a^5/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^4-1/24/d/a^2/\tan(1/2*d*x+1/2*c)^3+5/8/d/a^2/\tan(1/2*d*x+1/2*c)-3/2/d/a^4/\tan(1/2*d*x+1/2*c)*b^2+1/4/d/a^3*b/\tan(1/2*d*x+1/2*c)^2+3/d/a^3*b*\ln(\tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*\ln(\tan(1/2*d*x+1/2*c))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.10123, size = 2639, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*(4*(2*a^4 - 3*a^2*b^2)*\cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*\cos(d*x + c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*\cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*a*b^2)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2 + 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x + c))*\sqrt{-a^2 + b^2}))/ (b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2) - 6*(a^4 - 2*a^2*b^2)*\cos(d*x + c) + 3*((3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*\cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*b^3)*\cos(d*x + c))*\sin(d*x + c))/ (a^5*b*d*\cos(d*x + c)^4 - 2*a^5*b*d*\cos(d*x + c)^2 + a^5*b*d - (a^6*d*\cos(d*x + c)^2 - a^6*d)*\sin(d*x + c)), -1/6*(4*(2*a^4 - 3*a^2*b^2)*\cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*\cos(d*x + c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*\cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*a*b^2)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*\cos(d*x + c) + 3*((3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*\cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*\cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*b^3)*\cos(d*x + c))*\sin(d*x + c))/ (a^5*b*d*\cos(d*x + c)^4 - 2*a^5*b*d*\cos(d*x + c)^2 + a^5*b*d - (a^6*d*\cos(d*x + c)^2 - a^6*d)*\sin(d*x + c)) \end{aligned}$$

]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c + dx)}{(a + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**2, x)

Giac [A] time = 1.85931, size = 481, normalized size = 2.02

$$\frac{24(3a^2b-4b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5} + \frac{48(a^4-5a^2b^2+4b^4)\left(\pi\left[\frac{dx+c}{2\pi}+\frac{1}{2}\right]\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^5} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*x + 1/2*c)^3))/d

$$3.191 \quad \int \frac{\cot^6(c+dx)}{(a+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=424

$$\frac{2(a^2 - 6b^2)(a^2 - b^2)^{3/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^7 d} - \frac{(-135a^2 b^2 + 38a^4 + 90b^4) \cot(c+dx)}{15a^6 d} + \frac{b(-40a^2 b^2 + 15a^4 + 24b^4)}{4a^7 d}$$

[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) + ((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((15*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 1.49342, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(a^2 - 6b^2)(a^2 - b^2)^{3/2} \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^7 d} - \frac{(-135a^2 b^2 + 38a^4 + 90b^4) \cot(c+dx)}{15a^6 d} + \frac{b(-40a^2 b^2 + 15a^4 + 24b^4)}{4a^7 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]

[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) + ((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((15*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))

Rule 2726

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6, x_Symbol] := -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4, x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m - 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N

$eQ[a^2 - b^2, 0] \&\& \text{NeQ}[m, 1] \&\& \text{IntegerQ}[2*m]$

Rule 3055

$\text{Int}[\left((a_{\cdot}) + (b_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]\right)^{(m_{\cdot})}\left((c_{\cdot}) + (d_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]\right)^{(n_{\cdot})}\left((A_{\cdot}) + (B_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})] + (C_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^2\right), x_{\text{Symbol}}] \rightarrow -\text{Simp}[\left((A*b^2 - a*b*B + a^2*C)\cos[e + f*x]\right)^{(m+1)}(c + d*\sin[e + f*x])^{(n+1)}/(f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}(c + d*\sin[e + f*x])^n*\text{Simp}[(m+1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m+n+2) - (c*(A*b^2 - a*b*B + a^2*C) + (m+1)*(b*c - a*d)*(A*b - a*B + b*C))*\sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m+n+3)*\sin[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& ((\text{EqQ}[a, 0] \&\& \text{IntegerQ}[m] \&\& !\text{IntegerQ}[n]) || !(\text{IntegerQ}[2*n] \&\& \text{LtQ}[n, -1] \&\& ((\text{IntegerQ}[n] \&\& !\text{IntegerQ}[m]) || \text{EqQ}[a, 0])))$

Rule 3001

$\text{Int}[\left((A_{\cdot}) + (B_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]\right)/\left(\left((a_{\cdot}) + (b_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]\right)\left((c_{\cdot}) + (d_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]\right)\right), x_{\text{Symbol}}] \rightarrow \text{Dist}[(A*b - a*B)/(b*c - a*d), \text{Int}[1/(a + b*\sin[e + f*x]), x], x] + \text{Dist}[(B*c - A*d)/(b*c - a*d), \text{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3770

$\text{Int}[\csc[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})], x_{\text{Symbol}}] \rightarrow -\text{Simp}[\text{ArcTanh}[\cos[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2660

$\text{Int}[\left((a_{\cdot}) + (b_{\cdot})\sin[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})]\right)^{-1}, x_{\text{Symbol}}] \rightarrow \text{With}\{e = \text{FreeFactors}[\tan[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a*e^2*x^2), x], x, \tan[(c + d*x)/2]/e], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 618

$\text{Int}[\left((a_{\cdot}) + (b_{\cdot})(x_{\cdot}) + (c_{\cdot})(x_{\cdot})^2\right)^{-1}, x_{\text{Symbol}}] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\left((a_{\cdot}) + (b_{\cdot})(x_{\cdot})^2\right)^{-1}, x_{\text{Symbol}}] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] || \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2} dx &= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{(2a^4-12a^2b^2+9b^4)\cot(c+dx)\csc^3(c+dx)}{6a^3b^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= -\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad^2} \\
&= -\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d}
\end{aligned}$$

Mathematica [A] time = 1.59178, size = 361, normalized size = 0.85

$$1920(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+240b(-40a^2b^2+15a^4+24b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-240b(-40a^2b^2+15a^4+24b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]

[Out] -(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 - 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] + 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(960*a^7*d)

Maple [B] time = 0.108, size = 897, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cot(dx+c)^6/(a+b*\sin(dx+c))^2,x)$

[Out]
$$-7/96/d/a^2*\tan(1/2*d*x+1/2*c)^3+7/96/d/a^2/\tan(1/2*d*x+1/2*c)^3-1/160/d/a^2/\tan(1/2*d*x+1/2*c)^5+1/160/d/a^2*\tan(1/2*d*x+1/2*c)^5-2/d/a^2/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-15/4/d/a^3*b*\ln(\tan(1/2*d*x+1/2*c))-11/16/d/a^2/\tan(1/2*d*x+1/2*c)+11/16/d/a^2*\tan(1/2*d*x+1/2*c)+16/d/a^3/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^2-1/2/d/a^5*\tan(1/2*d*x+1/2*c)^2*b^3+5/2/d/a^6*b^4*\tan(1/2*d*x+1/2*c)+1/32/d/a^3*b/\tan(1/2*d*x+1/2*c)^4+1/2/d*b^3/a^5/\tan(1/2*d*x+1/2*c)^2-6/d/a^7*b^5*\ln(\tan(1/2*d*x+1/2*c))-2/d/a^6/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^5-1/8/d/a^4/\tan(1/2*d*x+1/2*c)^3*b^2-5/2/d/a^6/\tan(1/2*d*x+1/2*c)*b^4-1/32/d/a^3*b*\tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^2*\tan(1/2*d*x+1/2*c)^3-26/d/a^5/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^4+27/8/d/a^4/\tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^3*b/\tan(1/2*d*x+1/2*c)^2+10/d/a^5*b^3*\ln(\tan(1/2*d*x+1/2*c))+1/2/d/a^3*b*\tan(1/2*d*x+1/2*c)^2-27/8/d/a^4*b^2*\tan(1/2*d*x+1/2*c)+4/d/a^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^3+4/d/a^5/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*\tan(1/2*d*x+1/2*c)*b^4+12/d/a^7/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^6-2/d/a^7/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*\tan(1/2*d*x+1/2*c)*b^6-2/d/a^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)*b^2*\tan(1/2*d*x+1/2*c)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cot(dx+c)^6/(a+b*\sin(dx+c))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 5.53225, size = 4674, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cot(dx+c)^6/(a+b*\sin(dx+c))^2,x, \text{algorithm}="fricas")$

[Out]
$$[1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*\cos(dx + c)^5 - 40*(7*a^6 - 27*a^4*b^2 + 18*a^2*b^4)*\cos(dx + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(dx + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(dx + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(dx + c)^2 - (a^5 - 7*a^3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*\cos(dx + c)^4 - 2*(a^5 - 7*a^3*b^2 + 6*a*b^4)*\cos(dx + c)^2)*\sin(dx + c))*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(dx + c)^2 - 2*a*b*\sin(dx + c) - a^2 - b^2 + 2*(a*\cos(dx + c))*\sin(dx + c) + b*\cos(dx + c))*\sqrt{-a^2 + b^2})/(b^2*\cos(dx + c)^2 - 2*a*b*\sin(dx + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*\cos(dx + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(dx + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(dx + c)$$

$$\begin{aligned}
&^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*\cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*\cos(d*x + c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c))*\sin(d*x + c))/(a^7*b*d*\cos(d*x + c)^6 - 3*a^7*b*d*\cos(d*x + c)^4 + 3*a^7*b*d*\cos(d*x + c)^2 - a^7*b*d - (a^8*d*\cos(d*x + c)^4 - 2*a^8*d*\cos(d*x + c)^2 + a^8*d)*\sin(d*x + c)), 1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*\cos(d*x + c)^5 - 40*(7*a^6 - 27*a^4*b^2 + 18*a^2*b^4)*\cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*\cos(d*x + c)^2 - (a^5 - 7*a^3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*\cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^2 + 6*a*b^4)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*\cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*\cos(d*x + c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*\cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*\cos(d*x + c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*\cos(d*x + c))*\sin(d*x + c))/(a^7*b*d*\cos(d*x + c)^6 - 3*a^7*b*d*\cos(d*x + c)^4 + 3*a^7*b*d*\cos(d*x + c)^2 - a^7*b*d - (a^8*d*\cos(d*x + c)^4 - 2*a^8*d*\cos(d*x + c)^2 + a^8*d)*\sin(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.3698, size = 805, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")

```
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 1620*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 - (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

$$3.192 \quad \int \frac{\tan^5(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=321

$$-\frac{a^5}{2d(a^2-b^2)^3(a+b \sin(c+dx))^2} - \frac{a^4(a^2+5b^2)}{d(a^2-b^2)^4(a+b \sin(c+dx))} + \frac{a^3(13a^2b^2+a^4+10b^4)\log(a+b \sin(c+dx))}{d(a^2-b^2)^5}$$

[Out] $-\left(\left(8a^2 - 5ab - b^2\right)\text{Log}\left[1 - \text{Sin}\left[c + d*x\right]\right]\right)/\left(16\left(a + b\right)^5*d\right) - \left(\left(8a^2 + 5ab - b^2\right)\text{Log}\left[1 + \text{Sin}\left[c + d*x\right]\right]\right)/\left(16\left(a - b\right)^5*d\right) + \left(a^3\left(a^4 + 13a^2*b^2 + 10*b^4\right)\text{Log}\left[a + b*\text{Sin}\left[c + d*x\right]\right]\right)/\left(\left(a^2 - b^2\right)^5*d\right) - a^5/\left(2\left(a^2 - b^2\right)^3*d\left(a + b*\text{Sin}\left[c + d*x\right]\right)^2\right) - \left(a^4\left(a^2 + 5*b^2\right)\right)/\left(\left(a^2 - b^2\right)^4*d\left(a + b*\text{Sin}\left[c + d*x\right]\right)\right) + \left(\text{Sec}\left[c + d*x\right]^4\left(a\left(a^2 + 3*b^2\right) - b\left(3*a^2 + b^2\right)*\text{Sin}\left[c + d*x\right]\right)\right)/\left(4\left(a^2 - b^2\right)^3*d\right) - \left(\text{Sec}\left[c + d*x\right]^2\left(8*a^3\left(a^2 + 5*b^2\right) - b\left(2*7*a^4 + 22*a^2*b^2 - b^4\right)*\text{Sin}\left[c + d*x\right]\right)\right)/\left(8\left(a^2 - b^2\right)^4*d\right)$

Rubi [A] time = 0.877422, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{2d(a^2-b^2)^3(a+b \sin(c+dx))^2} - \frac{a^4(a^2+5b^2)}{d(a^2-b^2)^4(a+b \sin(c+dx))} + \frac{a^3(13a^2b^2+a^4+10b^4)\log(a+b \sin(c+dx))}{d(a^2-b^2)^5}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]

[Out] $-\left(\left(8a^2 - 5ab - b^2\right)\text{Log}\left[1 - \text{Sin}\left[c + d*x\right]\right]\right)/\left(16\left(a + b\right)^5*d\right) - \left(\left(8a^2 + 5ab - b^2\right)\text{Log}\left[1 + \text{Sin}\left[c + d*x\right]\right]\right)/\left(16\left(a - b\right)^5*d\right) + \left(a^3\left(a^4 + 13a^2*b^2 + 10*b^4\right)\text{Log}\left[a + b*\text{Sin}\left[c + d*x\right]\right]\right)/\left(\left(a^2 - b^2\right)^5*d\right) - a^5/\left(2\left(a^2 - b^2\right)^3*d\left(a + b*\text{Sin}\left[c + d*x\right]\right)^2\right) - \left(a^4\left(a^2 + 5*b^2\right)\right)/\left(\left(a^2 - b^2\right)^4*d\left(a + b*\text{Sin}\left[c + d*x\right]\right)\right) + \left(\text{Sec}\left[c + d*x\right]^4\left(a\left(a^2 + 3*b^2\right) - b\left(3*a^2 + b^2\right)*\text{Sin}\left[c + d*x\right]\right)\right)/\left(4\left(a^2 - b^2\right)^3*d\right) - \left(\text{Sec}\left[c + d*x\right]^2\left(8*a^3\left(a^2 + 5*b^2\right) - b\left(2*7*a^4 + 22*a^2*b^2 - b^4\right)*\text{Sin}\left[c + d*x\right]\right)\right)/\left(8\left(a^2 - b^2\right)^4*d\right)$

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 1647

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629


```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\tan^5(c + dx)}{(a + b \sin(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3(b^2-x^2)^3} dx, x, b \sin(c + dx)\right)}{d}$$

$$= \frac{\sec^4(c + dx) (a(a^2 + 3b^2) - b(3a^2 + b^2) \sin(c + dx))}{4(a^2 - b^2)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3 b^6 (3a^2 + b^2) - a^2 b^4 (4a^4 + 3a^2 b^2 - b^4)}{(a^2 - b^2)^3} - \frac{a^2 b^4 (4a^4 + 3a^2 b^2 - b^4)}{(a^2 - b^2)^3}}{(a^2 - b^2)^3} dx, x, b \sin(c + dx)\right)}{d}$$

$$= \frac{\sec^4(c + dx) (a(a^2 + 3b^2) - b(3a^2 + b^2) \sin(c + dx))}{4(a^2 - b^2)^3 d} - \frac{\sec^2(c + dx) (8a^3(a^2 + 5b^2) - b(8a^2 + 5ab - b^2) \log(1 - \sin(c + dx)))}{8(a^2 - b^2)^3 d}$$

$$= \frac{\sec^4(c + dx) (a(a^2 + 3b^2) - b(3a^2 + b^2) \sin(c + dx))}{4(a^2 - b^2)^3 d} - \frac{\sec^2(c + dx) (8a^3(a^2 + 5b^2) - b(8a^2 + 5ab - b^2) \log(1 + \sin(c + dx)))}{8(a^2 - b^2)^3 d}$$

$$= -\frac{(8a^2 - 5ab - b^2) \log(1 - \sin(c + dx))}{16(a + b)^5 d} - \frac{(8a^2 + 5ab - b^2) \log(1 + \sin(c + dx))}{16(a - b)^5 d} + \frac{a^3(a^4 + 5a^2 b^2 + 5ab^3)}{d(a^2 - b^2)^5}$$

Mathematica [A] time = 6.34144, size = 304, normalized size = 0.95

$$\frac{a^5}{2d(a^2 - b^2)^3(a + b \sin(c + dx))^2} - \frac{a^4(a^2 + 5b^2)}{d(a^2 - b^2)^4(a + b \sin(c + dx))} + \frac{a^3(13a^2 b^2 + a^4 + 10b^4) \log(a + b \sin(c + dx))}{d(a^2 - b^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) - ((8*a^2 +
5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^2*b
b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) + 1/(16*(a + b)^3*
d*(1 - Sin[c + d*x])^2) - (7*a + b)/(16*(a + b)^4*d*(1 - Sin[c + d*x])) + 1
/(16*(a - b)^3*d*(1 + Sin[c + d*x])^2) - (7*a - b)/(16*(a - b)^4*d*(1 + Sin
[c + d*x])) - a^5/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 +
5*b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.116, size = 465, normalized size = 1.5

$$\frac{a^5}{2d(a + b)^3(a - b)^3(a + b \sin(dx + c))^2} + \frac{a^7 \ln(a + b \sin(dx + c))}{d(a + b)^5(a - b)^5} + 13 \frac{a^5 \ln(a + b \sin(dx + c)) b^2}{d(a + b)^5(a - b)^5} + 10 \frac{a^3 \ln(a + b \sin(dx + c))}{d(a + b)^5(a - b)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^2+1/d*a^7/(a+b)^5/(a-b)^5*ln(a+
b*sin(d*x+c))+13/d*a^5/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*b^2+10/d*a^3/(a+b
)^5/(a-b)^5*ln(a+b*sin(d*x+c))*b^4-1/d*a^6/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))
-5/d*a^4/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))*b^2+1/16/d/(a+b)^3/(sin(d*x+c)-1)
^2+1/16/d/(a+b)^4/(sin(d*x+c)-1)*b+7/16/d/(a+b)^4/(sin(d*x+c)-1)*a-1/2/d/(a
+b)^5*ln(sin(d*x+c)-1)*a^2+5/16/d/(a+b)^5*ln(sin(d*x+c)-1)*a*b+1/16/d/(a+b)
^5*ln(sin(d*x+c)-1)*b^2+1/16/d/(a-b)^3/(1+sin(d*x+c))^2+1/16/d/(a-b)^4/(1+s
in(d*x+c))*b-7/16/d/(a-b)^4/(1+sin(d*x+c))*a-1/2/d/(a-b)^5*ln(1+sin(d*x+c))
*a^2-5/16/d/(a-b)^5*ln(1+sin(d*x+c))*a*b+1/16/d/(a-b)^5*ln(1+sin(d*x+c))*b^
2
```

Maxima [B] time = 1.30265, size = 986, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(a^7 + 13*a^5*b^2 + 10*a^3*b^4)*log(b*sin(d*x + c) + a)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^2 + 5*a*b - b^
2)*log(sin(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5) - (8*a^2 - 5*a*b - b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(18*a^7 + 72*a^5*b^2 + 6*a^3*b^4 +
(8*a^6*b + 67*a^4*b^3 + 22*a^2*b^5 - b^7)*sin(d*x + c)^5 + 2*(6*a^7 + 41*a
^5*b^2 + 2*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - (5*a^6*b + 159*a^4*b^3 + 27*a^
2*b^5 + b^7)*sin(d*x + c)^3 - 4*(8*a^7 + 37*a^5*b^2 + 4*a^3*b^4 - a*b^6)*si
n(d*x + c)^2 - (a^6*b - 86*a^4*b^3 - 11*a^2*b^5)*sin(d*x + c))/(a^10 - 4*a^
8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a
^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^
6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 -
4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*
a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b
^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c))/d
```

Fricas [B] time = 6.42709, size = 2206, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^9 - 16*a^7*b^2 + 24*a^5*b^4 - 16*a^3*b^6 + 4*a*b^8 - 4*(6*a^9 +
35*a^7*b^2 - 39*a^5*b^4 - 3*a^3*b^6 + a*b^8)*cos(d*x + c)^4 - 16*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 - 16*((a^7*b^2 + 13*a^5*b^4 +
10*a^3*b^6)*cos(d*x + c)^6 - 2*(a^8*b + 13*a^6*b^3 + 10*a^4*b^5)*cos(d*x +
c)^4*sin(d*x + c) - (a^9 + 14*a^7*b^2 + 23*a^5*b^4 + 10*a^3*b^6)*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) + ((8*a^7*b^2 + 45*a^6*b^3 + 104*a^5*b^4 + 12
5*a^4*b^5 + 80*a^3*b^6 + 23*a^2*b^7 - b^9)*cos(d*x + c)^6 - 2*(8*a^8*b + 45
*a^7*b^2 + 104*a^6*b^3 + 125*a^5*b^4 + 80*a^4*b^5 + 23*a^3*b^6 - a*b^8)*cos
(d*x + c)^4*sin(d*x + c) - (8*a^9 + 45*a^8*b + 112*a^7*b^2 + 170*a^6*b^3 +
184*a^5*b^4 + 148*a^4*b^5 + 80*a^3*b^6 + 22*a^2*b^7 - b^9)*cos(d*x + c)^4)*
log(sin(d*x + c) + 1) + ((8*a^7*b^2 - 45*a^6*b^3 + 104*a^5*b^4 - 125*a^4*b^
```

$$5 + 80a^3b^6 - 23a^2b^7 + b^9) \cos(dx + c)^6 - 2(8a^8b - 45a^7b^2 + 104a^6b^3 - 125a^5b^4 + 80a^4b^5 - 23a^3b^6 + ab^8) \cos(dx + c)^4 \sin(dx + c) - (8a^9 - 45a^8b + 112a^7b^2 - 170a^6b^3 + 184a^5b^4 - 148a^4b^5 + 80a^3b^6 - 22a^2b^7 + b^9) \cos(dx + c)^4 \log(-\sin(dx + c) + 1) - 2(2a^8b - 8a^6b^3 + 12a^4b^5 - 8a^2b^7 + 2b^9 + (8a^8b + 59a^6b^3 - 45a^4b^5 - 23a^2b^7 + b^9) \cos(dx + c)^4 - (11a^8b - 36a^6b^3 + 42a^4b^5 - 20a^2b^7 + 3b^9) \cos(dx + c)^2) \sin(dx + c) / ((a^{10}b^2 - 5a^8b^4 + 10a^6b^6 - 10a^4b^8 + 5a^2b^{10} - b^{12}) d \cos(dx + c)^6 - 2(a^{11}b - 5a^9b^3 + 10a^7b^5 - 10a^5b^7 + 5a^3b^9 - ab^{11}) d \cos(dx + c)^4 \sin(dx + c) - (a^{12} - 4a^{10}b^2 + 5a^8b^4 - 5a^6b^6 + 4a^4b^8 - b^{12}) d \cos(dx + c)^4)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**5/(a+b*sin(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 3.70755, size = 790, normalized size = 2.46

$$\frac{16(a^7b + 13a^5b^3 + 10a^3b^5) \log(|b \sin(dx+c)+a|)}{a^{10}b - 5a^8b^3 + 10a^6b^5 - 10a^4b^7 + 5a^2b^9 - b^{11}} - \frac{(8a^2 + 5ab - b^2) \log(|\sin(dx+c)+1|)}{a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5} - \frac{(8a^2 - 5ab - b^2) \log(|\sin(dx+c)-1|)}{a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5} - \frac{2(8a^6b \sin(dx+c)^5 + 67a^4b^3 \sin(dx+c)^5 + 22a^2b^5 \sin(dx+c)^5 - b^7 \sin(dx+c)^5 + 12a^7 \sin(dx+c)^4 + 82a^5b^2 \sin(dx+c)^4 + 4a^3b^4 \sin(dx+c)^4 - 2a^6b \sin(dx+c)^4 - 5a^6b \sin(dx+c)^3 - 159a^4b^3 \sin(dx+c)^3 - 27a^2b^5 \sin(dx+c)^3 - b^7 \sin(dx+c)^3 - 32a^7 \sin(dx+c)^2 - 148a^5b^2 \sin(dx+c)^2 - 16a^3b^4 \sin(dx+c)^2 + 4a^6b \sin(dx+c)^2 - a^6b \sin(dx+c) + 86a^4b^3 \sin(dx+c) + 11a^2b^5 \sin(dx+c) + 18a^7 + 72a^5b^2 + 6a^3b^4) / ((a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8) (b \sin(dx+c)^3 + a \sin(dx+c)^2 - b \sin(dx+c) - a)^2) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^5/(a+b*sin(dx+c))^3,x, algorithm="giac")

[Out] 1/16*(16*(a^7*b + 13*a^5*b^3 + 10*a^3*b^5)*log(abs(b*sin(dx + c) + a)))/(a^10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - (8*a^2 + 5*a*b - b^2)*log(abs(sin(dx + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(abs(sin(dx + c) - 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(8*a^6*b*sin(dx + c)^5 + 67*a^4*b^3*sin(dx + c)^5 + 22*a^2*b^5*sin(dx + c)^5 - b^7*sin(dx + c)^5 + 12*a^7*sin(dx + c)^4 + 82*a^5*b^2*sin(dx + c)^4 + 4*a^3*b^4*sin(dx + c)^4 - 2*a^6*b*sin(dx + c)^4 - 5*a^6*b*sin(dx + c)^3 - 159*a^4*b^3*sin(dx + c)^3 - 27*a^2*b^5*sin(dx + c)^3 - b^7*sin(dx + c)^3 - 32*a^7*sin(dx + c)^2 - 148*a^5*b^2*sin(dx + c)^2 - 16*a^3*b^4*sin(dx + c)^2 + 4*a^6*b*sin(dx + c)^2 - a^6*b*sin(dx + c) + 86*a^4*b^3*sin(dx + c) + 11*a^2*b^5*sin(dx + c) + 18*a^7 + 72*a^5*b^2 + 6*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(dx + c)^3 + a*sin(dx + c)^2 - b*sin(dx + c) - a)^2)/d

$$3.193 \quad \int \frac{\tan^3(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=232

$$\frac{a^3}{2d(a^2-b^2)^2(a+b \sin(c+dx))^2} + \frac{a^2(a^2+3b^2)}{d(a^2-b^2)^3(a+b \sin(c+dx))} - \frac{a(8a^2b^2+a^4+3b^4) \log(a+b \sin(c+dx))}{d(a^2-b^2)^4} + \frac{\sec^2(c+dx)}{d(a^2-b^2)^4}$$

[Out] $((2*a - b)*\text{Log}[1 - \text{Sin}[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*\text{Log}[1 + \text{Sin}[c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*\text{Sin}[c + d*x])^2) + (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*\text{Sin}[c + d*x])) + (\text{Sec}[c + d*x])^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*\text{Sin}[c + d*x])/((2*(a^2 - b^2)^3*d)$

Rubi [A] time = 0.482381, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{2d(a^2-b^2)^2(a+b \sin(c+dx))^2} + \frac{a^2(a^2+3b^2)}{d(a^2-b^2)^3(a+b \sin(c+dx))} - \frac{a(8a^2b^2+a^4+3b^4) \log(a+b \sin(c+dx))}{d(a^2-b^2)^4} + \frac{\sec^2(c+dx)}{d(a^2-b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]

[Out] $((2*a - b)*\text{Log}[1 - \text{Sin}[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*\text{Log}[1 + \text{Sin}[c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*\text{Sin}[c + d*x])^2) + (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*\text{Sin}[c + d*x])) + (\text{Sec}[c + d*x])^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*\text{Sin}[c + d*x])/((2*(a^2 - b^2)^3*d)$

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 1647

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3(b^2-x^2)^2} dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{\sec^2(c+dx)(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx))}{2(a^2-b^2)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^4(3a^2+b^2)-a^2b^2(2a^4-3a^2b^2-}{(a^2-b^2)^3} - \frac{a^2b^2(2a^4-3a^2b^2-}{(a^2-b^2)^3}}{(a+x)^3} dx, x, b\sin(c+dx)\right)}{(a^2-b^2)^3 d} \\
&= \frac{\sec^2(c+dx)(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx))}{2(a^2-b^2)^3 d} + \frac{\text{Subst}\left(\int \left(\frac{b^2(-2a+b)}{2(a+b)^4(b-x)} - \frac{2a^3b^2}{(a^2-b^2)^2(a+x)}\right) dx, x, b\sin(c+dx)\right)}{(a^2-b^2)^3 d} \\
&= \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4 d} - \frac{a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4 d}
\end{aligned}$$

Mathematica [A] time = 2.189, size = 196, normalized size = 0.84

$$\frac{\frac{2a^3}{(a^2-b^2)^2(a+b\sin(c+dx))^2} + \frac{4a^2(a^2+3b^2)}{(a^2-b^2)^3(a+b\sin(c+dx))} - \frac{4a(8a^2b^2+a^4+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4} - \frac{1}{(a+b)^3(\sin(c+dx)-1)} + \frac{1}{(a-b)^3(\sin(c+dx)+1)} + \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4 d} - \frac{a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]

[Out] (((2*a - b)*Log[1 - Sin[c + d*x]])/(a + b)^4 + ((2*a + b)*Log[1 + Sin[c + d*x]])/(a - b)^4 - (4*a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^4 - 1/((a + b)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^3*(1 + Sin[c + d*x])) + (2*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (4*a^2*(a^2 + 3*b^2))/(a^2 - b^2)^3*(a + b*Sin[c + d*x]))/(4*d)

Maple [A] time = 0.109, size = 323, normalized size = 1.4

$$\frac{a^3}{2d(a+b)^2(a-b)^2(a+b\sin(dx+c))^2} - \frac{a^5 \ln(a+b\sin(dx+c))}{d(a+b)^4(a-b)^4} - 8 \frac{a^3 \ln(a+b\sin(dx+c)) b^2}{d(a+b)^4(a-b)^4} - 3 \frac{a \ln(a+b\sin(dx+c))}{d(a+b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x)

[Out] 1/2/d*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-1/d*a^5/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))-8/d*a^3/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^2-3/d*a/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^4+1/d*a^4/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+3/d*a^2/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*b^2-1/4/d/(a+b)^3/(sin(d*x+c)-1)+1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*a-1/4/d/(a+b)^4*ln(sin(d*x+c)-1)*b+1/4/d/(a-b)^3/(1+sin(d*x+c))+1/2/d/(a-b)^4*ln(1+sin(d*x+c))*a+1/4/d/(a-b)^4*ln(1+sin(d*x+c))*b

Maxima [A] time = 1.87626, size = 595, normalized size = 2.56

$$\frac{4(a^5+8a^3b^2+3ab^4)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{(2a+b)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(2a-b)\log(\sin(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{2(4a^5+8a^3b^2+3ab^4)\log(b\sin(dx+c)+a)}{a^8-3a^6b^2+3a^4b^4-a^2b^6-(a^6b^2-3a^4b^4+3a^2b^6-b^8)\sin(dx+c)}$$

$4d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out]
$$-1/4*(4*(a^5 + 8*a^3*b^2 + 3*a*b^4)*\log(b*\sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (2*a + b)*\log(\sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*\log(\sin(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(4*a^5 + 8*a^3*b^2 - (2*a^4*b + 9*a^2*b^3 + b^5)*\sin(d*x + c)^3 - (3*a^5 + 10*a^3*b^2 - a*b^4)*\sin(d*x + c)^2 + (a^4*b + 11*a^2*b^3)*\sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*\sin(d*x + c)^4 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*\sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*\sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*\sin(d*x + c))/d$$

Fricas [B] time = 3.96383, size = 1727, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out]
$$-1/4*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 2*(3*a^7 + 7*a^5*b^2 - 11*a^3*b^4 + a*b^6)*\cos(d*x + c)^2 + 4*((a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*\cos(d*x + c)^4 - 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*\cos(d*x + c)^2*\sin(d*x + c) - (a^7 + 9*a^5*b^2 + 11*a^3*b^4 + 3*a*b^6)*\cos(d*x + c)^2)*\log(b*\sin(d*x + c) + a) - ((2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b^6 + b^7)*\cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^5 + a*b^6)*\cos(d*x + c)^2*\sin(d*x + c) - (2*a^7 + 9*a^6*b + 18*a^5*b^2 + 23*a^4*b^3 + 22*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) - ((2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*a*b^6 - b^7)*\cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*\cos(d*x + c)^2*\sin(d*x + c) - (2*a^7 - 9*a^6*b + 18*a^5*b^2 - 23*a^4*b^3 + 22*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - b^7)*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (2*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 - b^7)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*\cos(d*x + c)^4 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*\cos(d*x + c)^2*\sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*\cos(d*x + c)^2)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**3,x)

[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**3, x)

Giac [B] time = 2.10026, size = 626, normalized size = 2.7

$$\frac{4(a^5b+8a^3b^3+3ab^5)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9} - \frac{(2a+b)\log(|\sin(dx+c)+1|)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(2a-b)\log(|\sin(dx+c)-1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{2(a^5\sin(dx+c)^2+8a^3b^2\sin(dx+c)^2+3ab^4\sin(dx+c)^2)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out]
$$-1/4*(4*(a^5*b + 8*a^3*b^3 + 3*a*b^5)*\log(\text{abs}(b*\sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (2*a + b)*\log(\text{abs}(\sin(d*x + c) + 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*\log(\text{abs}(\sin(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(a^5*\sin(d*x + c)^2 + 8*a^3*b^2*\sin(d*x + c)^2 + 3*a*b^4*\sin(d*x + c)^2 - 3*a^4*b*\sin(d*x + c) + 2*a^2*b^3*\sin(d*x + c) + b^5*\sin(d*x + c) - 6*a^3*b^2 - 6*a*b^4)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(\sin(d*x + c)^2 - 1) - 2*(3*a^5*b^2*\sin(d*x + c)^2 + 24*a^3*b^4*\sin(d*x + c)^2 + 9*a*b^6*\sin(d*x + c)^2 + 8*a^6*b*\sin(d*x + c) + 52*a^4*b^3*\sin(d*x + c) + 12*a^2*b^5*\sin(d*x + c) + 6*a^7 + 26*a^5*b^2 + 4*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*\sin(d*x + c) + a)^2))/d$$

$$3.194 \quad \int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=149

$$\frac{a}{2d(a^2-b^2)(a+b \sin(c+dx))^2} - \frac{a^2+b^2}{d(a^2-b^2)^2(a+b \sin(c+dx))} + \frac{a(a^2+3b^2) \log(a+b \sin(c+dx))}{d(a^2-b^2)^3} - \frac{\log(1-\sin(c+dx))}{2d(a+b \sin(c+dx))}$$

[Out] $-\text{Log}[1 - \text{Sin}[c + d*x]]/(2*(a + b)^3*d) - \text{Log}[1 + \text{Sin}[c + d*x]]/(2*(a - b)^3*d) + (a*(a^2 + 3*b^2)*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*(a^2 - b^2)*d*(a + b*\text{Sin}[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b*\text{Sin}[c + d*x]))$

Rubi [A] time = 0.132163, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 801}

$$\frac{a}{2d(a^2-b^2)(a+b \sin(c+dx))^2} - \frac{a^2+b^2}{d(a^2-b^2)^2(a+b \sin(c+dx))} + \frac{a(a^2+3b^2) \log(a+b \sin(c+dx))}{d(a^2-b^2)^3} - \frac{\log(1-\sin(c+dx))}{2d(a+b \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[c + d*x]/(a + b*\text{Sin}[c + d*x])^3, x]$

[Out] $-\text{Log}[1 - \text{Sin}[c + d*x]]/(2*(a + b)^3*d) - \text{Log}[1 + \text{Sin}[c + d*x]]/(2*(a - b)^3*d) + (a*(a^2 + 3*b^2)*\text{Log}[a + b*\text{Sin}[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*(a^2 - b^2)*d*(a + b*\text{Sin}[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b*\text{Sin}[c + d*x]))$

Rule 2721

$\text{Int}[(a + b*\sin[e + f*x])^m*\tan[e + f*x]^p, x_Symbol] :> \text{Dist}[1/f, \text{Subst}[\text{Int}[(x^p*(a + x)^m)/(b^2 - x^2)^{(p+1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[(p+1)/2]$

Rule 801

$\text{Int}[(d + e*x)^m*(f + g*x)/((a + c*x^2)^2), x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)}{(a+b \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3(b^2-x^2)} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^3(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^3} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)^2} + \frac{a^3+3ab^2}{(a-b)^3(a+b)^3(a+x)} - \frac{1}{2(a-b)^3(b+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\log(1-\sin(c+dx))}{2(a+b)^3d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} + \frac{a(a^2+3b^2) \log(a+b \sin(c+dx))}{(a^2-b^2)^3d} - \frac{1}{2(a^2-b^2)^3d} \end{aligned}$$

Mathematica [A] time = 2.02021, size = 213, normalized size = 1.43

$$\frac{\frac{2b}{(a^2-b^2)(a+b\sin(c+dx))} - \frac{4ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2} + a \left(\frac{b \left(\frac{(a^2-b^2)(-5a^2-4ab\sin(c+dx)+b^2)}{(a+b\sin(c+dx))^2} + 2(3a^2+b^2)\log(a+b\sin(c+dx)) \right)}{(a^2-b^2)^3} \right) + \frac{\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\log(\sin(c+dx))}{(a-b)^3}}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]

[Out] $(-\text{Log}[1 - \text{Sin}[c + d*x]]/(a + b)^2 + \text{Log}[1 + \text{Sin}[c + d*x]]/(a - b)^2 - (4*a*b*\text{Log}[a + b*\text{Sin}[c + d*x]])/(a^2 - b^2)^2 + (2*b)/((a^2 - b^2)*(a + b*\text{Sin}[c + d*x])) + a*(\text{Log}[1 - \text{Sin}[c + d*x]]/(a + b)^3 - \text{Log}[1 + \text{Sin}[c + d*x]]/(a - b)^3 + (b*(2*(3*a^2 + b^2)*\text{Log}[a + b*\text{Sin}[c + d*x]] + ((a^2 - b^2)*(-5*a^2 + b^2 - 4*a*b*\text{Sin}[c + d*x]))/(a + b*\text{Sin}[c + d*x]^2)))/(a^2 - b^2)^3))/(2*b*d)$

Maple [A] time = 0.1, size = 198, normalized size = 1.3

$$\frac{a}{2d(a+b)(a-b)(a+b\sin(dx+c))^2} - \frac{a^2}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))} - \frac{b^2}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^3,x)

[Out] $-1/2/d*a/(a+b)/(a-b)/(a+b*\sin(d*x+c))^2 - 1/d/(a+b)^2/(a-b)^2/(a+b*\sin(d*x+c))*a^2 - 1/d/(a+b)^2/(a-b)^2/(a+b*\sin(d*x+c))*b^2 + 1/d*a^3/(a+b)^3/(a-b)^3*\ln(a+b*\sin(d*x+c)) + 3/d*a/(a+b)^3/(a-b)^3*\ln(a+b*\sin(d*x+c))*b^2 - 1/2/d/(a+b)^3*\ln(\sin(d*x+c)-1) - 1/2*\ln(1+\sin(d*x+c))/(a-b)^3/d$

Maxima [A] time = 1.91543, size = 308, normalized size = 2.07

$$\frac{2(a^3+3ab^2)\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{3a^3+ab^2+2(a^2b+b^3)\sin(dx+c)}{a^6-2a^4b^2+a^2b^4+(a^4b^2-2a^2b^4+b^6)\sin(dx+c)^2+2(a^5b-2a^3b^3+ab^5)\sin(dx+c)} - \frac{\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} - \frac{\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2-b^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $1/2*(2*(a^3 + 3*a*b^2)*\log(b*\sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (3*a^3 + a*b^2 + 2*(a^2*b + b^3)*\sin(d*x + c))/(a^6 - 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*\sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*\sin(d*x + c)) - \log(\sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - \log(\sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d$

Fricas [B] time = 2.6488, size = 995, normalized size = 6.68

$$3a^5 - 2a^3b^2 - ab^4 - 2(a^5 + 4a^3b^2 + 3ab^4 - (a^3b^2 + 3ab^4)\cos(dx+c)^2 + 2(a^4b + 3a^2b^3)\sin(dx+c))\log(b\sin(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{2}*(3*a^5 - 2*a^3*b^2 - a*b^4 - 2*(a^5 + 4*a^3*b^2 + 3*a*b^4 - (a^3*b^2 + 3*a*b^4)*\cos(d*x + c)^2 + 2*(a^4*b + 3*a^2*b^3)*\sin(d*x + c))*\log(b*\sin(d*x + c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*\cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*\sin(d*x + c))*\log(\sin(d*x + c) + 1) + (a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*\cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*\sin(d*x + c))*\log(-\sin(d*x + c) + 1) + 2*(a^4*b - b^5)*\sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*\cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*\sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^4*b^4 - b^8)*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**3,x)

[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**3, x)

Giac [A] time = 1.58231, size = 347, normalized size = 2.33

$$\frac{2(a^3b+3ab^3)\log(|b\sin(dx+c)+a|)}{a^6b-3a^4b^3+3a^2b^5-b^7} - \frac{\log(|\sin(dx+c)+1|)}{a^3-3a^2b+3ab^2-b^3} - \frac{\log(|\sin(dx+c)-1|)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a^3b^2\sin(dx+c)^2+9ab^4\sin(dx+c)^2+8a^4b\sin(dx+c)+18a^2b^3\sin(dx+c)-2b^5}{(a^6-3a^4b^2+3a^2b^4-b^6)(b\sin(dx+c)+a)^2}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{2}*(2*(a^3*b + 3*a*b^3)*\log(\text{abs}(b*\sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - \log(\text{abs}(\sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - \log(\text{abs}(\sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3*b^2*\sin(d*x + c)^2 + 9*a*b^4*\sin(d*x + c)^2 + 8*a^4*b*\sin(d*x + c) + 18*a^2*b^3*\sin(d*x + c) - 2*b^5*\sin(d*x + c) + 6*a^5 + 7*a^3*b^2 - a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*\sin(d*x + c) + a)^2))/d$

$$3.195 \quad \int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=75

$$\frac{1}{a^2 d (a + b \sin(c + dx))} - \frac{\log(a + b \sin(c + dx))}{a^3 d} + \frac{\log(\sin(c + dx))}{a^3 d} + \frac{1}{2 a d (a + b \sin(c + dx))^2}$$

[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.060479, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 44}

$$\frac{1}{a^2 d (a + b \sin(c + dx))} - \frac{\log(a + b \sin(c + dx))}{a^3 d} + \frac{\log(\sin(c + dx))}{a^3 d} + \frac{1}{2 a d (a + b \sin(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]

[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{(a+b \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3 x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\log(\sin(c+dx))}{a^3 d} - \frac{\log(a+b \sin(c+dx))}{a^3 d} + \frac{1}{2 a d (a+b \sin(c+dx))^2} + \frac{1}{a^2 d (a+b \sin(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.261819, size = 60, normalized size = 0.8

$$\frac{a(3a+2b \sin(c+dx))}{(a+b \sin(c+dx))^2} - 2 \log(a+b \sin(c+dx)) + 2 \log(\sin(c+dx))$$

$$2a^3 d$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]

[Out] (2*Log[Sin[c + d*x]] - 2*Log[a + b*Sin[c + d*x]] + (a*(3*a + 2*b*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2)/(2*a^3*d)

Maple [A] time = 0.046, size = 74, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{a^3d} - \frac{\ln(a+b\sin(dx+c))}{a^3d} + \frac{1}{2da(a+b\sin(dx+c))^2} + \frac{1}{a^2d(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^3,x)

[Out] ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1/a^2/d/(a+b*sin(d*x+c))

Maxima [A] time = 1.54714, size = 109, normalized size = 1.45

$$\frac{\frac{2b\sin(dx+c)+3a}{a^2b^2\sin(dx+c)^2+2a^3b\sin(dx+c)+a^4} - \frac{2\log(b\sin(dx+c)+a)}{a^3} + \frac{2\log(\sin(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((2*b*sin(d*x + c) + 3*a)/(a^2*b^2*sin(d*x + c)^2 + 2*a^3*b*sin(d*x + c) + a^4) - 2*log(b*sin(d*x + c) + a)/a^3 + 2*log(sin(d*x + c))/a^3)/d

Fricas [B] time = 2.00358, size = 365, normalized size = 4.87

$$\frac{2ab\sin(dx+c) + 3a^2 + 2(b^2\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2)\log(b\sin(dx+c) + a) - 2(b^2\cos(dx+c)^2 - 2a^3b^2d\cos(dx+c)^2 - 2a^4bd\sin(dx+c) - (a^5 + a^3b^2)d)}{2(a^3b^2d\cos(dx+c)^2 - 2a^4bd\sin(dx+c) - (a^5 + a^3b^2)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(2*a*b*sin(d*x + c) + 3*a^2 + 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*log(b*sin(d*x + c) + a) - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*b^2*d*cos(d*x + c)^2 - 2*a^4*b*d*sin(d*x + c) - (a^5 + a^3*b^2)*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**3, x)

Giac [A] time = 1.74614, size = 93, normalized size = 1.24

$$-\frac{\frac{2 \log(|b \sin(dx+c)+a|)}{a^3} - \frac{2 \log(|\sin(dx+c)|)}{a^3} - \frac{2 ab \sin(dx+c)+3 a^2}{(b \sin(dx+c)+a)^2 a^3}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] $-\frac{1}{2} \frac{2 \log(\text{abs}(b \sin(d*x + c) + a)) / a^3 - 2 \log(\text{abs}(\sin(d*x + c))) / a^3 - (2 * a * b * \sin(d*x + c) + 3 * a^2) / ((b * \sin(d*x + c) + a)^2 * a^3)}{d}$

3.196 $\int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^3} dx$

Optimal. Leaf size=145

$$-\frac{a^2 - 3b^2}{a^4 d (a + b \sin(c + dx))} - \frac{a^2 - b^2}{2a^3 d (a + b \sin(c + dx))^2} - \frac{(a^2 - 6b^2) \log(\sin(c + dx))}{a^5 d} + \frac{(a^2 - 6b^2) \log(a + b \sin(c + dx))}{a^5 d} + \dots$$

[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 0.133294, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$-\frac{a^2 - 3b^2}{a^4 d (a + b \sin(c + dx))} - \frac{a^2 - b^2}{2a^3 d (a + b \sin(c + dx))^2} - \frac{(a^2 - 6b^2) \log(\sin(c + dx))}{a^5 d} + \frac{(a^2 - 6b^2) \log(a + b \sin(c + dx))}{a^5 d} + \dots$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]

[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*Sin[c + d*x]))

Rule 2721

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)}{(a+b \sin(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^3} dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^3 x^3} - \frac{3b^2}{a^4 x^2} + \frac{-a^2+6b^2}{a^5 x} + \frac{a^2-b^2}{a^3(a+x)^3} + \frac{a^2-3b^2}{a^4(a+x)^2} + \frac{a^2-6b^2}{a^5(a+x)}\right) dx, x, b \sin(c+dx)\right)}{d} \\ &= \frac{3b \csc(c+dx)}{a^4 d} - \frac{\csc^2(c+dx)}{2a^3 d} - \frac{(a^2 - 6b^2) \log(\sin(c+dx))}{a^5 d} + \frac{(a^2 - 6b^2) \log(a + b \sin(c+dx))}{a^5 d} \end{aligned}$$

Mathematica [A] time = 0.931238, size = 121, normalized size = 0.83

$$\frac{\frac{2a(a^2-3b^2)}{a+b\sin(c+dx)} + 2(a^2-6b^2)\log(\sin(c+dx)) - 2(a^2-6b^2)\log(a+b\sin(c+dx)) + \frac{a^2(a-b)(a+b)}{(a+b\sin(c+dx))^2} + a^2\csc^2(c+dx) - 6a^2}{2a^5d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]

[Out] -(-6*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 6*b^2)*Log[Sin[c + d*x]]) - 2*(a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]] + (a^2*(a - b)*(a + b))/(a + b*Sin[c + d*x])^2 + (2*a*(a^2 - 3*b^2))/(a + b*Sin[c + d*x])/(2*a^5*d)

Maple [A] time = 0.111, size = 194, normalized size = 1.3

$$\frac{\ln(a + b \sin(dx + c))}{a^3 d} - 6 \frac{\ln(a + b \sin(dx + c)) b^2}{d a^5} - \frac{1}{a^2 d (a + b \sin(dx + c))} + 3 \frac{b^2}{d a^4 (a + b \sin(dx + c))} - \frac{1}{2 d a (a + b \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x)

[Out] ln(a+b*sin(d*x+c))/a^3/d-6/d/a^5*ln(a+b*sin(d*x+c))*b^2-1/a^2/d/(a+b*sin(d*x+c))+3/d/a^4/(a+b*sin(d*x+c))*b^2-1/2/a/d/(a+b*sin(d*x+c))^2+1/2/d/a^3/(a+b*sin(d*x+c))^2*b^2-1/2/d/a^3/sin(d*x+c)^2-ln(sin(d*x+c))/a^3/d+6/d/a^5*ln(sin(d*x+c))*b^2+3/d/a^4*b/sin(d*x+c)

Maxima [A] time = 1.92069, size = 211, normalized size = 1.46

$$\frac{4a^2b\sin(dx+c)-2(a^2b-6b^3)\sin(dx+c)^3-a^3-3(a^3-6ab^2)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+2a^5b\sin(dx+c)^3+a^6\sin(dx+c)^2} + \frac{2(a^2-6b^2)\log(b\sin(dx+c)+a)}{a^5} - \frac{2(a^2-6b^2)\log(\sin(dx+c))}{a^5}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((4*a^2*b*sin(d*x + c) - 2*(a^2*b - 6*b^3)*sin(d*x + c)^3 - a^3 - 3*(a^3 - 6*a*b^2)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + 2*a^5*b*sin(d*x + c)^3 + a^6*sin(d*x + c)^2) + 2*(a^2 - 6*b^2)*log(b*sin(d*x + c) + a)/a^5 - 2*(a^2 - 6*b^2)*log(sin(d*x + c))/a^5)/d

Fricas [B] time = 2.3421, size = 911, normalized size = 6.28

$$4a^4 - 18a^2b^2 - 3(a^4 - 6a^2b^2)\cos(dx+c)^2 - 2((a^2b^2 - 6b^4)\cos(dx+c)^4 + a^4 - 5a^2b^2 - 6b^4 - (a^4 - 4a^2b^2 - 12b^4))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

```
[Out] -1/2*(4*a^4 - 18*a^2*b^2 - 3*(a^4 - 6*a^2*b^2)*cos(d*x + c)^2 - 2*((a^2*b^2 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*((a^2*b^2 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x + c)) - 2*(a^3*b + 6*a*b^3 + (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*b^2*d*cos(d*x + c)^4 - (a^7 + 2*a^5*b^2)*d*cos(d*x + c)^2 + (a^7 + a^5*b^2)*d - 2*(a^6*b*d*cos(d*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.80037, size = 208, normalized size = 1.43

$$\frac{2(a^2 - 6b^2) \log(|\sin(dx+c)|)}{a^5} - \frac{2(a^2b - 6b^3) \log(|b \sin(dx+c) + a|)}{a^5b} + \frac{2a^2b \sin(dx+c)^3 - 12b^3 \sin(dx+c)^3 + 3a^3 \sin(dx+c)^2 - 18ab^2 \sin(dx+c)^2 - 4a^2b \sin(dx+c) + 2a^2b \sin(dx+c)}{(b \sin(dx+c)^2 + a \sin(dx+c))^2 a^4}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 6*b^2)*log(abs(sin(d*x + c)))/a^5 - 2*(a^2*b - 6*b^3)*log(abs(b*sin(d*x + c) + a))/(a^5*b) + (2*a^2*b*sin(d*x + c)^3 - 12*b^3*sin(d*x + c)^3 + 3*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x + c)^2 - 4*a^2*b*sin(d*x + c) + a^3)/((b*sin(d*x + c)^2 + a*sin(d*x + c))^2*a^4))/d
```


$$3.197 \quad \int \frac{\cot^5(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=221

$$\frac{-6a^2b^2 + a^4 + 5b^4}{a^6d(a + b \sin(c + dx))} + \frac{(a^2 - b^2)^2}{2a^5d(a + b \sin(c + dx))^2} + \frac{(a^2 - 3b^2) \csc^2(c + dx)}{a^5d} - \frac{2b(3a^2 - 5b^2) \csc(c + dx)}{a^6d} + \frac{(-12a^2b^2}{a^6d}$$

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 - 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.211009, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2721, 894}

$$\frac{-6a^2b^2 + a^4 + 5b^4}{a^6d(a + b \sin(c + dx))} + \frac{(a^2 - b^2)^2}{2a^5d(a + b \sin(c + dx))^2} + \frac{(a^2 - 3b^2) \csc^2(c + dx)}{a^5d} - \frac{2b(3a^2 - 5b^2) \csc(c + dx)}{a^6d} + \frac{(-12a^2b^2}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 - 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3} dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^3} dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^3x^5} - \frac{3b^4}{a^4x^4} + \frac{2b^2(-a^2+3b^2)}{a^5x^3} + \frac{2(3a^2b^2-5b^4)}{a^6x^2} + \frac{a^4-12a^2b^2+15b^4}{a^7x} - \frac{(a^2-b^2)^2}{a^5(a+x)^3} + \frac{-a^4+6a^2b^2-5b^4}{a^6(a+x)^2} + \frac{a^4-12a^2b^2+15b^4}{a^7(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6d} + \frac{(a^2-3b^2)\csc^2(c+dx)}{a^5d} + \frac{b\csc^3(c+dx)}{a^4d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{(a^4-12a^2b^2+15b^4)\log(\sin(c+dx))}{4a^7d}$$

Mathematica [A] time = 5.32652, size = 195, normalized size = 0.88

$$\frac{4a(-6a^2b^2+a^4+5b^4)}{a+b\sin(c+dx)} + \frac{2(a^3-ab^2)^2}{(a+b\sin(c+dx))^2} + 4a^2(a^2-3b^2)\csc^2(c+dx) - 8ab(3a^2-5b^2)\csc(c+dx) + 4(-12a^2b^2+a^4+15b^4)\log(\sin(c+dx))}{4a^7d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]

[Out] (-8*a*b*(3*a^2 - 5*b^2)*Csc[c + d*x] + 4*a^2*(a^2 - 3*b^2)*Csc[c + d*x]^2 + 4*a^3*b*Csc[c + d*x]^3 - a^4*Csc[c + d*x]^4 + 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]] - 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]] + (2*(a^3 - a*b^2)^2)/(a + b*Sin[c + d*x])^2 + (4*a*(a^4 - 6*a^2*b^2 + 5*b^4))/(a + b*Sin[c + d*x]))/(4*a^7*d)

Maple [A] time = 0.114, size = 348, normalized size = 1.6

$$-\frac{\ln(a+b\sin(dx+c))}{a^3d} + 12\frac{\ln(a+b\sin(dx+c))b^2}{da^5} - 15\frac{\ln(a+b\sin(dx+c))b^4}{da^7} + \frac{1}{a^2d(a+b\sin(dx+c))} - 6\frac{1}{da^4(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x)

[Out] -ln(a+b*sin(d*x+c))/a^3/d+12/d/a^5*ln(a+b*sin(d*x+c))*b^2-15/d/a^7*ln(a+b*sin(d*x+c))*b^4+1/a^2/d/(a+b*sin(d*x+c))-6/d/a^4/(a+b*sin(d*x+c))*b^2+5/d/a^6/(a+b*sin(d*x+c))*b^4+1/2/a/d/(a+b*sin(d*x+c))^2-1/d/a^3/(a+b*sin(d*x+c))^2*b^2+1/2/d/a^5/(a+b*sin(d*x+c))^2*b^4-1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d*x+c)^2-3/d/a^5/sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a^3/d-12/d/a^5*ln(sin(d*x+c))*b^2+15/d/a^7*ln(sin(d*x+c))*b^4+1/d/a^4*b/sin(d*x+c)^3-6/d/a^4*b/sin(d*x+c)+10/d*b^3/a^6/sin(d*x+c)

Maxima [A] time = 1.61205, size = 319, normalized size = 1.44

$$\frac{2a^4b\sin(dx+c)+4(a^4b-12a^2b^3+15b^5)\sin(dx+c)^5-a^5+6(a^5-12a^3b^2+15ab^4)\sin(dx+c)^4-4(a^4b-5a^2b^3)\sin(dx+c)^3+(4a^5-5a^3b^2)\sin(dx+c)^2-4(a^4-12a^2b^2+15b^4)\sin(dx+c)}{a^6b^2\sin(dx+c)^6+2a^7b\sin(dx+c)^5+a^8\sin(dx+c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")

```
[Out] 1/4*((2*a^4*b*sin(d*x + c) + 4*(a^4*b - 12*a^2*b^3 + 15*b^5)*sin(d*x + c)^5
- a^5 + 6*(a^5 - 12*a^3*b^2 + 15*a*b^4)*sin(d*x + c)^4 - 4*(4*a^4*b - 5*a^
2*b^3)*sin(d*x + c)^3 + (4*a^5 - 5*a^3*b^2)*sin(d*x + c)^2)/(a^6*b^2*sin(d*
x + c)^6 + 2*a^7*b*sin(d*x + c)^5 + a^8*sin(d*x + c)^4) - 4*(a^4 - 12*a^2*b
^2 + 15*b^4)*log(b*sin(d*x + c) + a)/a^7 + 4*(a^4 - 12*a^2*b^2 + 15*b^4)*lo
g(sin(d*x + c))/a^7)/d
```

Fricas [B] time = 2.42375, size = 1708, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*a^6 - 77*a^4*b^2 + 90*a^2*b^4 + 6*(a^6 - 12*a^4*b^2 + 15*a^2*b^4)*c
os(d*x + c)^4 - (16*a^6 - 149*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^2 + 4*((a
^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*a^2*b^4
- 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 + (2*a^6
- 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a^3*b^3
+ 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 12
*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a)
- 4*((a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*
a^2*b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 +
(2*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a
^3*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5
*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x
+ c)) - 2*(5*a^5*b + 14*a^3*b^3 - 30*a*b^5 - 2*(a^5*b - 12*a^3*b^3 + 15*a*b
^5)*cos(d*x + c)^4 - 2*(2*a^5*b + 19*a^3*b^3 - 30*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4
+ (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos
(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 2.10396, size = 441, normalized size = 2.

$$\frac{12(a^4 - 12a^2b^2 + 15b^4) \log(|\sin(dx+c)|)}{a^7} - \frac{12(a^4b - 12a^2b^3 + 15b^5) \log(|b \sin(dx+c) + a|)}{a^7b} + \frac{6(3a^4b^2 \sin(dx+c)^2 - 36a^2b^4 \sin(dx+c)^2 + 45b^6 \sin(dx+c)^2 + 8a^5)}{(b \sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 12*a^2*b^2 + 15*b^4)*log(abs(sin(d*x + c)))/a^7 - 12*(a^4*b
- 12*a^2*b^3 + 15*b^5)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 6*(3*a^4*b^2
*sin(d*x + c)^2 - 36*a^2*b^4*sin(d*x + c)^2 + 45*b^6*sin(d*x + c)^2 + 8*a^5
*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 100*a*b^5*sin(d*x + c) + 6*a^6
- 50*a^4*b^2 + 56*a^2*b^4)/((b*sin(d*x + c) + a)^2*a^7) - (25*a^4*sin(d*x +
c)^4 - 300*a^2*b^2*sin(d*x + c)^4 + 375*b^4*sin(d*x + c)^4 + 72*a^3*b*sin(
d*x + c)^3 - 120*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 36*a^2*b^2*
sin(d*x + c)^2 - 12*a^3*b*sin(d*x + c) + 3*a^4)/(a^7*sin(d*x + c)^4))/d
```

$$3.198 \quad \int \frac{\tan^4(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=474

$$\frac{a^4 (2a^2 + b^2) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \frac{8a^4 b^2 \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \dots$$

[Out] $(8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))$

Rubi [A] time = 0.871767, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2731, 2650, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^4 (2a^2 + b^2) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \frac{8a^4 b^2 \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(c+dx) \right) + b}{\sqrt{a^2 - b^2}} \right)}{d (a^2 - b^2)^{9/2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]

[Out] $(8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))$

Rule 2731

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m]/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, p/2]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n

+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] & & EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2754

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3} dx &= \int \left(\frac{1}{4(a+b)^3(-1+\sin(c+dx))^2} + \frac{3a}{4(a+b)^4(-1+\sin(c+dx))} + \frac{1}{4(a-b)^3(1+\sin(c+dx))} \right) dx \\
&= -\frac{(3a) \int \frac{1}{1+\sin(c+dx)} dx}{4(a-b)^4} + \frac{\int \frac{1}{(1+\sin(c+dx))^2} dx}{4(a-b)^3} + \frac{(3a) \int \frac{1}{-1+\sin(c+dx)} dx}{4(a+b)^4} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2} dx}{4(a+b)^3} \\
&= \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))^2} - \frac{3a \cos(c+dx)}{4(a+b)^4 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^3 d(1+\sin(c+dx))} \\
&= \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))^2} - \frac{3a \cos(c+dx)}{4(a+b)^4 d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))} \\
&= \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))^2} - \frac{3a \cos(c+dx)}{4(a+b)^4 d(1-\sin(c+dx))} \\
&= \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))^2} - \frac{3a \cos(c+dx)}{4(a+b)^4 d(1-\sin(c+dx))} \\
&= \frac{8a^4 b^2 \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{\cos(c+dx)}{12(a+b)^3 d(1-\sin(c+dx))^2} \\
&= \frac{8a^4 b^2 \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{12a^2 b^2 (a^2 + b^2) \tan^{-1} \left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2} d} + \frac{a^4 (2a^2 + b^2)}{(a^2-b^2)^{9/2} d}
\end{aligned}$$

Mathematica [A] time = 1.043, size = 351, normalized size = 0.74

$$\frac{96a^2(21a^2b^2+2a^4+12b^4) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}} \right)}{(a^2-b^2)^{9/2}} - \frac{\sec^3(c+dx)(22a^5b^2 \sin(c+dx)-91a^5b^2 \sin(3(c+dx))-17a^5b^2 \sin(5(c+dx))-264a^3b^4 \sin(c+dx)-244a^3b^4 \sin(3(c+dx))-12a^3b^4 \sin(5(c+dx))-17a^5b^2 \sin(5(c+dx))-76a^3b^4 \sin(5(c+dx))-12a^3b^6 \sin(5(c+dx)))}{(a^2-b^2)^4(a+b\sin(c+dx))^2}}{(96*d)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]

[Out] ((96*a^2*(2*a^4 + 21*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(9/2) - (Sec[c + d*x]^3*(-264*a^6*b - 358*a^4*b^3 + 8*a^2*b^5 - 16*b^7 - 8*(44*a^6*b + 55*a^4*b^3 + 8*a^2*b^5 - 2*b^7)*Cos[2*(c + d*x)] - 2*(28*a^6*b + 89*a^4*b^3 - 12*a^2*b^5)*Cos[4*(c + d*x)] + 22*a^5*b^2*Sin[c + d*x] - 264*a^3*b^4*Sin[c + d*x] + 32*a*b^6*Sin[c + d*x] + 32*a^7*Sin[3*(c + d*x)] - 91*a^5*b^2*Sin[3*(c + d*x)] - 244*a^3*b^4*Sin[3*(c + d*x)] - 12*a*b^6*Sin[3*(c + d*x)] - 17*a^5*b^2*Sin[5*(c + d*x)] - 76*a^3*b^4*Sin[5*(c + d*x)] - 12*a*b^6*Sin[5*(c + d*x)]))/(a^2 - b^2)^4*(a + b*Sin[c + d*x])^2))/(96*d)

Maple [B] time = 0.131, size = 922, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x)`

[Out]
$$\begin{aligned} & -1/3/d/(a+b)^3/(\tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^3/(\tan(1/2*d*x+1/2*c)-1) \\ & ^2+1/d/(a+b)^4/(\tan(1/2*d*x+1/2*c)-1)*a-1/2/d/(a+b)^4/(\tan(1/2*d*x+1/2*c)-1) \\ & *b+5/d*a^5/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b \\ & +a)^2*\tan(1/2*d*x+1/2*c)^3*b^2+6/d*a^3/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c)^2 \\ & *a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c)^3*b^4+4/d*a^6/(a-b)^4/(a \\ & +b)^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c) \\ & ^2*b+15/d*a^4/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c) \\ & *b+a)^2*\tan(1/2*d*x+1/2*c)^2*b^3+14/d*b^5/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c) \\ & ^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a^2*\tan(1/2*d*x+1/2*c)^2+11/d*a^5/(a-b)^4 \\ & /(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c) \\ & *b^2+22/d*a^3/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c) \\ & *b+a)^2*\tan(1/2*d*x+1/2*c)*b^4+4/d*a^6/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c) \\ & ^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*b+7/d*a^4/(a-b)^4/(a+b)^4/(\tan(1/2*d*x+1/2*c) \\ & ^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*b^3+2/d*a^6/(a-b)^4/(a+b)^4/(a^2-b^2)^{(1/2)} \\ & *arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})+21/d*a^4/(a-b)^4/(a+b)^4 \\ & /(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c) \\ & *b^2+12/d*b^4/(a-b)^4/(a+b)^4/(a^2-b^2)^{(1/2)}*arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c) \\ & +2*b)/(a^2-b^2)^{(1/2)})*a^2-1/3/d/(a-b)^3/(\tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^3 \\ & /(\tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^4/(\tan(1/2*d*x+1/2*c)+1)*a+1/2/d/(a-b)^4 \\ & /(\tan(1/2*d*x+1/2*c)+1)*b \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 2.85301, size = 2765, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 - 2*(28*a^8*b \\ & + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^7)*\cos(d*x + c)^4 - 4*(8*a^8*b - 25* \\ & a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9)*\cos(d*x + c)^2 - 3*((2*a^6*b^2 + 2 \\ & 1*a^4*b^4 + 12*a^2*b^6)*\cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b \\ & ^5)*\cos(d*x + c)^3*\sin(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2 \\ & *b^6)*\cos(d*x + c)^3)*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(d*x + c)^2 - \\ & 2*a*b*\sin(d*x + c) - a^2 - b^2 + 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x \\ & + c))*\sqrt{-a^2 + b^2}))/ (b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b \\ & ^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^2 \\ & + 59*a^5*b^4 - 64*a^3*b^6 - 12*a*b^8)*\cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b^2 \\ & ^2 + 3*a^5*b^4 + 5*a^3*b^6 - 3*a*b^8)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^10* \end{aligned}$$


```

b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x +
c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11
)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^
8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6*(2*a^8*b - 8*a^6*b^3 + 12*a^4
*b^5 - 8*a^2*b^7 + 2*b^9 - (28*a^8*b + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^
7)*cos(d*x + c)^4 - 2*(8*a^8*b - 25*a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9
)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 21*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5
- 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^
8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^
7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^2 + 59*a^5*b^4 - 64*a^
3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b^2 + 3*a^5*b^4 + 5*a^3
*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a
^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a
^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin
(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)
*d*cos(d*x + c)^3)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 3.39856, size = 853, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```

[Out] 1/3*(3*(2*a^6 + 21*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(5*a^5*b^2*tan(1/
2*d*x + 1/2*c)^3 + 6*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1/2*d*x +
1/2*c)^2 + 15*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^5*tan(1/2*d*x + 1/
2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/2*c) + 22*a^3*b^4*tan(1/2*d*x + 1/2*c)
+ 4*a^6*b + 7*a^4*b^3)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 2*(3*a^5*tan(1/
2*d*x + 1/2*c)^5 + 24*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b^3*tan(1/2*d*x + 1/2*
c)^4 - 3*b^5*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*tan(1/2*d*x + 1/2*c)^3 - 56*a^
3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*b*tan
(1/2*d*x + 1/2*c)^2 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*
x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c) + 9*a*b^4*tan(1/2*d*x + 1/2*c)
- 15*a^4*b - 20*a^2*b^3 - b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
b^8)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d

```

$$3.199 \quad \int \frac{\tan^2(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=350

$$\frac{a^2(2a^2+b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{4a^2b^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{1}{2d(a^2-b^2)^{7/2}}$$

[Out] $(-4a^2b^2 \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) - (a^2(2a^2 + b^2) \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) - (2b^2(3a^2 + b^2) \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) + \operatorname{Cos}[c + dx]/(2(a + b)^3 d(1 - \operatorname{Sin}[c + dx])) - \operatorname{Cos}[c + dx]/(2(a - b)^3 d(1 + \operatorname{Sin}[c + dx])) - (a^2 b \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^2 d(a + b \operatorname{Sin}[c + dx])^2) - (3a^3 b \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^3 d(a + b \operatorname{Sin}[c + dx])) - (2a^2 b^3 \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^3 d(a + b \operatorname{Sin}[c + dx]))$

Rubi [A] time = 0.538886, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2731, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^2(2a^2+b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{4a^2b^2 \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}} - \frac{1}{2d(a^2-b^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tan}[c + dx]^2/(a + b \operatorname{Sin}[c + dx])^3, x]$

[Out] $(-4a^2b^2 \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) - (a^2(2a^2 + b^2) \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) - (2b^2(3a^2 + b^2) \operatorname{ArcTan}[(b + a \operatorname{Tan}[(c + dx)/2])]/\operatorname{Sqrt}[a^2 - b^2])/((a^2 - b^2)^{(7/2)*d}) + \operatorname{Cos}[c + dx]/(2(a + b)^3 d(1 - \operatorname{Sin}[c + dx])) - \operatorname{Cos}[c + dx]/(2(a - b)^3 d(1 + \operatorname{Sin}[c + dx])) - (a^2 b \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^2 d(a + b \operatorname{Sin}[c + dx])^2) - (3a^3 b \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^3 d(a + b \operatorname{Sin}[c + dx])) - (2a^2 b^3 \operatorname{Cos}[c + dx])/(2(a^2 - b^2)^3 d(a + b \operatorname{Sin}[c + dx]))$

Rule 2731

$\operatorname{Int}[(a + b \sin(e + f(x)))^m \tan(e + f(x))^p, x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(\operatorname{Sin}[e + f(x)]^p (a + b \operatorname{Sin}[e + f(x)])^m)/(1 - \operatorname{Sin}[e + f(x)]^2)^{p/2}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{IntegersQ}[m, p/2]$

Rule 2648

$\operatorname{Int}[(a + b \sin(c + d(x)))^{-1}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{Cos}[c + dx]/(d(b + a \operatorname{Sin}[c + dx])), x] /;$ $\operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2664

$\operatorname{Int}[(a + b \sin(c + d(x)))^{n-1}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(b \operatorname{Cos}[c + dx](a + b \operatorname{Sin}[c + dx])^{n-1})/(d(n-1)(a^2 - b^2)), x] + \operatorname{Dist}[1$

```

/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]

```

Rule 2754

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

```

Rule 2660

```

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]

```

Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3} dx &= \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{a^2}{(a^2-b^2)(a+b\sin(c+dx))} \right) dx \\
&= \frac{\int \frac{1}{1+\sin(c+dx)} dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)} dx}{2(a+b)^3} - \frac{(2ab^2) \int \frac{1}{(a+b\sin(c+dx))^2} dx}{(a^2-b^2)^2} - \frac{a^2 \int \frac{1}{(a+b\sin(c+dx))^3} dx}{a^2-b^2} - \frac{a^2 \int \frac{1}{(a+b\sin(c+dx))^3} dx}{a^2-b^2} \\
&= \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} - \frac{a^2 b \cos(c+dx)}{2(a^2-b^2)^2 d(a+b\sin(c+dx))} \\
&= \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} - \frac{a^2 b \cos(c+dx)}{2(a^2-b^2)^2 d(a+b\sin(c+dx))} \\
&= -\frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} \\
&= -\frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} \\
&= -\frac{4a^2 b^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} - \frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} + \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} \\
&= -\frac{4a^2 b^2 \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} - \frac{a^2(2a^2+b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d} - \frac{2b^2(3a^2+b^2) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2} d}
\end{aligned}$$

Mathematica [A] time = 3.29141, size = 212, normalized size = 0.61

$$\frac{2(11a^2b^2+2a^4+2b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{ab \cos(c+dx)(b(3a^2+4b^2) \sin(c+dx)+4a^3+3ab^2)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right) \left(\frac{2}{(a-b)^3 \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right) \right)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]

[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)

Maple [B] time = 0.115, size = 766, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\tan(dx+c)^2/(a+b*\sin(dx+c))^3,x)$

[Out]
$$\begin{aligned} & -1/d/(a+b)^3/(\tan(1/2*d*x+1/2*c)-1)-5/d/(a-b)^3/(a+b)^3/(\tan(1/2*d*x+1/2*c) \\ & ^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c)^3*a^3*b^2-2/d*b^4/(a-b) \\ & ^3/(a+b)^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a*\tan(1/2*d* \\ & x+1/2*c)^3-4/d/(a-b)^3/(a+b)^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c) \\ & *b+a)^2*\tan(1/2*d*x+1/2*c)^2*a^4*b-11/d*b^3/(a-b)^3/(a+b)^3/(\tan(1/2*d*x+1/ \\ & 2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a^2*\tan(1/2*d*x+1/2*c)^2-6/d*b^5/(a-b) \\ & ^3/(a+b)^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+ \\ & 1/2*c)^2-11/d/(a-b)^3/(a+b)^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)* \\ & b+a)^2*\tan(1/2*d*x+1/2*c)*a^3*b^2-10/d*b^4/(a-b)^3/(a+b)^3/(\tan(1/2*d*x+1/2 \\ & *c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a*\tan(1/2*d*x+1/2*c)-4/d/(a-b)^3/(a+b)^ \\ & 3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a^4*b-3/d*b^3/(a-b)^3 \\ & /(a+b)^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*a^2-2/d/(a-b)^ \\ & 3/(a+b)^3/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2) \\ & ^{(1/2)})*a^4-11/d*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/ \\ & 2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*a^2-2/d*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^{(1 \\ & /2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-1/d/(a-b)^3/(t \\ & an(1/2*d*x+1/2*c)+1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\tan(dx+c)^2/(a+b*\sin(dx+c))^3,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 2.32169, size = 2067, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\tan(dx+c)^2/(a+b*\sin(dx+c))^3,x, \text{algorithm}="fricas")$

[Out]
$$\begin{aligned} & [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11 \\ & *a^2*b^5 + 2*b^7)*\cos(dx + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*\cos(dx \\ & x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*\cos(dx + c)*\sin(dx + c) - (\\ & 2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*\cos(dx + c))*\sqrt{-a^2 + b^2}*\log \\ & (((2*a^2 - b^2)*\cos(dx + c)^2 - 2*a*b*\sin(dx + c) - a^2 - b^2 + 2*(a*\cos(\\ & dx + c)*\sin(dx + c) + b*\cos(dx + c))*\sqrt{-a^2 + b^2}))/ (b^2*\cos(dx + c) \\ & ^2 - 2*a*b*\sin(dx + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - \\ & 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*\cos(dx + c)^2)*\sin(dx + c))/((a \\ & ^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^{10})*d*\cos(dx + c)^3 - 2*(a^ \\ & 9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*\cos(dx + c)*\sin(dx + c \\ &) - (a^{10} - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^{10})*d*\cos(dx \\ & + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 - \\ & 11*a^2*b^5 + 2*b^7)*\cos(dx + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*\cos \\ & (dx + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*\cos(dx + c)*\sin(dx + c) \\ & - (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*\cos(dx + c))*\sqrt{a^2 - b^2}* \\ & rctan(-(a*\sin(dx + c) + b)/(\sqrt{a^2 - b^2}*\cos(dx + c))) - (2*a^7 - 6*a^ \end{aligned}$$

$$5b^2 + 6a^3b^4 - 2ab^6 - 5(a^5b^2 + a^3b^4 - 2ab^6)\cos(dx + c)^2 \sin(dx + c) / ((a^8b^2 - 4a^6b^4 + 6a^4b^6 - 4a^2b^8 + b^{10})d\cos(dx + c)^3 - 2(a^9b - 4a^7b^3 + 6a^5b^5 - 4a^3b^7 + ab^9)d\cos(dx + c)\sin(dx + c) - (a^{10} - 3a^8b^2 + 2a^6b^4 + 2a^4b^6 - 3a^2b^8 + b^{10})d\cos(dx + c))]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)**2/(a+b*sin(dx+c))**3,x)

[Out] Integral(tan(c + dx)**2/(a + b*sin(c + dx))**3, x)

Giac [A] time = 1.9558, size = 518, normalized size = 1.48

$$\frac{(2a^4 + 11a^2b^2 + 2b^4) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sqrt{a^2 - b^2}} + \frac{2 \left(a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 3ab^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 3a^2b - b^3 \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right)} + \frac{5a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 2a^3b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2/(a+b*sin(dx+c))^3,x, algorithm="giac")

[Out] $-\left((2a^4 + 11a^2b^2 + 2b^4) \left(\pi \operatorname{floor} \left(\frac{1}{2} (dx + c) / \pi + \frac{1}{2} \right) \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right) / \left((a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sqrt{a^2 - b^2} \right) + 2 \left(a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 3ab^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 3a^2b - b^3 \right) / \left((a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right) \right) + (5a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 2a^3b^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 4a^4b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 11a^2b^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 6b^5 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 11a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 10a^2b^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 4a^4b + 3a^2b^3) / \left((a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right) \right) + a^2 \right) / d$

$$3.200 \quad \int \frac{\cot^2(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=202

$$\frac{(-9a^2b^2 + 2a^4 + 6b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}} - \frac{(5a^2 - 6b^2) \cot(c+dx)}{2a^3d(a^2-b^2)} + \frac{(2a^2 - 3b^2) \cot(c+dx)}{2a^2d(a^2-b^2)(a+b \sin(c+dx))} + \frac{3b \tan(c+dx)}{2a^2d(a^2-b^2)}$$

[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a + b*Sin[c + d*x]^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])))

Rubi [A] time = 0.788453, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2723, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{(-9a^2b^2 + 2a^4 + 6b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}} - \frac{(5a^2 - 6b^2) \cot(c+dx)}{2a^3d(a^2-b^2)} + \frac{(2a^2 - 3b^2) \cot(c+dx)}{2a^2d(a^2-b^2)(a+b \sin(c+dx))} + \frac{3b \tan(c+dx)}{2a^2d(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]

[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a + b*Sin[c + d*x]^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])))

Rule 2723

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^2, x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3055

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3001

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Ssin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rule 2660

```

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]

```

Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3} dx &= \int \frac{\csc^2(c+dx)(1-\sin^2(c+dx))}{(a+b\sin(c+dx))^3} dx \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)(3(a^2-b^2)-2(a^2-b^2)\sin^2(c+dx))}{(a+b\sin(c+dx))^2} dx}{2a(a^2-b^2)} \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)(5a^4-11a^2b^2+6b^4-a^2)}{(a+b\sin(c+dx))^2} dx}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} \\
&= -\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} \\
&= -\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} \\
&= \frac{3b \tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} \\
&= \frac{3b \tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))} \\
&= -\frac{(2a^4-9a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{3/2}d} + \frac{3b \tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] time = 5.78381, size = 195, normalized size = 0.97

$$\frac{2(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{ab(4b^2-3a^2)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - \frac{\phantom{2(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}}{2a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]

[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]] - 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 + (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])) + a*Tan[(c + d*x)/2])/(2*a^4*d)

Maple [B] time = 0.116, size = 729, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x)

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-5/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3*b^2+6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)/a^2*tan(1/2*d*x+1/2*c)^2+10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-11/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)*b^2+14/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*b+5/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)-2/d/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+9/d/a^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-6/d/a^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.54849, size = 3044, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 17*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x + c)
```

```

c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
- a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.55189, size = 458, normalized size = 2.27

$$\frac{2(2a^4 - 9a^2b^2 + 6b^4) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6 - a^4b^2) \sqrt{a^2 - b^2}} + \frac{2 \left(5a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 6ab^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 4a^4b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 3a^2b^3 \right)}{(a^6 - a^4b^2) \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```

[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
- a)/(a^4*tan(1/2*d*x + 1/2*c))/d

```

$$3.201 \quad \int \frac{\cot^4(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=289

$$\frac{(-19a^2b^2 + 2a^4 + 20b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^6 d \sqrt{a^2 - b^2}} + \frac{(17a^2 - 60b^2) \cot(c + dx)}{6a^5 d} - \frac{b(9a^2 - 20b^2) \tanh^{-1}(\cos(c + dx))}{2a^6 d} - \frac{(a^2 - b^2) \cot(c + dx)}{2a^6 d}$$

[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6*a^3*b*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 1.07153, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{(-19a^2b^2 + 2a^4 + 20b^4) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{a^6 d \sqrt{a^2 - b^2}} + \frac{(17a^2 - 60b^2) \cot(c + dx)}{6a^5 d} - \frac{b(9a^2 - 20b^2) \tanh^{-1}(\cos(c + dx))}{2a^6 d} - \frac{(a^2 - b^2) \cot(c + dx)}{2a^6 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]

[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6*a^3*b*d*(a + b*Sin[c + d*x]))

Rule 2724

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)^4, x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m + 1))*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x] - Simp[((3*a^2 + b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*Sin[e + f*x]^2, x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 3055

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2, x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*SIN[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3} dx &= \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)(2(3a^2-10b^2)-2ab\sin(c+dx))}{(a+b\sin(c+dx))^3} dx}{6} \\
&= \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{(3a^2-20b^2)\cot(c+dx)\csc(c+dx)}{6a^3bd(a+b\sin(c+dx))} \\
&= -\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{3ad(a+b\sin(c+dx))} \\
&= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))} \\
&= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))} \\
&= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} \\
&= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} \\
&= \frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}d} - \frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}
\end{aligned}$$

Mathematica [A] time = 6.21126, size = 459, normalized size = 1.59

$$\frac{(9a^2b-20b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{(20b^3-9a^2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))} + \frac{a^2b\cos(c+dx)}{2a^6d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]

[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Sin[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)

Maple [B] time = 0.134, size = 780, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)+5/d/a^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d*
b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c
)^2-1/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)^2*b^3-18/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2*b^5+11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*b-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*b^3+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))-19/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/d/a^3/tan(1/2*d*x+1/2*c
)^3+5/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^4*b
/tan(1/2*d*x+1/2*c)^2+9/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*ln(ta
n(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.54187, size = 4578, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b
- 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
+ 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
```

$$\begin{aligned}
& 0*a*b^6)*\cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*\cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*\cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^5)*\cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*\cos(d*x + c))*\sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*\cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*\cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*\cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*\cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*\sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*\cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*\cos(d*x + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*\cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*\cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*\cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*\cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*\cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*\cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*\cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*\cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*\cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*\cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^5)*\cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*\cos(d*x + c))*\sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*\cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*\cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*\cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*\cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*\sin(d*x + c))]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c + dx)}{(a + b \sin(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**3, x)

Giac [A] time = 2.05061, size = 609, normalized size = 2.11

$$\frac{12(9a^2b - 20b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^6} + \frac{24(2a^4 - 19a^2b^2 + 20b^4) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + b}{\sqrt{a^2 - b^2}}\right) \right)}{\sqrt{a^2 - b^2} a^6} + \frac{24\left(5a^3b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 10ab^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^3}{\sqrt{a^2 - b^2} a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")


```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

$$3.202 \quad \int \frac{\cot^6(c+dx)}{(a+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=492

$$\frac{\sqrt{a^2 - b^2} (-29a^2b^2 + 2a^4 + 42b^4) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}} \right)}{a^8 d} - \frac{(-645a^2b^2 + 91a^4 + 630b^4) \cot(c+dx)}{30a^7 d} + \frac{b(-200a^2b^2 + 42b^4)}{30a^7 d}$$

[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b^2*d*(a + b*Sin[c + d*x]))

Rubi [A] time = 2.12688, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{\sqrt{a^2 - b^2} (-29a^2b^2 + 2a^4 + 42b^4) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) + b}{\sqrt{a^2 - b^2}} \right)}{a^8 d} - \frac{(-645a^2b^2 + 91a^4 + 630b^4) \cot(c+dx)}{30a^7 d} + \frac{b(-200a^2b^2 + 42b^4)}{30a^7 d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]

[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b^2*d*(a + b*Sin[c + d*x]))

Rule 2726

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)/tan[(e_) + (f_)*(x_)]^6, x_Symbol] := -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x)]/Sin[e + f*x]^4, x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*x]^6), x]

```
x]^2), x] + Simp[(a*cos[e + f*x]*(a + b*sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*sin[e + f*x]^3), x] - Simp[(b*(m - 4)*cos[e + f*x]*(a + b*sin[e + f*x]
)^(m + 1))/(20*a^2*f*sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3} dx &= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{5ad} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4)\cot(c+dx)}{60a^3b^2d(a+b\sin(c+dx))^2} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4)\cot(c+dx)}{60a^3b^2d(a+b\sin(c+dx))^2} \\
&= -\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)}{12b^2d(a+b\sin(c+dx))^2} \\
&= \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} \\
&= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd} \\
&= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd} \\
&= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} \\
&= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} \\
&= -\frac{\sqrt{a^2-b^2}(2a^4-29a^2b^2+42b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d}
\end{aligned}$$

Mathematica [A] time = 1.70473, size = 448, normalized size = 0.91

$$-\frac{3840(-31a^4b^2+71a^2b^4+2a^6-42b^6)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 480b(-200a^2b^2+45a^4+168b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 480b(-200a^2b^2+45a^4+168b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]

[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 + 9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)] - 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b

$$\frac{\sin(5(c+dx)) + 3975a^3b^3\sin(5(c+dx)) - 3780ab^5\sin(5(c+dx))}{(b+a\csc(c+dx))^2(3840a^8d)}$$

Maple [B] time = 0.138, size = 1252, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x)`

[Out]
$$\begin{aligned} & -13/d/a^6/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*b^5-21/d/a^8* \\ & b^5*\ln(\tan(1/2*d*x+1/2*c))+1/4/d/a^5*b^2*\tan(1/2*d*x+1/2*c)^3-5/4/d/a^6*\tan \\ & (1/2*d*x+1/2*c)^2*b^3+15/2/d/a^7*b^4*\tan(1/2*d*x+1/2*c)+3/64/d/a^4*b/\tan(1/ \\ & 2*d*x+1/2*c)^4-3/64/d/a^4*b*\tan(1/2*d*x+1/2*c)^4-15/2/d/a^7/\tan(1/2*d*x+1/2 \\ & *c)*b^4-5/d/a^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2 \\ & *d*x+1/2*c)^3*b^2-1/160/d/a^3/\tan(1/2*d*x+1/2*c)^5+1/160/d/a^3*\tan(1/2*d*x+ \\ & 1/2*c)^5-45/8/d/a^4*b*\ln(\tan(1/2*d*x+1/2*c))+19/d/a^5/(\tan(1/2*d*x+1/2*c)^2 \\ & *a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c)^3*b^4-4/d*b/(\tan(1/2*d*x+ \\ & 1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2/a^2*\tan(1/2*d*x+1/2*c)^2+9/d/a^4/(\tan \\ & (1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c)^2*b^3+2 \\ & 1/d/a^6/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/2 \\ & *c)^2*b^5-11/d/a^3/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(\\ & 1/2*d*x+1/2*c)*b^2+49/d/a^5/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+ \\ & a)^2*\tan(1/2*d*x+1/2*c)*b^4+31/d/a^4/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/ \\ & 2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^2-71/d/a^6/(a^2-b^2)^{(1/2)}*\arctan(1/2* \\ & (2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^4+27/4/d/a^5/\tan(1/2*d*x+1/ \\ & 2*c)*b^2-3/4/d/a^4*b/\tan(1/2*d*x+1/2*c)^2+25/d/a^6*b^3*\ln(\tan(1/2*d*x+1/2*c \\ &))+3/4/d/a^4*b*\tan(1/2*d*x+1/2*c)^2-27/4/d/a^5*b^2*\tan(1/2*d*x+1/2*c)-4/d/a \\ & ^2/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*b+17/d/a^4/(\tan(1/2* \\ & d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*b^3-2/d/a^2/(a^2-b^2)^{(1/2)}*\arct \\ & an(1/2*(2*a*\tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^{(1/2)})-7/96/d/a^3*\tan(1/2*d*x \\ & +1/2*c)^3+7/96/d/a^3/\tan(1/2*d*x+1/2*c)^3+11/16/d/a^3*\tan(1/2*d*x+1/2*c)-11 \\ & /16/d/a^3/\tan(1/2*d*x+1/2*c)-14/d/a^7/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x \\ & +1/2*c)*b+a)^2*\tan(1/2*d*x+1/2*c)^3*b^6+5/4/d/a^6*b^3/\tan(1/2*d*x+1/2*c)^2- \\ & 26/d/a^8/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan(1/2*d*x+1/ \\ & 2*c)^2*b^7-38/d/a^7/(\tan(1/2*d*x+1/2*c)^2*a+2*\tan(1/2*d*x+1/2*c)*b+a)^2*\tan \\ & (1/2*d*x+1/2*c)*b^6+42/d/a^8/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*d*x+1/ \\ & 2*c)+2*b)/(a^2-b^2)^{(1/2)})*b^6-1/4/d/a^5/\tan(1/2*d*x+1/2*c)^3*b^2 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 6.51664, size = 6091, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*\cos(d*x + c)^7 - 4*(92*a^7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*\cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*\cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*\cos(d*x + c)^6 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 + 126*b^6)*\cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2 + 2*(a*\cos(d*x + c)*\sin(d*x + c) + b*\cos(d*x + c))*\sqrt{-a^2 + b^2})/(b^2*\cos(d*x + c)^2 - 2*a*b*\sin(d*x + c) - a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*\cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*\cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*\cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*\cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*\cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*\cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*\cos(d*x + c)^3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*\cos(d*x + c))*\sin(d*x + c))/(2*a^9*b*d*\cos(d*x + c)^6 - 6*a^9*b*d*\cos(d*x + c)^4 + 6*a^9*b*d*\cos(d*x + c)^2 - 2*a^9*b*d + (a^8*b^2*d*\cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*\cos(d*x + c)^4 + (2*a^10 + 3*a^8*b^2)*d*\cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*\sin(d*x + c)), -1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*\cos(d*x + c)^7 - 4*(92*a^7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*\cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*\cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*\cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*\cos(d*x + c)^6 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 + 126*b^6)*\cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*\cos(d*x + c)^2)*\sin(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\sin(d*x + c) + b)/(\sqrt{a^2 - b^2}*\cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*\cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*\cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*\cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(1/2*\cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*\cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*\cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*\cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/2*\cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5) \end{aligned}$$

$$\begin{aligned} & * \cos(dx + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*\cos(dx + c)^3 \\ & + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*\cos(dx + c)*\sin(dx + c) / (\\ & 2*a^9*b*d*\cos(dx + c)^6 - 6*a^9*b*d*\cos(dx + c)^4 + 6*a^9*b*d*\cos(dx + c) \\ &)^2 - 2*a^9*b*d + (a^8*b^2*d*\cos(dx + c)^6 - (a^{10} + 3*a^8*b^2)*d*\cos(dx \\ & + c)^4 + (2*a^{10} + 3*a^8*b^2)*d*\cos(dx + c)^2 - (a^{10} + a^8*b^2)*d*\sin(dx \\ & + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)**6/(a+b*sin(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 1.85019, size = 987, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^6/(a+b*sin(dx+c))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) \\ &))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*\text{floor}(1/2*(d*x \\ & + c)/pi + 1/2)*\text{sgn}(a) + \arctan((a*\tan(1/2*d*x + 1/2*c) + b)/\sqrt{a^2 - b^2} \\ &))/(\sqrt{a^2 - b^2})*a^8) + 960*(5*a^5*b^2*\tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b \\ & ^4*\tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*\tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*\tan(1 \\ & /2*d*x + 1/2*c)^2 - 9*a^4*b^3*\tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*\tan(1/2*d \\ & *x + 1/2*c)^2 + 26*b^7*\tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*\tan(1/2*d*x + 1/ \\ & 2*c) - 49*a^3*b^4*\tan(1/2*d*x + 1/2*c) + 38*a*b^6*\tan(1/2*d*x + 1/2*c) + 4* \\ & a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*\tan(1/2*d*x + 1/2*c)^2 + 2*b*\tan(1/2*d \\ & *x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*\tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b \\ & ^3*\tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*\tan(1/2*d*x + 1/2*c)^5 - 660*a^5*\tan(\\ & 1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*\tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*\tan(1 \\ & /2*d*x + 1/2*c)^4 - 720*a^4*b*\tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*\tan(1/2 \\ & *d*x + 1/2*c)^3 + 70*a^5*\tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*\tan(1/2*d*x + \\ & 1/2*c)^2 + 45*a^4*b*\tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*\tan(1/2*d*x + 1/2*c \\ &)^5) - (6*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 45*a^{11}*b*\tan(1/2*d*x + 1/2*c)^4 - \\ & 70*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 240*a^{10}*b^2*\tan(1/2*d*x + 1/2*c)^3 + 720* \\ & a^{11}*b*\tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*\tan(1/2*d*x + 1/2*c)^2 + 660*a \\ & ^{12}*\tan(1/2*d*x + 1/2*c) - 6480*a^{10}*b^2*\tan(1/2*d*x + 1/2*c) + 7200*a^8*b^4 \\ & * \tan(1/2*d*x + 1/2*c))/a^{15})/d \end{aligned}$$

3.203 $\int (a + b \sin(e + fx))^3 (g \tan(e + fx))^p dx$

Optimal. Leaf size=271

$$\frac{3a^2b \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2}; \frac{p+4}{2}; \sin^2(e + fx)\right)}{fg(p+2)} + \frac{a^3 (g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; \sin^2(e + fx)\right)}{fg(p+1)}$$

[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rubi [A] time = 0.379927, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^2b \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2}; \frac{p+4}{2}; \sin^2(e + fx)\right)}{fg(p+2)} + \frac{a^3 (g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; \sin^2(e + fx)\right)}{fg(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*SIN[e + f*x])^3*(g*TAN[e + f*x])^p,x]

[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*TAN[e + f*x])^p, (a + b*SIN[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*TAN[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2602

Int[((a_)*sin[(e_.) + (f_.)*(x_)]^(m_))*((b_)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[(a*cos[e + f*x]^(n + 1)*(b*tan[e + f*x]^(n + 1)))/(b*(a*sin[e + f*x]^(n + 1))), Int[(a*sin[e + f*x]^(m + n))/cos[e + f*x]^n, x] , x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]

Rule 2577

Int[(cos[(e_.) + (f_.)*(x_)]*(b_))^(n_)*((a_)*sin[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*cos[e + f*x])^(2*FracPart[(n - 1)/2])*(a*sin[e + f*x]^(m + 1))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rule 2591

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int (a + b \sin(e + fx))^3 (g \tan(e + fx))^p dx &= \int (a^3 (g \tan(e + fx))^p + 3a^2 b \sin(e + fx) (g \tan(e + fx))^p + 3ab^2 \sin^2(e + fx) (g \tan(e + fx))^p + b^3 \sin^3(e + fx) (g \tan(e + fx))^p) dx \\ &= a^3 \int (g \tan(e + fx))^p dx + (3a^2 b) \int \sin(e + fx) (g \tan(e + fx))^p dx + (3ab^2) \int \sin^2(e + fx) (g \tan(e + fx))^p dx + b^3 \int \sin^3(e + fx) (g \tan(e + fx))^p dx \\ &= \frac{(a^3 g) \text{Subst}\left(\int \frac{x^p}{g^2 + x^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(3ab^2 g) \text{Subst}\left(\int \frac{x^{2+p}}{(g^2 + x^2)^2} dx, x, g \tan(e + fx)\right)}{f} \\ &= \frac{a^3 {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{3a^2 b \cos^2(e + fx) (g \tan(e + fx))^{1+p}}{fg(1+p)} \end{aligned}$$

Mathematica [C] time = 18.4028, size = 4791, normalized size = 17.68

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]

[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2])*(g*Tan[e + f*x])^p*(-(b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 - a^3*Sin[e + f*x]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + ((3*I)/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 - (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[e + f*x]^3*(a^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p - I*a^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^3*((I/8)*b^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p - (I/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (I/8)*b^3*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + (I/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (I/8)*b^3*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)

$$\begin{aligned}
& e + f*x)]*Tan[e + f*x]^p + (b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8) + Sin[e \\
& + f*x]^2*((-3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + ((3*I)/2)*a^2*b*S \\
& in[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((-3*a*b^2* \\
& Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin[3 \\
& *(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*T \\
& an[e + f*x]^p)/4) + Cos[2*(e + f*x)]^2*((-3*b^3*Sin[3*(e + f*x)]*Tan[e + f* \\
& x]^p)/8 - (3*a*b^2*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I) \\
& /8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)] \\
& *(((3*I)/8)*b^3*Tan[e + f*x]^p - ((3*I)/4)*a*b^2*Sin[e + f*x]*Tan[e + f*x] \\
& ^p - (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/8)) + Cos[3*(e + f*x)]*((-I/8) \\
& *b^3*Tan[e + f*x]^p - I*a^3*Sin[e + f*x]^3*Tan[e + f*x]^p - (3*b^3*Sin[2*(e \\
& + f*x)]*Tan[e + f*x]^p)/8 + ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^ \\
& p + (b^3*Sin[2*(e + f*x)]^3*Tan[e + f*x]^p)/8 + Sin[e + f*x]^2*(((-3*I)/2)* \\
& a^2*b*Tan[e + f*x]^p - (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2) + Sin[e \\
& + f*x]*(((3*I)/4)*a*b^2*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*Tan[e \\
& + f*x]^p)/2 + ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)) + Cos[e + \\
& f*x]^2*((3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + 3*a^3*Sin[e + f*x]*S \\
& in[3*(e + f*x)]*Tan[e + f*x]^p - ((3*I)/2)*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e \\
& + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)]*(((3*I)/2)*a^2*b*Tan[e + f*x]^p + \\
& (3*I)*a^3*Sin[e + f*x]*Tan[e + f*x]^p + (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + \\
& f*x]^p)/2) + Cos[2*(e + f*x)]*(((3*I)/2)*a^2*b*Cos[3*(e + f*x)]*Tan[e + f* \\
& x]^p - (3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2)) + Cos[e + f*x]*(((3*I) \\
& /4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*I)*a^3*Sin[e + f*x]^2*Sin[3* \\
& (e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[\\
& e + f*x]^p)/2 - ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + \\
& f*x]^p + Cos[2*(e + f*x)]^2*((-3*a*b^2*Cos[3*(e + f*x)]*Tan[e + f*x]^p)/4 \\
& + ((3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((3*I)*a^ \\
& 2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p + 3*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e + f \\
& *x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*((-3*a*b^2*Tan[e + f*x]^p)/4 - 3*a^ \\
& 3*Sin[e + f*x]^2*Tan[e + f*x]^p + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Tan[e + \\
& f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4 + Sin[e + f*x]*(-3*a \\
& ^2*b*Tan[e + f*x]^p + (3*I)*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)) + Cos[2 \\
& *(e + f*x)]*(((3*I)/2)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*I)*a^2*b \\
& *Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*S \\
& in[3*(e + f*x)]*Tan[e + f*x]^p)/2 + Cos[3*(e + f*x)]*((3*a*b^2*Tan[e + f*x] \\
& ^p)/2 + 3*a^2*b*Sin[e + f*x]*Tan[e + f*x]^p - ((3*I)/2)*a*b^2*Sin[2*(e + f \\
& x)]*Tan[e + f*x]^p)) + Cos[2*(e + f*x)]*((3*b^3*Sin[3*(e + f*x)]*Tan[e + f \\
& *x]^p)/8 + (3*a^2*b*Sin[e + f*x]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3 \\
& *I)/4)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*b^3*Sin[2* \\
& (e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 + Sin[e + f*x]*((3*a*b^2*Si \\
& n[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin[3*(\\
& e + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*(((3*I)/8)*b^3*Tan[e + f*x]^p \\
& + ((3*I)/2)*a^2*b*Sin[e + f*x]^2*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]*T \\
& an[e + f*x]^p)/4 - ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p + Sin[e \\
& + f*x]*(((3*I)/2)*a*b^2*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Tan[e + \\
& f*x]^p)/2))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p \\
& *Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 \\
& + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*Appel \\
& lF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - \\
& 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan \\
& [(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + \\
& f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, \\
& Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, \\
& Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2)))*Tan[(e + f*x)/2]))*Tan[e + f*x] \\
& ^(-1 + p))/((1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f \\
& *x)/2]^2)^p*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x) \\
& /2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, \\
& (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF \\
& 1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1
\end{aligned}$$

+ p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]))*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*(-(1 + p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 2*b*(((1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Sec[(e + f*x)/2]^2)/2 + 6*a*b*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 6*a*b*(2 + p)*((-3*(1 + p)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (1 + p)*Tan[(e + f*x)/2]*(3*a^2*(-2*(1 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + 4*b^2*(-3*(1 + p/2)*AppellF1[2 + p/2, p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + (4*(1 + p/2)*AppellF1[2 + p/2, p, 5, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) - ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2))))*Tan[e + f*x]^p)/((1 + p)*(2 + p)))))

Maple [F] time = 1.514, size = 0, normalized size = 0.

$$\int (a + b \sin(fx + e))^3 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)

[Out] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a)^3 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3ab^2 \cos(fx + e)^2 - a^3 - 3ab^2 + \left(b^3 \cos(fx + e)^2 - 3a^2b - b^3\right) \sin(fx + e)\right) (g \tan(fx + e))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a)^3 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)

3.204 $\int (a + b \sin(e + fx))^2 (g \tan(e + fx))^p dx$

Optimal. Leaf size=186

$$\frac{a^2 (g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{2ab \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+2)}$$

[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rubi [A] time = 0.242273, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2 (g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{2ab \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*SIN[e + f*x])^2*(g*TAN[e + f*x])^p,x]

[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*TAN[e + f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*TAN[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*TAN[e + f*x])^(3 + p))/(f*g^3*(3 + p))

Rule 2722

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*TAN[e + f*x])^p, (a + b*SIN[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*TAN[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2602

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a*COS[e + f*x]^(n+1)*(b*TAN[e + f*x])^(n+1))/(b*(a*SIN[e + f*x])^(n+1)), Int[(a*SIN[e + f*x])^(m+n)/COS[e + f*x]^n, x]

, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]

Rule 2577

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^ (n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2]))*(a*SIN[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rule 2591

Int[sin[(e_.) + (f_.)*(x_.)]^ (m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^ (n_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int (a + b \sin(e + fx))^2 (g \tan(e + fx))^p dx &= \int (a^2 (g \tan(e + fx))^p + 2ab \sin(e + fx) (g \tan(e + fx))^p + b^2 \sin^2(e + fx) (g \tan(e + fx))^p) dx \\ &= a^2 \int (g \tan(e + fx))^p dx + (2ab) \int \sin(e + fx) (g \tan(e + fx))^p dx + b^2 \int \sin^2(e + fx) (g \tan(e + fx))^p dx \\ &= \frac{(a^2 g) \text{Subst}\left(\int \frac{x^p}{g^2 + x^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(b^2 g) \text{Subst}\left(\int \frac{x^{2+p}}{(g^2 + x^2)^2} dx, x, g \tan(e + fx)\right)}{f} \\ &= \frac{a^2 {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{2ab \cos^2(e + fx) (g \tan(e + fx))^{1+p}}{fg(1+p)} \end{aligned}$$

Mathematica [C] time = 14.2085, size = 2464, normalized size = 13.25

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]

[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[e + f*x])^p*(-(b^2*Cos[2*(e + f*x)]^3*Tan[e + f*x]^p)/4 + (I/4)*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p + I*a^2*Sin[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p + (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/2 - (I/4)*b^2*Sin[2*(e + f*x)]^3*Tan[e + f*x]^p + Cos[e + f*x]^2*(a^2*Cos[2*(e + f*x)]*Tan[e + f*x]^p - I*a^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^2*((b^2*Tan[e + f*x]^p)/2 + a*b*Sin[e + f*x]*Tan[e + f*x]^p - (I/4)*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*(I*a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p) + Cos[2*(e + f*x)]*(-(b^2*Tan[e + f*x]^p)/4 - a*b*Sin[e + f*x]*Tan[e + f*x]^p - a^2*Sin[e + f*x]^2*Tan[e + f*x]^p - (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4) + Cos[e + f*x]*((-I)*a*b*Cos[2*(e + f*x)]^2*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + 2*a^2*Sin[e + f*x]*Sin[2*(e + f*x)]*Tan[e + f*x]^p - I*a*b*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p

```

] ^p + Cos[2*(e + f*x)]*(I*a*b*Tan[e + f*x]^p + (2*I)*a^2*Sin[e + f*x]*Tan[e
+ f*x]^p)))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*
Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 +
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*AppellF1[(
1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*(2
+ p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^(-1 + p))/((1 + p)*(2 +
p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a^2*(2 + p)*
AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2
+ p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e +
f*x]^p)/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p
)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2]))*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 +
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*AppellF1[(
1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*(2
+ p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (
2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*(-(1 +
p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*
AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*b*((a*(1 +
p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2)/2 + a*(1 + p)*Tan[(e + f*x)/2]*((-2*(1 + p/2)*Appell
F1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1
+ p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/(2 + p/2)) + b*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1 + p
)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1
+ p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - b*(2 + p)*((-3*(1 + p)*AppellF1[1 + (
1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)
/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p))))*Tan[e + f*x]^p)/((1 + p)*(2 + p)
)))

```

Maple [F] time = 1.446, size = 0, normalized size = 0.

$$\int (a + b \sin(fx + e))^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)

[Out] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a)^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2 \cos(fx + e)^2 - 2ab \sin(fx + e) - a^2 - b^2\right)(g \tan(fx + e))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(g*tan(f*x + e))^p, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g \tan(e + fx))^p (a + b \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)

[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a)^2 (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)

3.205 $\int (a + b \sin(e + fx))(g \tan(e + fx))^p dx$

Optimal. Leaf size=129

$$\frac{a(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{b \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2};\right)}{fg(p+2)}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p))
```

Rubi [A] time = 0.147865, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2722, 3476, 364, 2602, 2577}

$$\frac{a(g \tan(e + fx))^{p+1} {}_2F_1\left(1, \frac{p+1}{2}; \frac{p+3}{2}; -\tan^2(e + fx)\right)}{fg(p+1)} + \frac{b \sin(e + fx) \cos^2(e + fx)^{\frac{p+1}{2}} (g \tan(e + fx))^{p+1} {}_2F_1\left(\frac{p+1}{2}, \frac{p+2}{2};\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 2722

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

Rule 364

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[e + f*x]^(n+1)*(b*Tan[e + f*x])^(n+1))/(b*(a*Sin[e + f*x])^(n+1)), Int[(a*Sin[e + f*x])^(m+n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2]))*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned} \int (a + b \sin(e + fx))(g \tan(e + fx))^p dx &= \int (a(g \tan(e + fx))^p + b \sin(e + fx)(g \tan(e + fx))^p) dx \\ &= a \int (g \tan(e + fx))^p dx + b \int \sin(e + fx)(g \tan(e + fx))^p dx \\ &= \frac{(ag) \operatorname{Subst}\left(\int \frac{x^p}{g^2+x^2} dx, x, g \tan(e + fx)\right)}{f} + \frac{(b \cos^{1+p}(e + fx) \sin^{-1-p}(e + fx))}{f} \\ &= \frac{a {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right) (g \tan(e + fx))^{1+p}}{fg(1+p)} + \frac{b \cos^2(e + fx)^{\frac{1+p}{2}} {}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right)}{fg(1+p)} \end{aligned}$$

Mathematica [C] time = 8.26249, size = 849, normalized size = 6.58

$$f \left(-16p \cos\left(\frac{1}{2}(e + fx)\right) \csc^3(e + fx) \sec(e + fx) \left(a(p + 2)F_1\left(\frac{p+1}{2}; p, 1; \frac{p+3}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right)\right), -\tan^2\left(\frac{1}{2}(e + fx)\right) \right) + 2b(p + 1) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]
```

```
[Out] (2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e + f*x)/2]^2*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*p*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*(1 + p)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(b*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (a*(2 + p)*(-AppellF1[(3 + p)/2, p, 2, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[(3 + p)/2, 1 + p, 1, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (2*b*(2 + p)*(-2*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/(4 + p)))
```

Maple [F] time = 0.904, size = 0, normalized size = 0.

$$\int (a + b \sin(fx + e)) (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)

[Out] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a) (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin(fx + e) + a\right) \left(g \tan(fx + e)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (g \tan(e + fx))^p (a + b \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))**p,x)

[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a) (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)

3.206 $\int \frac{(g \tan(e+fx))^p}{a+b \sin(e+fx)} dx$

Optimal. Leaf size=284

$$\frac{b \cos(e+fx) \sin^2(e+fx)^{-p/2} (g \tan(e+fx))^p F_1\left(\frac{1-p}{2}; -\frac{p}{2}, 1; \frac{3-p}{2}; \cos^2(e+fx), \frac{b^2 \cos^2(e+fx)}{b^2-a^2}\right)}{f(p-1)(b^2-a^2)} + \frac{ag \sin^2(e+fx)^{\frac{1-p}{2}} (g \tan(e+fx))^p}{f(p-1)(b^2-a^2)}$$

[Out] (a*g*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + p)/2)*Hypergeometric2F1[(1 - p)/2, (1 - p)/2, (3 - p)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*(Sin[e + f*x]^2)^((1 - p)/2)*(g*Tan[e + f*x])^(-1 + p))/((a^2 - b^2)*f*(-1 + p)) + (b*AppellF1[(1 - p)/2, -p/2, 1, (3 - p)/2, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]*(g*Tan[e + f*x])^p)/((-a^2 + b^2)*f*(-1 + p)*(Sin[e + f*x]^2)^(p/2))

Rubi [F] time = 0.0480098, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(g \tan(e+fx))^p}{a+b \sin(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]

[Out] Defer[Int] [(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]), x]

Rubi steps

$$\int \frac{(g \tan(e+fx))^p}{a+b \sin(e+fx)} dx = \int \frac{(g \tan(e+fx))^p}{a+b \sin(e+fx)} dx$$

Mathematica [B] time = 13.4983, size = 864, normalized size = 3.04

$$a^2 b f (p+1)(p+2)(a+b \sin(e+fx)) \left(\frac{\sec^2(e+fx) \left((a^2-b^2)(p+1) F_1\left(\frac{p+2}{2}; -\frac{1}{2}, 1; \frac{p+4}{2}; -\tan^2(e+fx), \frac{(b^2-a^2) \tan^2(e+fx)}{a^2}\right) \tan(e+fx) + a \left(b(p+2) {}_2F_1\left(1, \frac{p+2}{2}; \frac{p+4}{2}; \tan^2(e+fx)\right) \right)}{a^2 b (p+2)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]

[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*((a^2 - b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a

$$\begin{aligned} &^2*\tan[e + f*x] + a*(b*(2 + p)*\text{Hypergeometric2F1}[1, (1 + p)/2, (3 + p)/2, \\ &(-1 + b^2/a^2)*\tan[e + f*x]^2] - a*(1 + p)*\text{Hypergeometric2F1}[1/2, 1 + p/2, \\ &2 + p/2, -\tan[e + f*x]^2*\tan[e + f*x]))/(a^2*b*f*(1 + p)*(2 + p)*(a + b*\text{S} \\ &\text{in}[e + f*x])*((\text{Sec}[e + f*x]^2*\tan[e + f*x]^p*((a^2 - b^2)*(1 + p)*\text{AppellF1}[\\ &(2 + p)/2, -1/2, 1, (4 + p)/2, -\tan[e + f*x]^2, ((-a^2 + b^2)*\tan[e + f*x]^ \\ &2)/a^2]*\tan[e + f*x] + a*(b*(2 + p)*\text{Hypergeometric2F1}[1, (1 + p)/2, (3 + p) \\ &/2, (-1 + b^2/a^2)*\tan[e + f*x]^2] - a*(1 + p)*\text{Hypergeometric2F1}[1/2, 1 + p \\ &/2, 2 + p/2, -\tan[e + f*x]^2*\tan[e + f*x]))/(a^2*b*(2 + p)) + (\tan[e + f* \\ &x]^{(1 + p)}*((a^2 - b^2)*(1 + p)*\text{AppellF1}[(2 + p)/2, -1/2, 1, (4 + p)/2, -\tan \\ &[e + f*x]^2, ((-a^2 + b^2)*\tan[e + f*x]^2)/a^2]*\text{Sec}[e + f*x]^2 + (a^2 - b^ \\ &2)*(1 + p)*\tan[e + f*x]*((2*(-a^2 + b^2)*(2 + p)*\text{AppellF1}[1 + (2 + p)/2, -1 \\ &/2, 2, 1 + (4 + p)/2, -\tan[e + f*x]^2, ((-a^2 + b^2)*\tan[e + f*x]^2)/a^2]*\text{S} \\ &\text{ec}[e + f*x]^2*\tan[e + f*x]))/(a^2*(4 + p)) + ((2 + p)*\text{AppellF1}[1 + (2 + p)/2 \\ &, 1/2, 1, 1 + (4 + p)/2, -\tan[e + f*x]^2, ((-a^2 + b^2)*\tan[e + f*x]^2)/a^2 \\ &]*\text{Sec}[e + f*x]^2*\tan[e + f*x]))/(4 + p)) + a*(-(a*(1 + p)*\text{Hypergeometric2F1}[\\ &1/2, 1 + p/2, 2 + p/2, -\tan[e + f*x]^2]*\text{Sec}[e + f*x]^2) - 2*a*(1 + p/2)*(1 \\ &+ p)*\text{Sec}[e + f*x]^2*(-\text{Hypergeometric2F1}[1/2, 1 + p/2, 2 + p/2, -\tan[e + f*x] \\ &]^2] + 1/\text{Sqrt}[1 + \tan[e + f*x]^2])) + b*(1 + p)*(2 + p)*\text{Csc}[e + f*x]*\text{Sec}[e + \\ &f*x]*(-\text{Hypergeometric2F1}[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*\tan[e + f \\ &*x]^2] + (1 - (-1 + b^2/a^2)*\tan[e + f*x]^2)^{-1}))))/(a^2*b*(1 + p)*(2 + p \\ &)))) \end{aligned}$$

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{a + b \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)

[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{b \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(g \tan(fx + e))^p}{b \sin(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(e + fx))^p}{a + b \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{b \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

$$3.207 \quad \int \frac{(g \tan(e+fx))^p}{(a+b \sin(e+fx))^2} dx$$

Optimal. Leaf size=737

$$\frac{2ab \cos(e+fx) \sin^2(e+fx)^{-q/2} (g \tan(e+fx))^q F_1\left(\frac{1-q}{2}; -\frac{q}{2}, 2; \frac{3-q}{2}; \cos^2(e+fx), \frac{b^2 \cos^2(e+fx)}{b^2-a^2}\right)}{f(q-1)(a^2-b^2)^2} + \frac{a^2 \sin(e+fx) \cos(e+fx)}{f(q-1)(a^2-b^2)^2}$$

[Out] (a^2*Cos[e + f*x]*(1 - Cos[e + f*x]^2)^((-1 + q)/2)*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^(-2 + (3 - q)/2 + (-1 + q)/2)*((2*(a^2 - b^2) + b^2*(1 + q))*Cos[e + f*x]^2)*HurwitzLerchPhi[-((a^2*Cot[e + f*x]^2)/(a^2 - b^2)), 1, (1 - q)/2] - b^2*(-1 + q)*Cos[e + f*x]^2*HurwitzLerchPhi[-((a^2*Cot[e + f*x]^2)/(a^2 - b^2)), 1, (3 - q)/2])*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - q)/2)*(g*Tan[e + f*x])^q/(2*(a^2 - b^2)^2*(-a^2 + b^2)*f) - (a^2*Cos[e + f*x]*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + q)/2)*Hypergeometric2F1[(1 - q)/2, (1 - q)/2, (3 - q)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - q)/2)*(g*Tan[e + f*x])^q/((a^2 - b^2)^2*f*(-1 + q)) + (b^2*Cos[e + f*x]*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + q)/2)*Hypergeometric2F1[(1 - q)/2, (1 - q)/2, (3 - q)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - q)/2)*(g*Tan[e + f*x])^q/((a^2 - b^2)^2*f*(-1 + q)) - (2*a*b*AppellF1[(1 - q)/2, -q/2, 2, (3 - q)/2, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]*(g*Tan[e + f*x])^q/((a^2 - b^2)^2*f*(-1 + q))*(Sin[e + f*x]^2)^(q/2))

Rubi [F] time = 0.045093, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(g \tan(e+fx))^p}{(a+b \sin(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]

[Out] Defer[Int] [(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2, x]

Rubi steps

$$\int \frac{(g \tan(e+fx))^p}{(a+b \sin(e+fx))^2} dx = \int \frac{(g \tan(e+fx))^p}{(a+b \sin(e+fx))^2} dx$$

Mathematica [A] time = 14.2268, size = 908, normalized size = 1.23

$$a^3 (a^2 - b^2) f(p+1)(p+2)(a + b \sin(e + fx))^2 \left(\frac{\sec^2(e+fx) \left(a^{p+2} \left((a^2+b^2) {}_2F_1 \left(1, \frac{p+1}{2}; \frac{p+3}{2}; \frac{(b^2-a^2) \tan^2(e+fx)}{a^2} \right) - 2b^2 {}_2F_1 \left(2, \frac{p+1}{2}; \frac{p+3}{2}; \frac{(b^2-a^2) \tan^2(e+fx)}{a^2} \right) \right)}{a^3 (a^2 - b^2)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Tan[e + f*x])^p/(a + b*SIN[e + f*x])^2,x]

[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2*b^2*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])/ (a^3*(a^2 - b^2)*f*(1 + p)*(2 + p)*(a + b*SIN[e + f*x])^2*((Sec[e + f*x]^2*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2*b^2*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])/ (a^3*(a^2 - b^2)*(2 + p)) + (Tan[e + f*x]^(1 + p)*(2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 + 2*b*(-a^2 + b^2)*(1 + p)*Tan[e + f*x]*((4*(-a^2 + b^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -1/2, 3, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/ (a^2*(4 + p)) + ((2 + p)*AppellF1[1 + (2 + p)/2, 1/2, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/ (4 + p)) + a*(2 + p)*(-2*b^2*(1 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 - ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2)^(-2)) + (a^2 + b^2)*(1 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 - ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2)^(-1))))/ (a^3*(a^2 - b^2)*(1 + p)*(2 + p)))

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{(g \tan(fx + e))^p}{(a + b \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)

[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan (fx + e))^p}{(b \sin (fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(g \tan (fx + e))^p}{b^2 \cos (fx + e)^2 - 2 ab \sin (fx + e) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(g*tan(f*x + e))^p/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan (e + fx))^p}{(a + b \sin (e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e))**2,x)

[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x))**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g \tan (fx + e))^p}{(b \sin (fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)

3.208 $\int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx$

Optimal. Leaf size=25

$$\text{Unintegrable}((g \tan(e + fx))^p (a + b \sin(e + fx))^m, x)$$

[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]

Rubi [A] time = 0.0403311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]

[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]

Rubi steps

$$\int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx = \int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx$$

Mathematica [A] time = 2.69509, size = 0, normalized size = 0.

$$\int (a + b \sin(e + fx))^m (g \tan(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]

[Out] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]

Maple [A] time = 0.862, size = 0, normalized size = 0.

$$\int (a + b \sin(fx + e))^m (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p, x)

[Out] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(fx + e) + a)^m (g \tan(fx + e))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin (f x+e)+a\right)^m\left(g \tan (f x+e)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")

[Out] integral((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sin (f x+e)+a)^m\left(g \tan (f x+e)\right)^p d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")

[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+`') or type(expn,'*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```